

# Managing intermittency in the electricity market

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4 Conclusion

## Motivation

- To reduce greenhouse gas emissions, it is recommended to shift to renewables-based electricity production
- Renewables such as wind and solar are **intermittent** (variable + uncertain)
- Renewables-based electricity is **intermittent** and **inflexible**
- Renewables intermittency challenges the “supply-matching-demand” exercise of the electricity industry
- Disruptions in this balance have technical and economical impacts
- **Flexibility** on the supply and/or **demand side** of the electricity market as a solution to managing renewables intermittency

## Related Literature - Demand flexibility

To manage demand intermittency and optimal capacities

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Can we take into account more diversified retail contracts to integrate intermittent renewable technologies?

# This paper

## What we do?

- Theoretical framework for integrating intermittent renewable technologies into an electricity mix with conventional energy
- Demand-side flexibility be implemented through retailers offering diversified electricity delivery contracts at different prices
- Diversity of the contracts be depicted through **base state-contingent electricity delivery contracts**



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- Demand-side flexibility be implemented through retailers offering diversified electricity delivery contracts at different prices
- Diversity of the contracts be depicted through **base state-contingent electricity delivery contracts**

## What we find?

- Model is consistent with a partial equilibrium model
- Welfare is constraint efficient
- Conditions when changing the base delivery contracts improves: welfare, integration of the renewable capacity and both

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## General features

### Intermittency:

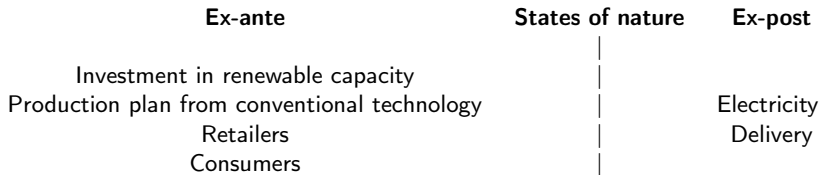
- Set of states of nature:  $s \in \{1, \dots, S\}$
- State-contingent electricity production traded on perfectly competitive state-contingent wholesale markets
- State-contingent expected prices:  $\mathbf{p} = (p_1, \dots, p_S) \in \mathbb{R}^S$

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### Decision making:



# Electricity Retailing (1)

- Retailers propose diversified delivery contracts built as from **base state-contingent delivery contracts**
- Example:
  - Time-of-Use retail contract
  - 1 off-peak period: night ( $s_1$ )
  - 1 peak period: day ( $s_2$ )
  - The base contracts can then be:

$$\begin{array}{cc} & k_1 & & k_2 \\ s_1 & \left( \begin{array}{c} a_1 \\ 0 \end{array} \right) & & \left( \begin{array}{c} 0 \\ a_2 \end{array} \right) \\ s_2 & & & \\ & \text{price : } q_1 & & \text{price : } q_2 \end{array}$$

- Linear combination of  $k_1$  and  $k_2$  can give a Flat delivery contract:

$$\begin{array}{c} k_3 \\ s_1 \left( \begin{array}{c} a_1 \\ a_1 \end{array} \right) \\ s_2 \\ \text{price : } q_3 \end{array}$$

## Electricity Retailing (2)

Description	Notation
Random electricity of 1 unit delivery contract $k$	$\mathbf{a}_k = (a_{1k}, \dots, a_{Sk}), a_{sk} \geq 0$
$K$ contracts	$K = \{1, \dots, K\}$
Electricity delivery of the $K$ contracts	$\mathbf{A} = [\mathbf{a}_k]_{k=1}^K \in \mathbb{R}_+^{SK}$
Portfolio of contracts offered	$\boldsymbol{\theta}_r = (\theta_1, \dots, \theta_K) \in \mathbb{R}^K$
Random electricity flow induced by portfolio	$\mathbf{A}\boldsymbol{\theta}_r \in \mathbb{R}^S$
Less contracts than states of nature	$K < S$
No redundant contract	$\text{rank}(\mathbf{A}) = K$
Always an asset delivering electricity in a state	$\forall s, \exists k, a_{sk} > 0$
Objective program	$\boldsymbol{\theta}_r^* \in \arg \max_{\boldsymbol{\theta}_r \in \mathbb{R}^K} (\mathbf{q}^*)' \boldsymbol{\theta}_r - (\mathbf{p}^*)' \mathbf{A}\boldsymbol{\theta}_r$

# Electricity Production

## Intermittent Technology

Description	Notation
Ex-ante capacity	$\kappa \in \mathbb{R}_+$
State-contingent production per unit capacity	$\mathbf{g} = (g_1, \dots, g_S) \in \mathbb{R}_+^S$
Increasing and convex investment cost function	$\mathcal{K}(\kappa)$ $(\partial\mathcal{K}(\kappa) > 0, \partial^2\mathcal{K}(\kappa) > 0 \text{ \& } \mathcal{K}(0) = 0)$
Objective program	$\kappa^* \in \arg \max_{\kappa \in \mathbb{R}_+} \kappa (\mathbf{p}^*)' \mathbf{g} - \mathcal{K}(\kappa)$

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## Conventional Technology

Description	Notation
State-contingent electricity production	$\mathbf{y} = (y_1, \dots, y_S) \in \mathbb{R}_+^S$
Increasing and convex production expected cost	$\sum_{s=1}^S c_s(y_s)$ $(\partial c_s(q_s) > 0, \partial^2 c_s(q_s) > 0, c_s(0) = 0)$
Objective program	$\mathbf{y}^* \in \arg \max_{\mathbf{y} \in \mathbb{R}_+^S} (\mathbf{p}^*)' \mathbf{y} - \sum_{s=1}^S c_s(y_s)$



# Electricity consumption

Description	Notation
Random electricity consumption	$\mathbf{x} = (x_1, \dots, x_S) \in \mathbb{R}_+^S$
Budget	$m_0 \in \mathbb{R}$
Money spent on other goods	$m \in \mathbb{R}$
Increasing and strictly concave utility function	$\mathcal{U}(\mathbf{x})$
Portfolio of contracts demanded	$\boldsymbol{\theta}_c \in \mathbb{R}^K$
Random electricity flow induced by portfolio	$\mathbf{x} = \mathbf{A}\boldsymbol{\theta}_c$
Objective program	$(\boldsymbol{\theta}_c^*, m^*) \in \arg \max_{(\boldsymbol{\theta}_c, m) \in \mathbb{R}^{K+1}} \mathcal{U}(\mathbf{A}\boldsymbol{\theta}_c) + m$ $\text{s.t. } \begin{cases} \mathbf{A}\boldsymbol{\theta}_c \geq 0 \\ (\mathbf{q}^*)' \boldsymbol{\theta}_c + m = m_0 \end{cases}$

## Partial Equilibrium: Definition

Equilibrium is  $(m^*, \theta_c^*, \theta_r^*, y^*, \kappa^*, p^*, q^*) \in \mathbb{R}^{K+1} \times \mathbb{R}^K \times \mathbb{R}_+^S \times \mathbb{R}_+ \times \mathbb{R}_+^S \times \mathbb{R}_+^K$  whereby:

- ex-ante, the consumers maximize utility and the retailers and producers maximize their profits
- the contract and contingent electricity markets clear:

$$\theta_r^* = \theta_c^* \quad \text{and} \quad y^* + \kappa^* \mathbf{g} = \mathbf{A}\theta_r^*$$

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- No-arbitrage condition shows that  $\mathbf{q} = \mathbf{A}'\mathbf{p}$  with  $\mathbf{p} \in \mathbb{R}_{++}^S$

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- The contingent electricity supply  $\mathbf{S} : \mathbb{R}_{++}^S \rightarrow \mathbb{R}_+^S$  is a differentiable function with the property that  $\partial\mathbf{S}(\mathbf{p})$  is positive definite and the boundary conditions are defined.

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### Proposition

*There exists a unique contingent price vector  $\mathbf{p}^* \in \mathbb{R}_{++}^S$  which clears the different state contingent electricity markets and an associated electricity delivery contract price vector  $\mathbf{q}^* = \mathbf{A}'\mathbf{p}^* \in \mathbb{R}_{++}^K$  which is free of arbitrage and clears the different contract markets.*



## Welfare Analysis

- Contract structure matters as there are less contracts than states of nature
- Potential contingent electricity consumptions restricted to the linear subspace generated by the columns of  $\mathbf{A}$ , i.e.  $\mathbf{x} \in \text{span}(\mathbf{A})$
- $\mathbf{x} = \mathbf{A}\boldsymbol{\theta}$   
With 2 contract structures  $\mathbf{A}$  and  $\tilde{\mathbf{A}}$  with property that  $\text{span}(\mathbf{A}) = \text{span}(\tilde{\mathbf{A}})$ , then  $\mathbf{A}$  is equivalent to  $\tilde{\mathbf{A}}$  in terms of electricity demand, i.e.  $\mathbf{A} \sim_e \tilde{\mathbf{A}}$

$$\mathbf{A} = \begin{bmatrix} \mathbf{B}\mathbf{C} \\ \mathbf{C} \end{bmatrix} \text{ with } \mathbf{C} \text{ any invertible matrix of dimension } K$$

$$\mathbf{x} = \begin{bmatrix} \mathbf{B} \\ \mathbf{I}_K \end{bmatrix} \boldsymbol{\theta} \Leftrightarrow \begin{cases} \boldsymbol{\theta} = (x_s)_{s=S-K+1}^K \\ \begin{bmatrix} \mathbf{I}_{S-K} & -\mathbf{B} \end{bmatrix} \mathbf{x} = \mathbf{0} \end{cases}$$

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# Welfare Analysis

$$SW(\mathbf{B}) = \max_{(\mathbf{y}, \kappa, \mathbf{x}) \in \mathbb{R}_+^{2S+1}} \mathcal{U}(\mathbf{x}) - \mathcal{C}(\mathbf{y}) - \mathcal{K}(\kappa) \text{ s.t. } \begin{cases} \mathbf{x} - \mathbf{y} - \kappa \mathbf{g} = \mathbf{0} \\ \begin{bmatrix} \mathbf{I}_{S-K} & -\mathbf{B} \end{bmatrix} \mathbf{x} = \mathbf{0} \end{cases}$$

$$\begin{cases} \partial \mathcal{U}(\mathbf{x}) - \boldsymbol{\lambda} - \begin{bmatrix} \mathbf{I}_{S-K} \\ -\mathbf{B}' \end{bmatrix} \cdot \boldsymbol{\mu} = \mathbf{0} \\ -\partial \mathcal{C}(\mathbf{y}) + \boldsymbol{\lambda} = \mathbf{0} \\ -\frac{d\mathcal{K}(\kappa)}{d\kappa} + \mathbf{g}' \cdot \boldsymbol{\lambda} = 0 \end{cases}$$

## Proposition

The competitive electricity production plan and allocation  $(\tilde{\mathbf{y}}, \tilde{\kappa}, \tilde{\mathbf{x}}) \in \mathbb{R}_+^{S+1} \times \mathbb{R}_+^S$  is constrained efficient.

## Comparative Statics

Improve welfare:

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### Improve integration of renewable capacity:

If at least two components of  $(\partial_{x_s} U(\mathbf{S}(\mathbf{p})) - p_s)_{s=1}^{S-K}$  are different from 0 and  $1 < K < S - 1$ , all the directions of price changes which improve investment in renewables can be reached, especially the one which is collinear to  $\mathbf{g}$  and which “maximizes” the penetration of renewables

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### Both of the above:

As long as  $\partial_B \kappa^*$  and  $\partial_B SW$  are not collinear with a negative coefficient

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## Conclusion

- Theoretical framework taking into account intermittency of renewables and demand-side flexibility through diversified retail contracts
- Shown existence and uniqueness of a competitive equilibrium of the contingent wholesale and retail markets
- Welfare is constraint efficient
- Characterized the conditions under which we can improve welfare, renewable capacity investment and both
- The results provide insights on how the role of retailers can be redefined so as to participate in demand-side flexibility
- The paper highlights the importance of accounting for intermittency in order to achieve renewable capacity objectives

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