A partial-equilibrium model of the electricity market

Clas Eriksson, Johan Linden and Christos Papahristodoulou

Mälardalen University College
Västerås, Sweden.

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Background

- Variable Renewable Electricity (VRE) changes in response to wind and sun radiation
- ‘Dispatchable’ Supply (DS; e.g hydro) is more controllable
- The intermittence of VRE is detrimental to its economic value
- Electricity prices at times when the VREs are supplying power in large quantities, are essential
Purpose

- We analyze the above issues in (an original) microeconomic model
- It takes a middle ground between small ‘static’ models and large numerical models:
  - Provides a higher transparency than larger and more realistic models
  - Lays out more details about all sectors of the industry, not just the marginal conditions (in simpler models)
Our approach

- DS consists of a merit order, reflecting their different marginal costs
- VRE has lower marginal costs and is thus delivering first
- We analyze a large number of fictive equilibria numerically
- Use them to examine the welfare effects of the increasing share of VRE
- Focus on variability in VRE and demand (and the correlation between them)
Electricity demand I

Utility and constraint

- $I$ consumers use electricity over $T$ hours
- Utility (‘benefit’) is a function of electricity consumption, $x$, and money for other goods, $y$.
- For individual $i$ at time $t$:

$$F_i(x_{it}, y_{it}) = \theta_{it} u_i(x_{it} - \gamma_{it}) + \mu_i y_{it}.$$  

The function $u_i$ is strictly concave, with $\lim_{x_{it} \to \gamma_{it}} u_i' = \infty$.

- The **subsistence consumption level of electricity** is $\gamma_{it}$
- $(\mu_i = \theta_{it} = 1$ from here??????)
Utility and constraint (cont’d)

- Summing over all the \( T = 8760 \) hours of a year:

\[
U_i = \sum_{t=1}^{T} \left[ \theta_{it} u_i (x_{it} - \gamma_{it}) + \mu_i y_{it} \right].
\]  

(1)

- Consumer \( i \) has the budget constraint

\[
\sum_{t=1}^{T} p_t x_{it} + \sum_{t=1}^{T} y_{it} = m_i,
\]

where \( p_t \) is the price of electricity and \( m_i \) is the income.
Electricity demand III

Optimal demand

- The Lagrangean of individual $i$

$$\mathcal{L}_i = \sum_{t=1}^{T} (\theta_{it} u_i (x_{it} - \gamma_{it}) + \mu_i y_{it})$$

$$-\lambda_i \left( \sum_{t=1}^{T} p_t x_{it} + \sum_{t=1}^{T} y_{it} - m_i \right)$$

- An optimal solution satisfies

$$\theta_{it} u_i' (x_{it} - \gamma_{it}) = \mu_i p_t, \quad t = 1, 2, \ldots, T.$$
Optimal demand (cont’d)

- Solve for the demand function

\[ x_{it} = d_i (\mu_i p_t / \theta_{it}) + \gamma_{it}, \]  

(2)

where we define \( d_i = (u')^{-1} \), with \( d'_i < 0 \).

- Demand is low if its price is high or if the marginal utility of money for other goods is high.
- A high \( \theta_{it} \) implies a higher electricity consumption.
- Consumption is positive even if \( p \to \infty \), because \( \gamma_{it} > 0 \) is always necessary.
Market demand

- Market demand at time $t$ is the sum of the demands from all consumers:

$$D_t = \sum_{i=1}^{I} x_{it} = \sum_{i=1}^{I} d_i (\mu_i p_t / \theta_{it}) + \gamma_t,$$

where $\gamma_t \equiv \sum_{i=1}^{I} \gamma_{it}$ is the total amount of subsistence consumption at time $t$.

- Note that $\gamma_t$ appears as a simple additive term. It is a central stochastic variable.
We will use the utility function

\[ u_i (x_{it} - \gamma_{it}) = \frac{(x_{it} - \gamma_{it})^{1-\frac{1}{\alpha}} - 1}{1 - \frac{1}{\alpha}} \]  

(3)

Thus

\[ u' (x_{it} - \gamma_{it}) = (x_{it} - \gamma_{it})^{-\frac{1}{\alpha}} \]

and the demand function (2) becomes

\[ x_{it} = \left( \frac{\theta_{it}}{\mu_i p_t} \right)^\alpha + \gamma_{it} \]  

(4)
Now market demand is

$$D_t = p_t^{-\alpha} \sum_{i=1}^{l} \left( \frac{\theta_{it}}{\mu_i} \right)^{\alpha} + \gamma_t. \quad (5)$$

Inverse market demand:

$$p_t = \left( \frac{\sum_{i=1}^{l} \left( \frac{\theta_{it}}{\mu_i} \right)^{\alpha}}{D_t - \gamma_t} \right)^{\frac{1}{\alpha}}. \quad (6)$$
Dispatchable supply (DS)

- DS is described by ‘stair cases’ indicating different marginal costs.
- Baseload ($b$), intermediate ($i$) and peak ($p$) generators (‘hydro’, ‘nuclear’ and ‘thermic’).
- We assume perfect competition, ignoring market power considerations.
- Marginal costs: $c_b$, $c_i$ and $c_p$ ($$/\text{MWh})$. 
Supply of VRE: $V_t$ (MWh)
A (stochastic) function of ‘weather’

Mean output: $\bar{V}$ (MW)

We focus on the market impact of the variability in $V_t$ and $\gamma_t$, and the correlation between them, $\rho$

Marginal cost is typically very low. We put it equal to zero
Equilibrium I

Quantities and Prices

- We define

\[ \tilde{D}_t (p_t) = p_t^{-\alpha} \sum_{i=1}^{l} \left( \frac{\theta_{it}}{\mu_i} \right)^\alpha, \]

so that

\[ D_t = \tilde{D}_t (p_t) + \gamma_t \]

- **Residual demand** is \( R_t = D_t - V_t \), or

\[ R_t = \tilde{D}_t + \gamma_t - V_t. \]

This part of demand is directed toward the dispatchable supply.
Equilibrium II

A model of the electricity market
Equilibrium III

Quantities and Prices (cont’d)

- In the Figure above the $\tilde{D}_t(p_t)$ curve stays put
- It coincides with $R_t$ if $\gamma_t = V_t$, leading to $E_0$
- Deviations from $\tilde{D}_t$ are determined by $\gamma_t - V_t$, which varies (stochastically) by the hour
- For $\gamma_1 > V_1$: $R_1 > \tilde{D}_t \Rightarrow E_1$
- For $\gamma_2 < V_2$: $R_2 < \tilde{D}_t \Rightarrow E_2$.
- For an even higher $V_t$: $E_3$.
  A high $V_t$ pushes $p$ down (‘self-cannibalism’)
- Alternatively, equilibrium at a vertical segment of the supply curve, e.g. $E_4$
Equilibrium IV

Value factors

- A value factor consists of two price ‘indexes’:
  1. The average electricity price:

\[ \bar{p} = \frac{\sum_{t=1}^{T} p_t}{T}, \]

  2. The average revenue of VRE

\[ p^v = \frac{\sum_{t=1}^{T} p_t V_t}{\sum_{t=1}^{T} V_t}. \]
Equilibrium V

Value factors (cont’d)

- The VRE value factor is then
  \[ v^V = \frac{p^V}{\bar{p}}. \]

  It is high if VRE produces much when \( p \) is high
- It is beneficial for \( V_t \) to be positively correlated with \( \gamma_t \)
- Otherwise, it often shifts the equilibrium down the merit-order curve and thereby lowers the price and its own ‘value’.
Value factors (cont’d)

- A common conjecture in the literature: $v^V$ falls monotonically with higher VRE penetration ($\bar{V} \uparrow$)
- We get a cyclical pattern
- This is because $v^V$ is an indicator of relative prices: as $p$ declines (due to $\bar{V} \uparrow$), $p^V$ and $\bar{p}$ follow each other downwards, although not perfectly because of the stairs and $\rho$
Producer surplus

Producer surplus:

$$\text{PS} = \text{Revenue} - \text{Variable Costs}$$

Profit:

$$\pi = \text{PS} - \text{Fixed Costs}$$

With zero marginal (variable) cost, the producer surplus of the VRE producers at time $t$ is $p_t V_t$.

To see profitability, we compare PS with (hypothetical) annualized fixed costs.
Equilibrium VIII

Indirect utility

- As VRE lowers the price, it increases indirect utility

\[
U = \sum_{i=1}^{I} U_i = \sum_{i=1}^{I} \mu_i m_i + \frac{\alpha}{1 - \alpha} \sum_{i=1}^{I} \sum_{t=1}^{T} \theta_{it}
\]

\[
- \frac{1}{1 - \alpha} \sum_{i=1}^{I} \sum_{t=1}^{T} \mu_i^{1-\alpha} \theta_{it}^\alpha p_t^{1-\alpha} - \sum_{i=1}^{I} \sum_{t=1}^{T} \mu_i p_t \gamma_{it}.
\]
For one simulation round we draw 8760 observations (one for each hour of a year) of $V_t$ and $\gamma_t$ from a jointly normal probability distribution.

One equilibrium point from each draw.

From that we compute (yearly) $v^V$, PS and CS.

This is repeated in 101 rounds, with different $\rho$ and $\bar{V}$.

Then $v^V$, PS and CS are compared between the rounds, to see the effects of changing $\rho$ and $\bar{V}$.
Basic example

- Dispatchable supply curve (benchmark):
  \[c_b = 10, \, c_i = 20, \, c_p = 30,\]
  \[\bar{S}_b = 20, \, \bar{S}_i = 20 \text{ and } \bar{S}_p = 40.\]
- Assume that \(\mu_i = \theta_{it} = 1, \, I = 60 \text{ and } \alpha = 0.06\)
- The default values of \(\sigma_V\) and \(\sigma_\gamma\) are both equal to 5.
Numerical simulations III

Basic example (cont’d)

Four cases:

(a) Benchmark
(b) Higher stairs
(c) $\sigma_\gamma$ doubled
(d) Both changes
Numerical simulations IV

(a) Benchmark

(b) Higher stairs
In the literature $v^V$ is often monotonously decreasing as $\bar{V} \uparrow$.

- We get waves.
- Downward part: more VRE capacity pushes more equilibria down to the next stair, reducing VRE revenue.
- Even further penetration: most equilibria at the lower level.
- $\Rightarrow$ smaller difference between the mean price of VRE electricity and the price in general.
- $\Rightarrow v^V \uparrow$.
Value factor

- The value factor decreases as the **correlation** between $V$ and $\gamma$ gets lower.
  (Lower curve)
- High correlation: VRE output often large when demand is high.
  Renewable electricity is then often sold at high prices
- **Stairs higher** (Figure (b)): $v^V$ levels lower
- Larger price reduction when much VRE pushes $p$ down to the next stair
Numerical simulations VII

(c) $\sigma_\gamma$ doubled

(d) Both Changes
Numerical simulations VIII

Value factor

- **Increasing the spread** of $\gamma$ (Figure (c)): the wave patterns (almost) disappear
- It matters less whether the mean $R$ is close to the next stair or not, when the equilibria are spread over more stairs
- The curve for $\rho > 0$ is moving up (above 1), while the curve for $\rho < 0$ is moving down
- If $\rho > 0$, the VRE is more often sold at a high price. This explains why $v^V$ climbs above 1.
In panel (d) the effect of the wider spread from panel (c) is reinforced by the higher steps.

If $\rho > 0$ it is more likely to get a very high price when VRE produces much, thus pushing the value factor a bit higher.

If $\rho < 0$ VRE often produces much when demand is low, thereby pushing the bottom curve down.
Consumers clearly benefit from the entrance of more VRE firms that produce electricity at low (zero) marginal cost.

At higher stairs utility starts lower, because the price level is generally higher.
Producer surplus

- Producer surplus fall monotonously as more VRE generators enter the market.

- Since the latter have lower marginal costs, they tend to reduce the prices (as well as quantities) for all suppliers, thus reducing profitability in the dispatchable sectors.

- The fall in profits is quite remarkable in percentage terms.