# The new benchmark for forecasts of the real price of crude oil

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*The views expressed in this paper are those of the authors and no responsibility for them should be attributed to the Bank of Canada.*

# What is a good forecast?

- Suppose we want to evaluate a forecast of $X_{t+h}$
- Common practice: compare accuracy of your forecast against

$$\textit{no-change benchmark} \equiv X_t$$

- $X_t$ is optimal forecast under the random walk hypothesis
  $\rightarrow$ accuracy-improvements over NCB imply that
  - series is predictable in general
  - our forecast is more useful than "naive" approach

- **But: this approach not informative for temporally aggregated data**
  - lower-frequency series constructed by *averaging* or summation
    (e.g., real commodity prices, interest rates, ...)

# The new benchmark

- Random walk hypothesis applies to high-frequency series
  - aggregated data are predictable by construction (Working 1960)
  - conventional NCB is not the optimal forecast

  $\rightarrow$ improvements over conventional NCB not informative about RWH

- New no-change benchmark:

  *last high-frequency observation*

  - optimal under random walk hypothesis
    $\rightarrow$ restores original interpretation of comparison with NCB
  - $\approx$ 45% improvement in MSPE for monthly/quarterly averages of daily data for 1-step-ahead prediction

# Application to the real price of crude oil

- Real price of crude oil typically based on averaged data
  (Kilian 2009; Baumeister & Hamilton 2019)
- Existing literature: model-based forecasts beat NCB
  (Baumeister & Kilian 2012, 2014, 2015; Alquist et al. 2013; Snudden, 2018; Funk, 2018; Garratt et al., 2019)

- New no-change benchmark based on monthly closing price of oil
  - $\approx$ 40% improvements in accuracy at 1-step-ahead prediction
  - using closing prices for estimation improves traditional models
  - but: most models do *not* beat the new benchmark

$\rightarrow$ oil prices are more difficult to predict than previously thought

# Intuition under the RW null hypothesis

- Forecaster's goal is to predict $X_{t+h}$ given time $t$ information

$$X_t \equiv \frac{1}{n}\sum_{i=1}^{n} y_{t,i}$$

$t$ = month; $i$ = day of month; $n$ = # of days in month

- Null hypothesis: daily observations follow random walk

$$y_{t,i} = y_{t,i-1} + \epsilon_{t,i}, \quad \text{for } i = 1, \cdots, n$$

$\epsilon_{t,i}$ is a mean-zero, *iid* error term with variance $\sigma_\epsilon^2$

  - What is the optimal forecast in this setting?
  - What are the consequences of using $X_t$ to evaluate forecasts?

# The optimal forecast under the RWH

- Optimal forecast in MSPE terms is the conditional expectation

$$E_t(X_{t+h}) = E(X_{t+h}|y_{1,1}; \cdots ; y_{t,n}; Z_1; \cdots ; Z_t)$$

- RWH: conditional expectation of each future daily obs

$$E_t(y_{t+h,i}) = y_{t,n} \quad \text{for all } h > 0,\ i = 1, \cdots , n$$

$\Rightarrow$ conditional expectation of each future average obs

$$E_t(X_{t+h}) = E_t(\frac{1}{n}\sum_{1}^{n} y_{t+h,n}) = y_{t,n}$$

- New benchmark is the optimal forecast under the RWH:

*last high-frequency observation in period t* $\equiv y_{t,n}$

# Comparing the no-change forecasts $X_t$ and $y_{t,n}$

- What is the MSPE of $y_{t,n}$ relative to the conventional NCB $X_t$?

  MSPE for conventional benchmark:

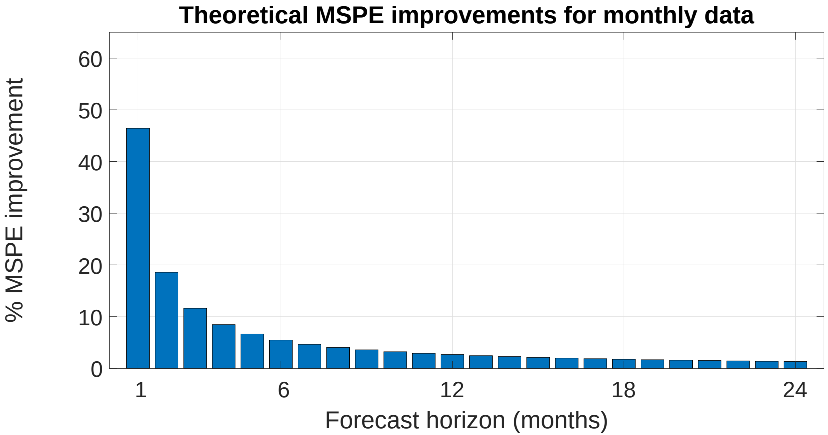
  $$E\left[(X_t - X_{t+h})^2\right] = \left((h-1)\cdot n + \frac{(n+1)\cdot(2n+1)}{6n} + \frac{(n-1)\cdot(2n-1)}{6n}\right)\sigma_\epsilon^2$$

  MSPE for new benchmark:

  $$E\left[(y_{t,n} - X_{t+h})^2\right] = \left((h-1)\cdot n + \frac{(n+1)\cdot(2n+1)}{6n}\right)\sigma_\epsilon^2$$

- $y_{t,n}$ has lower MSPE than $X_t$ under the RWH
- All forecasting gains occur at the 1-step ahead prediction
  - forecast gain is constant, forecast error increases with $h$
    $\Rightarrow$ relative improvements largest for short horizons

# MSPE ratios of the two NCBs



# Consequences for forecast comparisons

- Conventional NCB is not the optimal forecast under the RWH
- Moreover, $\Delta X_t$ is autocorrelated (Working 1960)

$$X_t - X_{t-1} = \frac{1}{n}\left[\sum_{i=1}^{n} i \cdot \epsilon_{t-1,i} + \sum_{j=1}^{n}(n+1-j) \cdot \epsilon_{t,j}\right]$$

$\Rightarrow$ improvements over no-change forecast $X_t$ are expected *even under the random walk null hypothesis*

$\Rightarrow$ improvements over no-change forecast $y_{t,n}$ are evidence against *the random walk null hypothesis*

- Using $y_{t,n}$ as a benchmark
  - maintains original spirit of comparisons with the NCB
  - more difficult to achieve when HF observations are persistent

# Application: Forecasting the real price of crude oil

- Forecast of the monthly real price of crude oil in standard setting (Baumeister & Kilian 2012, 2014, 2015; Alquist et al. 2013)
  - real oil price is deflated average of daily nominal prices

$$\bar{p}_t^r = \frac{\frac{1}{n}\sum_{i=1}^{n} p_{t,i}}{CPI_t}$$

  - goal: forecast $\bar{p}_{t+h}^r$ given month $t$ information

- The new benchmark: series of monthly closing prices

$$p_t^{r,closing} = \frac{p_{t,n}}{CPI_t}$$

# Econometric models

1. Univariate time series models
    - AR (log-level and percent changes )
    - ARMA(1,1)
    - ARFI

2. VAR models (Kilian & Murphy 2014)
    - percent change in global crude oil production
    - real economic activity indicator (Kilian 2009)
    - real price of oil
    - change in above-ground global crude oil inventories

    > unrestricted least-squares estimation

    > Gaussian BVAR with prior variance (Giannone, Lenza & Primiceri 2010)

3. Futures price curve

4. Equal-weight forecast combination

# Implementation

Following Baumeister & Kilian (2012):

- Real-time data
- Out-of-sample forecasts with expanding window
  - estimation period starts 1973M2
  - out-of-sample evaluation 1992M1 - 2018M12
- All models estimated with average prices
- Criteria: MSPE ratio and directional accuracy
  - displayed relative to conventional NCB
  - improvements over the new NCB are in **bold**
  - standard tests for inference

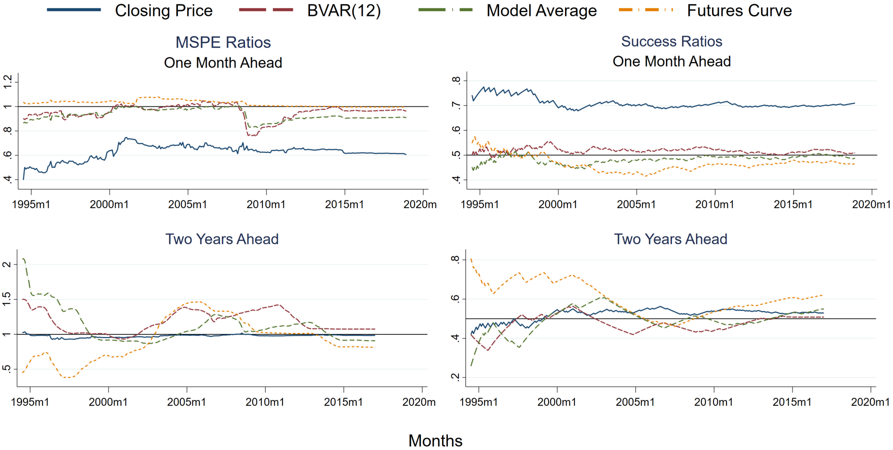
    (Diebold & Mariano 1995, Pesaran and Timmermann 2009)

# Baseline results: Real WTI prices, real-time data

| Months Ahead | Last WTI Close Price | BVAR(12) | VAR(12) | AR(12) | AR(12) %Δ | ARFI(1) | ARMA(1,1) | Futures Curve | Model Averaging |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | MSPE Ratios | | | | |
| 1 | 0.61*** | 0.97 | 1.01 | 0.94 | 0.95* | 0.93 | 0.92 | 1.00 | 0.91 |
| 3 | 0.89** | 1.00 | 1.00 | 0.97 | 0.99 | 0.96 | 0.95 | 0.97 | 0.92 |
| 6 | 0.95** | 1.05 | 1.04 | 1.00 | 1.04 | 0.99 | 0.95 | 0.96 | 0.95 |
| 12 | 0.96** | 1.10 | 1.11 | 1.00 | 1.10 | 1.00 | 0.94 | **0.85**** | 0.94 |
| 24 | 0.99 | 1.08 | 1.06 | 1.02 | 1.19 | 1.04 | 0.96 | **0.82*** | 0.91 |
| | | | | | Success Ratios | | | | |
| 1 | 0.71*** | 0.51 | 0.54** | 0.52 | 0.49 | 0.52 | 0.53 | 0.47 | 0.49 |
| 3 | 0.60*** | 0.53 | 0.54* | 0.49 | 0.57** | 0.49 | 0.50 | 0.49 | 0.52 |
| 6 | 0.56** | 0.53 | 0.55* | 0.48 | 0.54 | 0.49 | 0.46 | 0.53 | 0.51 |
| 12 | 0.59*** | 0.49 | 0.56** | 0.53 | 0.50 | 0.51 | 0.50 | 0.61*** | 0.51 |
| 24 | 0.53 | 0.51 | 0.55 | 0.57 | 0.47 | 0.55 | 0.56 | **0.62***** | 0.55 |

*Note:* ***, **, and * denote significant improvement over the average no-change forecast at the 1%, 5%, and 10% level. **Bold values indicate significant improvements over the last closing price no-change forecast at the 5 percent level.**

# Evolution of baseline WTI Real price forecasts: Real-time data




*Note:* Dynamic, recursive, out-of-sample forecasts 1992M1–2018M12. The forecast criteria reported include the recursive MSPE expressed as a ratio relative to the monthly average no-change forecast. All forecast criteria are evaluated in the levels of the real price of oil. The first 30 months are dropped to reduce starting-point effects.

# Interpreting baseline results

- Large(!) forecasting gains for short forecast horizons
- Actual vs. predicted MSPE ratios (RWH for daily data)

| Horizon (months) | 1 | 3 | 6 | 12 | 24 |
|---|---|---|---|---|---|
| Theoretical | 0.54 | 0.88 | 0.95 | 0.97 | 0.99 |
| Empirical (revised data) | 0.60 | 0.89 | 0.95 | 0.96 | 0.99 |

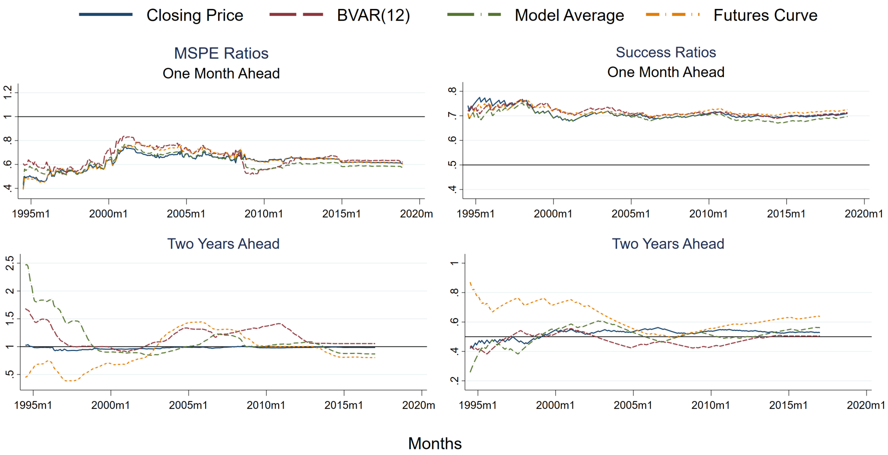
- Robustness: remarkably similar results for
  - various estimation / evaluation periods
  - different oil price series (Brent, RAC, nominal prices)
  - ex-post revised data, nominal price, quarterly data
- What about estimating models with closing prices?

# Models estimated with last closing price

| Months Ahead | Last WTI Close Price | BVAR(12) | VAR(12) | AR(2) | AR(1) %Δ | ARFI(2) | ARMA(1,1) | Last Future Price Curve | Model Averaging |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | MSPE Ratios | | | | |
| 1 | 0.61*** | 0.62** | 0.73* | 0.57*** | 0.62*** | 0.57*** | 0.57*** | 0.60*** | 0.58*** |
| 3 | 0.89** | 0.90 | 0.91 | 0.85 | 0.93* | 0.86 | 0.86 | 0.88** | 0.82* |
| 6 | 0.95** | 0.98 | 0.97 | 0.89 | 1.04 | 0.89 | 0.90 | 0.92* | 0.87 |
| 12 | 0.96** | 1.05 | 1.06 | 0.90 | 1.15 | 0.90 | 0.91 | **0.84**** | 0.87* |
| 24 | 0.99 | 1.06 | 1.06 | 0.95 | 1.48 | 0.95 | 0.97 | **0.80*** | 0.87 |
| | | | | | Success Ratios | | | | |
| 1 | 0.71*** | 0.71*** | 0.66*** | 0.71*** | 0.73*** | 0.72*** | 0.71*** | 0.73*** | 0.70*** |
| 3 | 0.60*** | 0.58** | 0.56** | 0.57** | 0.62*** | 0.60*** | 0.60*** | 0.60*** | 0.59*** |
| 6 | 0.56** | 0.58** | 0.58** | 0.52 | 0.58** | 0.51 | 0.52 | 0.55 | 0.55 |
| 12 | 0.59*** | 0.54 | 0.58** | 0.54 | 0.59** | 0.51 | 0.53 | **0.63***** | 0.54 |
| 24 | 0.53 | 0.51 | **0.57*** | 0.58 | 0.56 | **0.55** | 0.54 | **0.64***** | 0.56 |

*Note:* ***, **, and * denote significant improvement over the average no-change forecast at the 1%, 5%, and 10% level. **Bold values indicate significant improvements over the last closing price no-change forecast at the 5 percent level.**

# Evolution of real-time closing-price WTI forecasts




*Note:* Dynamic, recursive, out-of-sample forecasts 1992M1–2018M12. The forecast criteria reported include the recursive MSPE expressed as a ratio relative to the monthly average no-change forecast. All forecast criteria are evaluated in the levels of the real price of oil. The first 30 months are dropped to reduce starting-point effects.

# Take-away

- Aggregating higher-frequency data
  - introduces loss of information (e.g., Rossana & Seater 1995)
  - *changes interpretation of standard forecast comparisons*

→ use closing observations for estimation & *forecast evaluation*

- New benchmark for real price of crude oil changes assessments of models & oil price predictability:
  - daily oil prices are "random walkish"
  - real price of crude oil is predictable by construction
  - but most models do not beat the new benchmark

- Averaging can be desirable, but watch out for settings in which
  - series of interest is temporally aggregated
  - underlying data is persistent

Thank you!