A partial-equilibrium model of the electricity market

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Introduction I

Background

- Variable Renewable Electricity (VRE) changes in response to wind and sun radiation
- 'Dispatchable' Supply (DS; e.g hydro) is more controllable
- The intermittence of VRE is detrimental to its economic value
- Electricity prices at times when the VREs are supplying power in large quantities, are essential

Introduction II

Purpose

- We analyze the above issues in (an original) microeconomic model
- It takes a middle ground between small ‘static’ models and large numerical models:
  - Provides a higher transparency than larger and more realistic models
  - Lays out more details about all sectors of the industry, not just the marginal conditions (in simpler models)

Introduction III

Our approach

- DS consists of a merit order, reflecting their different marginal costs
- VRE has lower marginal costs and is thus delivering first
- We analyze a large number of fictive equilibria numerically
- Use them to examine the welfare effects of the increasing share of VRE
- Focus on variability in VRE and demand (and the correlation between them)

Electricity demand I

Utility and constraint

- I consumers use electricity over T hours
- Utility ('benefit') is a function of electricity consumption, x, and money for other goods, y.
- For individual i at time t:

$$F_i(x_{it}, y_{it}) = \theta_{it} u_i(x_{it} - \gamma_{it}) + \mu_i y_{it}.$$

  The function u_i is strictly concave, with $\lim_{x_{it} \to \gamma_{it}} u_i' = \infty$.
- The **subsistence consumption level of electricity** is γ_{it}
- ($\mu_i = \theta_{it} = 1$ from here??????)

Electricity demand II

Utility and constraint (cont'd)

- Summing over all the $T = 8760$ hours of a year:

$$U_i = \sum_{t=1}^{T} \left[\theta_{it} u_i \left(x_{it} - \gamma_{it}\right) + \mu_i y_{it}\right]. \quad (1)$$

- Consumer i has the budget constraint

$$\sum_{t=1}^{T} p_t x_{it} + \sum_{t=1}^{T} y_{it} = m_i,$$

where p_t is the price of electricity and m_i is the income

Electricity demand III

Optimal demand

- The Lagrangean of individual i

$$\mathcal{L}_i = \sum_{t=1}^{T} \left(\theta_{it} u_i \left(x_{it} - \gamma_{it}\right) + \mu_i y_{it}\right)$$

$$-\lambda_i \left(\sum_{t=1}^{T} p_t x_{it} + \sum_{t=1}^{T} y_{it} - m_i \right)$$

- An optimal solution satisfies

$$\theta_{it} u_i' \left(x_{it} - \gamma_{it}\right) = \mu_i p_t, \quad t = 1, 2, \ldots, T.$$

Electricity demand IV

Optimal demand (cont'd)

- Solve for the demand function

$$x_{it} = d_i\left(\mu_i p_t / \theta_{it}\right) + \gamma_{it}, \tag{2}$$

where we define $d_i = \left(u'\right)^{-1}$, with $d_i' < 0$.

- Demand is low if its price is high or if the marginal utility of money for other goods is high
- A high θ_{it} implies a higher electricity consumption
- Consumption is positive even if $p \to \infty$, because $\gamma_{it} > 0$ is always necessary

Electricity demand V

Market demand

- Market demand at time t is the sum of the demands from all consumers:

$$D_t = \sum_{i=1}^{I} x_{it} = \sum_{i=1}^{I} d_i \left(\mu_i p_t / \theta_{it} \right) + \gamma_t ,$$

  where $\gamma_t \equiv \sum_{i=1}^{I} \gamma_{it}$ is the total amount of subsistence consumption at time t.

- Note that γ_t appears as a simple additive term.
  It is a central stochastic variable

Electricity demand VI

Specification of utility function

- We will use the utility function

$$u_i\left(x_{it}-\gamma_{it}\right)=\frac{\left(x_{it}-\gamma_{it}\right)^{1-\frac{1}{\alpha}}-1}{1-\frac{1}{\alpha}} \tag{3}$$

- Thus

$$u'\left(x_{it}-\gamma_{it}\right)=\left(x_{it}-\gamma_{it}\right)^{-\frac{1}{\alpha}}$$

and the demand function (2) becomes

$$x_{it}=\left(\frac{\theta_{it}}{\mu_i p_t}\right)^{\alpha}+\gamma_{it}. \tag{4}$$

Electricity demand VII

Specification of utility function (cont'd)

- Now market demand is

$$D_t = p_t^{-\alpha} \sum_{i=1}^{I} \left(\frac{\theta_{it}}{\mu_i} \right)^{\alpha} + \gamma_t. \tag{5}$$

- Inverse market demand:

$$p_t = \left(\frac{\sum_{i=1}^{I} (\theta_{it}/\mu_i)^{\alpha}}{D_t - \gamma_t} \right)^{\frac{1}{\alpha}}. \tag{6}$$

Electricity supply I

Dispatchable supply (DS)

- DS is described by ‘stair cases’ indicating different marginal costs
- Baseload (b), intermediate (i) and peak (p) generators (‘hydro’, ‘nuclear’ and ‘thermic’)
- We assume perfect competition, ignoring market power considerations
- Marginal costs: c_b, c_i and c_p ($\$/\mathrm{MWh}$).

Electricity supply II

VRE supply

- Supply of VRE: V_t (MWh)
  A (stochastic) function of 'weather'
- Mean output: $\bar{V}$ (MW)
- We focus on the market impact of the variability in V_t and γ_t, and the correlation between them, ρ
- Marginal cost is typically very low.
  We put it equal to zero

Equilibrium I

Quantities and Prices

- We define

$$\tilde{D}_t\left(p_t\right) = p_t^{-\alpha} \sum_{i=1}^{I} \left(\frac{\theta_{it}}{\mu_i} \right)^{\alpha},$$

  so that

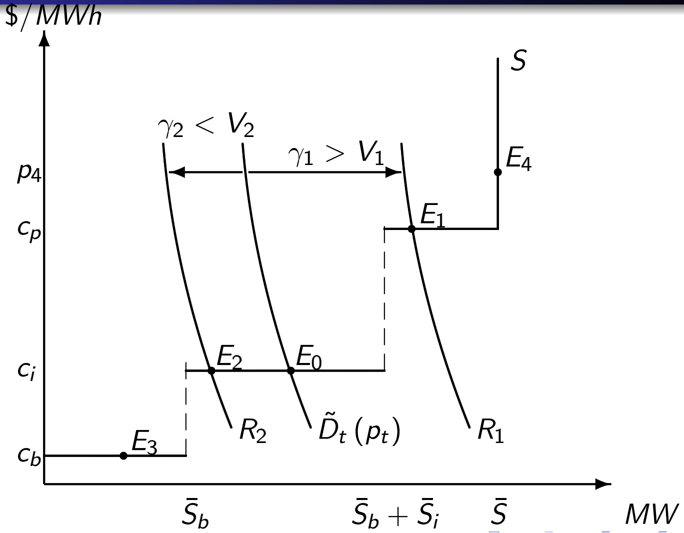
$$D_t = \tilde{D}_t\left(p_t\right) + \gamma_t$$

- **Residual demand** is $R_t = D_t - V_t$, or

$$R_t = \tilde{D}_t + \gamma_t - V_t.$$

  This part of demand is directed toward the dispatchable supply

Equilibrium II



Equilibrium III

Quantities and Prices (cont'd)

- In the Figure above the $\tilde{D}_t(p_t)$ curve stays put
- It coincides with R_t if $\gamma_t = V_t$, leading to E_0
- Deviations from $\tilde{D}_t$ are determined by $\gamma_t - V_t$, which varies (stochastically) by the hour
- For $\gamma_1 > V_1$: $R_1 > \tilde{D}_t \Rightarrow E_1$
- For $\gamma_2 < V_2$: $R_2 < \tilde{D}_t \Rightarrow E_2$.
- For an even higher V_t: E_3.
  A high V_t pushes p down ('self-cannibalism')
- Alternatively, equilibrium at a vertical segment of the supply curve, e.g. E_4

Equilibrium IV

Value factors

- A value factor consists of two price ‘indexes’:
  1. The average electricity price:

$$\bar{p} = \frac{\sum_{t=1}^{T} p_t}{T},$$

  2. The average revenue of VRE

$$p^V = \frac{\sum_{t=1}^{T} p_t V_t}{\sum_{t=1}^{T} V_t}.$$

Equilibrium V

Value factors (cont'd)

- The VRE *value factor* is then

$$v^V = \frac{p^V}{\bar{p}}.$$

  It is high if VRE produces much when p is high
- It is beneficial for V_t to be positively correlated with γ_t
- Otherwise, it often shifts the equilibrium down the merit-order curve and thereby lowers the price and its own 'value'.

Equilibrium VI

Value factors (cont'd)

- A common conjecture in the literature:
  v^V falls monotonically with higher VRE penetration ($\bar{V}\uparrow$)
- We get a cyclical pattern
- This is because v^V is an indicator of relative prices:
  as p declines (due to $\bar{V}\uparrow$), p^V and $\bar{p}$ follow each other downwards, although not perfectly because of the stairs and ρ

Equilibrium VII

Producer surplus

- Producer surplus:

$$\text{PS} = \text{Revenue} - \text{Variable Costs}$$

- Profit:

$$\pi = \text{PS} - \text{Fixed Costs}$$

- With zero marginal (variable) cost, the producer surplus of the VRE producers at time t is $p_t V_t$.
- To see profitability, we compare PS with (hypothetical) annualized fixed costs

Equilibrium VIII

Indirect utility

- As VRE lowers the price, it increase indirect utility

$$U = \sum_{i=1}^{I} U_i = \sum_{i=1}^{I} \mu_i m_i + \frac{\alpha}{1-\alpha} \sum_{i=1}^{I} \sum_{t=1}^{T} \theta_{it}$$

$$-\frac{1}{1-\alpha} \sum_{i=1}^{I} \sum_{t=1}^{T} \mu_i^{1-\alpha} \theta_{it}^{\alpha} p_t^{1-\alpha} - \sum_{i=1}^{I} \sum_{t=1}^{T} \mu_i p_t \gamma_{it}.$$

Numerical simulations I

- For one simulation round we draw 8760 observations (one for each hour of a year) of V_t and γ_t from a jointly normal probability distribution
- One equilibrium point from each draw
- From that we compute (yearly) v^V, PS and CS
- This is repeated in 101 rounds, with different ρ and $\bar{V}$
- Then v^V, PS and CS are compared between the rounds, to see the effects of changing ρ and $\bar{V}$

Numerical simulations II

Basic example

- Dispatchable supply curve (benchmark):
  $c_b = 10,\ c_i = 20,\ c_p = 30,$
  $\bar{S}_b = 20,\ \bar{S}_i = 20$ and $\bar{S}_p = 40.$
- Assume that $\mu_i = \theta_{it} = 1,\ I = 60$ and $\alpha = 0.06$
- The default values of σ_V and σ_γ are both equal to 5.

Numerical simulations III

Basic example (cont'd)

Four cases:

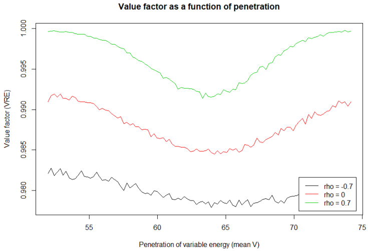
(a) Benchmark

(b) Higher stairs

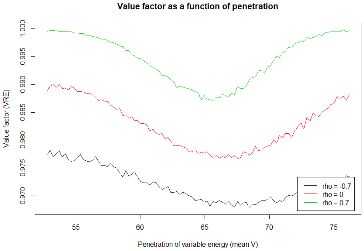
(c) σ_γ doubled

(d) Both changes

Numerical simulations IV



(a) Benchmark



(b) Higher stairs

Numerical simulations V

Value factor

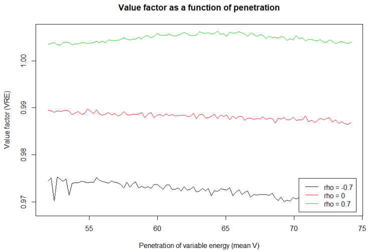
- In the literature v^V is often monotonously decreasing as $\bar{V} \uparrow$
- We get waves
- Downward part: more VRE capacity pushes more equilibria down to the next stair, reducing VRE revenue
- Even further penetration: most equilibria at the lower level
- $\Rightarrow$ smaller difference between the mean price of VRE electricity and the price in general
- $\Rightarrow v^V \uparrow$

Numerical simulations VI

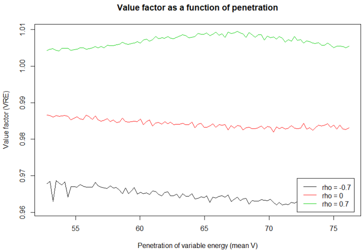
Value factor

- The value factor decreases as the **correlation** between V and γ gets lower.
  (Lower curve)
- High correlation: VRE output often large when demand is high.
  Renewable electricity is then often sold at high prices
- **Stairs higher** (Figure (b)): v^V levels lower
- Larger price reduction when much VRE pushes p down to the next stair

Numerical simulations VII



(c) σ_γ doubled



(d) Both Changes

Numerical simulations VIII

Value factor

- **Increasing the spread** of γ (Figure (c)): the wave patterns (almost) disappear
- It matters less whether the mean R is close to the next stair or not, when the equilibria are spread over more stairs
- The curve for $\rho > 0$ is moving up (above 1), while the curve for $\rho < 0$ is moving down
- If $\rho > 0$, the VRE is more often sold at a high price. This explains why v^V climbs above 1.

Numerical simulations IX

Value factor (cont'd)

- In panel (d) the effect of the wider spread from panel (c) is reinforced by the higher steps.
- If $\rho > 0$ it is more likely to get a very high price when VRE produces much, thus pushing the value factor a bit higher.
- If $\rho < 0$ VRE often produces much when demand is low, thereby pushing the bottom curve down

Numerical simulations X

Indirect utility

- Consumers clearly benefit from the entrance of more VRE firms that produce electricity at low (zero) marginal cost.
- At higher stairs utility starts lower, because the price level is generally higher.

Numerical simulations XI

Producer surplus

- Producer surplus fall monotonously as more VRE generators enter the market
- Since the latter have lower marginal costs, they tend to reduce the prices (as well as quantities) for all suppliers, thus reducing profitability in the dispatchable sectors.
- The fall in profits is quite remarkable in percentage terms.