Managing intermittency in the electricity market

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Energy, Covid and Climate Change
IAEE digital conference
Motivation

- To reduce greenhouse gas emissions, it is recommended to shift to renewables-based electricity production

- Renewables such as wind and solar are *intermittent* (variable + uncertain)

- Renewables-based electricity is *intermittent* and *inflexible*

- Renewables intermittency challenges the “supply-matching-demand” exercise of the electricity industry

- Disruptions in this balance have technical and economical impacts

- **Flexibility** on the supply and/or demand side of the electricity market as a solution to managing renewables intermittency
To manage demand intermittency and optimal capacities

Borenstein and Holland (2005) and Joskow and Tirole (2007): time-varying retail tariffs can make demand follow supply and help achieve optimal capacities
Related Literature - Demand flexibility

To manage demand intermittency and optimal capacities

*Borenstein and Holland (2005)* and *Joskow and Tirole (2007)*: time-varying retail tariffs can make demand follow supply and help achieve optimal capacities.

Can retail contracts be designed to unlock demand flexibility when supply is intermittent due to the integration of intermittent renewable technologies?
Related Literature - Demand flexibility

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To manage supply intermittency and optimal renewables capacities

*Ambec and Crampes (2012)* and *Rouillon (2015)*: first-best energy mix is reachable when consumers move from flat-rate tariff to Real Time Pricing.
**Related Literature - Demand flexibility**

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Can retail contracts be designed to unlock demand flexibility when supply is intermittent due to the integration of intermittent renewable technologies?

To manage supply intermittency and optimal renewables capacities

*Ambec and Crampes (2012) and Rouillon (2015)*: first-best energy mix is reachable when consumers move from flat-rate tariff to Real Time Pricing.

Can we take into account more diversified retail contracts to integrate intermittent renewable technologies?
This paper

What we do?

- Theoretical framework for integrating intermittent renewable technologies into an electricity mix with conventional energy

-Demand-side flexibility be implemented through retailers offering diversified electricity delivery contracts at different prices

-Diversity of the contracts be depicted through base state-contingent electricity delivery contracts
This paper

What we do?

- Theoretical framework for integrating intermittent renewable technologies into an electricity mix with conventional energy

- Demand-side flexibility be implemented through retailers offering diversified electricity delivery contracts at different prices

- Diversity of the contracts be depicted through base state-contingent electricity delivery contracts

What we find?

- Model is consistent with a partial equilibrium model

- Welfare is constraint efficient

- Conditions when changing the base delivery contracts improves: welfare, integration of the renewable capacity and both
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General features

Intermittency:

- Set of states of nature: $s \in \{1, \ldots, S\}$

- State-contingent electricity production traded on perfectly competitive state-contingent wholesale markets

- State-contingent expected prices: $p = (p_1, \ldots, p_S) \in \mathbb{R}^S$
## General features

**Intermittency:**
- Set of states of nature: $s \in \{1, \ldots, S\}$
- State-contingent electricity production traded on perfectly competitive state-contingent wholesale markets
- State-contingent expected prices: $\mathbf{p} = (p_1, \ldots, p_S) \in \mathbb{R}^S$

### Decision making:

<table>
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<tr>
<th>Ex-ante</th>
<th>States of nature</th>
<th>Ex-post</th>
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<td>Investment in renewable capacity</td>
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<td>Production plan from conventional technology</td>
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<td>Retailers</td>
<td></td>
<td></td>
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<tr>
<td>Consumers</td>
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</table>
Electricity Retailing (1)

Retailers propose diversified delivery contracts built as from base state-contingent delivery contracts

Example:
- Time-of-Use retail contract
- 1 off-peak period: night ($s_1$)
- 1 peak period: day($s_2$)
- The base contracts can then be:

\[
\begin{align*}
\text{price} : q_1 & \quad k_1 \\
\begin{pmatrix} s_1 \\ s_2 \end{pmatrix} & \begin{pmatrix} a_1 \\ 0 \end{pmatrix} \\
\text{price} : q_2 & \quad k_2 \\
\begin{pmatrix} 0 \\ a_2 \end{pmatrix} & \begin{pmatrix} \end{pmatrix}
\end{align*}
\]

- Linear combination of $k_1$ and $k_2$ can give a Flat delivery contract:

\[
\begin{align*}
\text{price} : q_3 & \quad k_3 \\
\begin{pmatrix} s_1 \\ s_2 \end{pmatrix} & \begin{pmatrix} a_1 \\ a_1 \end{pmatrix}
\end{align*}
\]
Electricity Retailing (2)

<table>
<thead>
<tr>
<th>Description</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Random electricity of 1 unit delivery contract $k$</td>
<td>$a_k = (a_{1k}, \ldots, a_{Sk}), a_{sk} \geq 0$</td>
</tr>
<tr>
<td>$K$ contracts</td>
<td>$K = {1, \ldots, K}$</td>
</tr>
<tr>
<td>Electricity delivery of the $K$ contracts</td>
<td>$A = [a_k]_k^{K} \in \mathbb{R}^{SK}$</td>
</tr>
<tr>
<td>Portfolio of contracts offered</td>
<td>$\theta_r = (\theta_1, \ldots, \theta_K) \in \mathbb{R}^K$</td>
</tr>
<tr>
<td>Random electricity flow induced by portfolio</td>
<td>$A\theta_r \in \mathbb{R}^S$</td>
</tr>
<tr>
<td>Less contracts than states of nature</td>
<td>$K &lt; S$</td>
</tr>
<tr>
<td>No redundant contract</td>
<td>rank$(A) = K$</td>
</tr>
<tr>
<td>Always an asset delivering electricity in a state</td>
<td>$\forall s, \exists k, a_{sk} &gt; 0$</td>
</tr>
<tr>
<td>Objective program</td>
<td>$\theta_r^* \in \arg \max_{\theta_r \in \mathbb{R}^K} (q^<em>)' \theta_r - (p^</em>)' A\theta_r$</td>
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</table>
## Electricity Production

### Intermittent Technology

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<td>Ex-ante capacity</td>
<td>$\kappa \in \mathbb{R}_+$</td>
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<tr>
<td>State-contingent production per unit capacity</td>
<td>$g = (g_1, \ldots, g_S) \in \mathbb{R}^S_+$</td>
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</table>
| Increasing and convex investment cost function | $K(\kappa)$  
  $(\partial K(\kappa) > 0, \partial^2 K(\kappa) > 0 \& K(0) = 0)$ |
| Objective program | $\kappa^* \in \arg \max_{\kappa \in \mathbb{R}_+} \kappa (p^*)' g - K(\kappa)$ |

**Conventional Technology**

**Description**

State-contingent electricity production

**Notation**

$y = (y_1, \ldots, y_S) \in \mathbb{R}^S_+$

Increasing and convex production expected cost

$\sum_{s=1}^{S} c_s(y_s)$  
$(\partial c_s(q_s) > 0, \partial^2 c_s(q_s) > 0, c_s(0) = 0)$

Objective program

$y^* \in \arg \max_{y \in \mathbb{R}^S_+} (p^*)' y - \sum_{s=1}^{S} c_s(y_s)$
Electricity Production

### Intermittent Technology

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| Increasing and convex investment cost function   | $K(\kappa)$  
  
  ($\partial K(\kappa) > 0$, $\partial^2 K(\kappa) > 0$ & $K(0) = 0$) |

Objective program

$$\kappa^* \in \arg \max_{\kappa \in \mathbb{R}^+} \kappa (p^*)' g - K(\kappa)$$

### Conventional Technology

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<tr>
<td>State-contingent electricity production</td>
<td>$y = (y_1, \ldots, y_S) \in \mathbb{R}^+_S$</td>
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| Increasing and convex production expected cost   | $\sum_{s=1}^S c_s (y_s)$  
  
  ($\partial c_s(q_s) > 0$, $\partial^2 c_s(q_s) > 0$, $c_s(0) = 0$) |

Objective program

$$y^* \in \arg \max_{y \in \mathbb{R}^+_S} (p^*)' y - \sum_{s=1}^S c_s (y_s)$$
### Electricity consumption

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<tr>
<td>Random electricity consumption</td>
<td>$x = (x_1, \ldots, x_S) \in \mathbb{R}_+^S$</td>
</tr>
<tr>
<td>Budget</td>
<td>$m_0 \in \mathbb{R}$</td>
</tr>
<tr>
<td>Money spent on other goods</td>
<td>$m \in \mathbb{R}$</td>
</tr>
<tr>
<td>Increasing and strictly concave utility function</td>
<td>$\mathcal{U}(x)$</td>
</tr>
<tr>
<td>Portfolio of contracts demanded</td>
<td>$\theta_c \in \mathbb{R}^K$</td>
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<tr>
<td>Random electricity flow induced by portfolio</td>
<td>$x = A\theta_c$</td>
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<tr>
<td>Objective program</td>
<td>$(\theta_c^<em>, m^</em>) \in \arg\max_{(\theta_c,m) \in \mathbb{R}^{K+1}} \mathcal{U}(A\theta_c) + m$</td>
</tr>
<tr>
<td></td>
<td>s.t. $A\theta_c \geq 0$</td>
</tr>
<tr>
<td></td>
<td>$(q^*)'\theta_c + m = m_0$</td>
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Partial Equilibrium: Definition

Equilibrium is \((m^*, \theta_c^*, \theta_r^*, y^*, \kappa^*, p^*, q^*)\) \(\in \mathbb{R}^{K+1} \times \mathbb{R}^K \times \mathbb{R}_+^S \times \mathbb{R}_+^S \times \mathbb{R}_+^K\) whereby:

- ex-ante, the consumers maximize utility and the retailers and producers maximize their profits

- the contract and contingent electricity markets clear:

\[
\theta_r^* = \theta_c^* \quad \text{and} \quad y^* + \kappa^* g = A \theta_r^*
\]
Partial Equilibrium: Definition

Equilibrium is \((m^*, \theta_c^*, \theta_r^*, y^*, \kappa^*, p^*, q^*) \in \mathbb{R}^{K+1} \times \mathbb{R}^K \times \mathbb{R}_+^S \times \mathbb{R}_+ \times \mathbb{R}_+^S \times \mathbb{R}_+^K\)

whereby:

- ex-ante, the consumers maximize utility and the retailers and producers maximize their profits
- the contract and contingent electricity markets clear:

\[\theta_r^* = \theta_c^* \quad \text{and} \quad y^* + \kappa^* \mathbf{g} = \mathbf{A} \theta_r^*\]
1. Introduction

2. Main assumptions

3. Results

4. Conclusion
Partial Equilibrium: Existence and Uniqueness

- No-arbitrage condition shows that $q = A'p$ with $p \in \mathbb{R}^S_{++}$
Partial Equilibrium: Existence and Uniqueness

- No-arbitrage condition shows that $q = A'p$ with $p \in \mathbb{R}^S_+$

- The state contingent demand of electricity is a differentiable function $D: \mathbb{R}^S_+ \rightarrow \mathbb{R}^S$ with the property that $\partial D(p)$ is a symmetric and negative semi-definite matrix and the boundary conditions are defined.
Partial Equilibrium: Existence and Uniqueness

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- The state contingent demand of electricity is a differentiable function \( D : \mathbb{R}_+^S \rightarrow \mathbb{R}^S \) with the property that \( \partial D(p) \) is a symmetric and negative semi-definite matrix and the boundary conditions are defined.

- The contingent electricity supply \( S : \mathbb{R}_+^S \rightarrow \mathbb{R}_+^S \) is a differentiable function with the property that \( \partial S(p) \) is positive definite and the boundary conditions are defined.
Partial Equilibrium: Existence and Uniqueness

- No-arbitrage condition shows that $q = A'p$ with $p \in \mathbb{R}^S_+$

- The state contingent demand of electricity is a differentiable function $D : \mathbb{R}^S_+ \to \mathbb{R}^S$ with the property that $\partial D(p)$ is a symmetric and negative semi-definite matrix and the boundary conditions are defined.

- The contingent electricity supply $S : \mathbb{R}^S_+ \to \mathbb{R}^S_+$ is a differentiable function with the property that $\partial S(p)$ is positive definite and the boundary conditions are defined.

**Proposition**

*There exists a unique contingent price vector $p^* \in \mathbb{R}^S_+$ which clears the different state contingent electricity markets and an associated electricity delivery contract price vector $q^* = A'p^* \in \mathbb{R}^K_+$ which is free of arbitrage and clears the different contract markets.*
Welfare Analysis

- Contract structure matters as there are less contracts than states of nature

- Potential contingent electricity consumptions restricted to the linear subspace generated by the columns of $A$, i.e. $x \in \text{span}(A)$

- $x = A\theta$
  
  With 2 contract structures $A$ and $\tilde{A}$ with property that $\text{span}(A) = \text{span}(\tilde{A})$, then $A$ is equivalent to $\tilde{A}$ in terms of electricity demand, i.e. $A \sim_e \tilde{A}$

$$A = \begin{bmatrix} BC \\ C \end{bmatrix} \text{ with } C \text{ any invertible matrix of dimension } K$$

$$x = \begin{bmatrix} B \\ I_K \end{bmatrix} \theta \iff \begin{cases} \theta = (x_s)_s^{S-K+1} \\ \begin{bmatrix} I_{S-K} & -B \end{bmatrix} x = 0 \end{cases}$$
Welfare Analysis

\[ SW(B) = \max_{(y, \kappa, x) \in \mathbb{R}^{2S+1}_+} \mathcal{U}(x) - C(y) - K(\kappa) \quad \text{s.t.} \quad \begin{cases} x - y - \kappa g = 0 \\ \left[ I_{S-K} - B \right] x = 0 \end{cases} \]

\[ \begin{cases} \partial \mathcal{U}(x) - \lambda - \left[ \begin{array}{c} I_{S-K} \\ -B' \end{array} \right] \cdot \mu = 0 \\ -\partial C(y) + \lambda = 0 \\ -\frac{dK(\kappa)}{d\kappa} + g' \cdot \lambda = 0 \end{cases} \]

Proposition

The competitive electricity production plan and allocation \((\tilde{y}, \tilde{\kappa}, \tilde{x}) \in \mathbb{R}^{S+1}_+ \times \mathbb{R}^S_+\) is constrained efficient.
Comparative Statics

Improve welfare:
As long as $\mu \neq 0$, any addition of a new contract to $A$ linearly independent of the existing ones improves welfare
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Improve welfare:
As long as $\mu \neq 0$, any addition of a new contract to $A$ linearly independent of the existing ones improves welfare.

Improve integration of renewable capacity:
If at least two components of $(\partial_s U(S(p)) - p_s)^{S-K}_{s=1}$ are different from 0 and $1 < K < S - 1$, all the directions of price changes which improve investment in renewables can be reached, especially the one which is collinear to $g$ and which “maximizes” the penetration of renewables.
Comparative Statics

Improve welfare:
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Improve integration of renewable capacity:
If at least two components of $(\partial_{x_s} U(S(p)) - p_s)_{s=1}^{S-K}$ are different from 0 and $1 < K < S - 1$, all the directions of price changes which improve investment in renewables can be reached, especially the one which is collinear to $g$ and which "maximizes" the penetration of renewables

Both of the above:
As long as $\partial_B \kappa^*$ and $\partial_B SW$ are not collinear with a negative coefficient
1 Introduction

2 Main assumptions

3 Results

4 Conclusion
Conclusion

- Theoretical framework taking into account intermittency of renewables and demand-side flexibility through diversified retail contracts

- Shown existence and uniqueness of a competitive equilibrium of the contingent wholesale and retail markets

- Welfare is constraint efficient

- Characterized the conditions under which we can improve welfare, renewable capacity investment and both

- The results provide insights on how the role of retailers can be redefined so as to participate in demand-side flexibility

- The paper highlights the importance of accounting for intermittency in order to achieve renewable capacity objectives
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