Utility-Scale Energy Storage in an Imperfectly Competitive Power Sector

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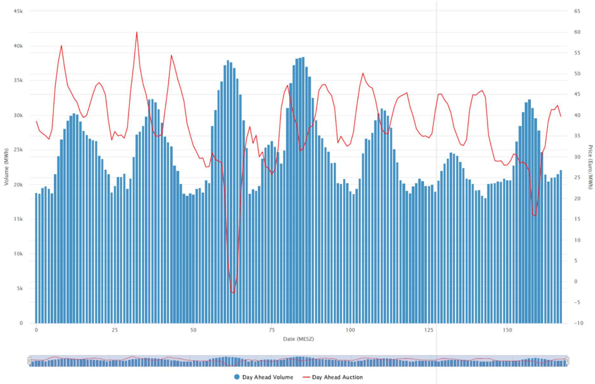
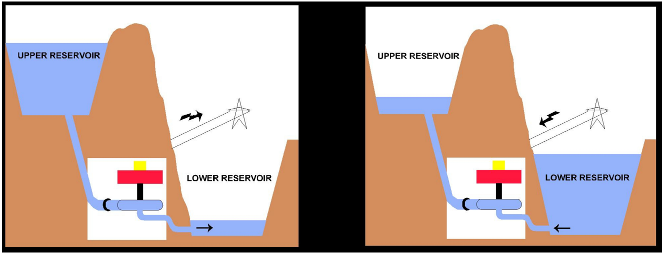
Introduction

Role of Flexible Generation (https://energy-charts.info/)

- Marginal costs: €30/MWh (lignite), €39/MWh (CCGT), €53/MWh (gas)
- Daily average prices: €22.85/MWh to €43.64/MWh



Is Energy Storage the Answer? (https://www.climatetechwiki.org/technology/jiqweb-ph)



Equilibrium Analysis of Storage

- Schill and Kemfert (2011) consider the use of storage in Germany without transmission constraints
- Sioshansi (2014) demonstrates when storage can reduce social welfare or increase GHG emissions (Sioshansi, 2011)
- Siddiqui et al. (2019) compare welfare impacts of ownership and market power
  - Cournot oligopoly: the merchant invests less than the welfare maximiser does to keep price differences high and benefit from temporal arbitrage (margin trading)
  - Perfect competition: the merchant invests in more capacity than a welfare maximiser does (volumetric trading)
- Nasrolahpour et al. (2016) use a bi-level model to assess storage investment by a merchant
- Dvorkin et al. (2018) consider transmission congestion but not strategic investors and market power at the lower level

Research Objective and Findings

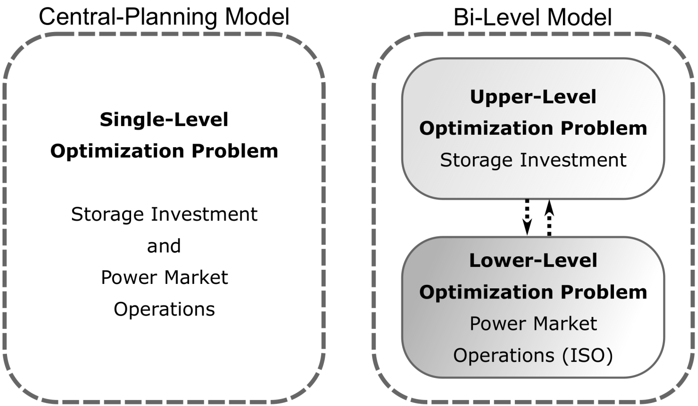
- **How are storage capacity and social welfare affected by the type of storage owner?**
- **Bi-level model for Western Europe with network and VRE: profit-maximising generators with a profit-maximising merchant investor (or a welfare-maximising entity)**
- **Market structure and spatio-temporal variations affect investment decisions more than the type of investor**
  - **Perfectly competitive lower level: 300 MWh of storage investment in Belgium and France with welfare transfer from producers to consumers**
  - **Cournot oligopolistic lower level: 100 MWh of storage investment in Germany and similar welfare transfers (although producers avoid losses)**
  - **Impact of investor type: welfare maximiser never invests less vis-à-vis the merchant under perfect competition, but low storage-investment cost may spur a merchant to adopt more capacity vis-à-vis the welfare maximiser under Cournot oligopoly**

Mathematical Formulation

Setup

- Inverse demand at each node, $D^{\text{int}}_{m,t,n} - D^{\text{slp}}_{m,t,n} q_{m,t,n}$
- DC load flow based on network transfer admittance, $H_{\ell,n}$, and susceptance, $B_{n,n'}$, with voltage angles, $v_{m,t,n}$
- Constant marginal costs, C^{conv}_u, with capacities, $\overline{G}^{\text{conv}}_{n,i',u}$
- VRE has capacities $\overline{G}^{e}_{n,i'}$ with availability factors, $A^{e}_{m,t,n}$
- Leader's problem
  - Maximise welfare (or, profit if merchant) by investing in storage capacity, $\sum_{y \in \mathcal{Y}} z_{n,j,y} \overline{R}^{d}_{y}$
  - Anticipate the response of followers
- Followers' problems
  - Power producers: maximise profit from net generation, $g^{\text{conv}}_{m,t,n,i',u} + g^{e}_{m,t,n,i'} + r^{\text{out}}_{m,t,n,i'} - r^{\text{in}}_{m,t,n,i'}$
  - ISO: maximise gross surplus by managing flows, $v_{m,t,n}$, and consumption, $q_{m,t,n}$
  - Merchant: maximise profit from storage operations, $r^{\text{out}}_{m,t,n,j} - r^{\text{in}}_{m,t,n,j}$

Playing Games

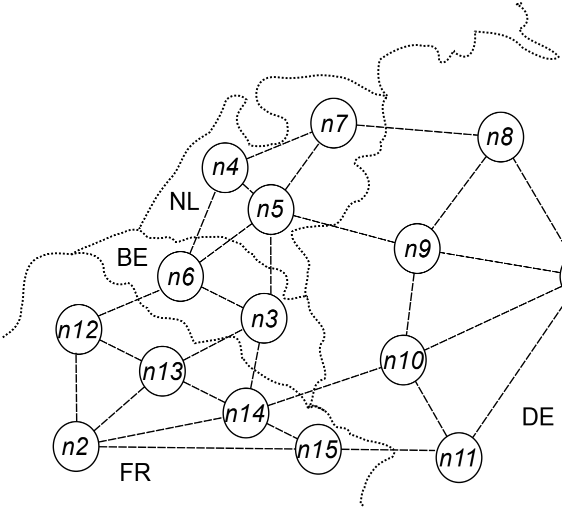


Mathematical Program with Primal and Dual Constraints (MPPDC)

- **Replace lower-level problems** (1)–(4), (5)–(10), **and** (11)–(20), $\forall i' \in \mathcal{I}'$, **by a single-agent quadratic programming (QP) problem using an extended-cost function**
- **Replace lower-level QP by:**
  - **Primal constraints**
  - **Dual constraints**
  - **QP strong duality (Dorn, 1960; Huppmann and Egerer, 2015)**
- **After resolving bilinear terms in strong-duality expression and merchant's upper-level objective function** (23) **via binary expansion, we render the bi-level problems as mixed-integer quadratically constrained quadratic programs (MIQCQPs)**
- **Also, we implement a benchmark central-planning problem that is a simple mixed-integer quadratric program (MIQP)**

Numerical Examples

Network Topology



Generation Technologies' Marginal Costs (with CO_2 price of 20€/t), Ramp Rates, and Emission Rates

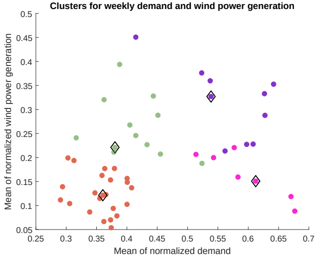
Type	Marginal cost (€/MWh)	Max hourly ramping rate (%)	CO_2 emissions per unit of electricity output (kg/kWh)
$u1$ (nuclear)	9	10	0
$u2$ (lignite)	30	10	0.94
$u3$ (coal)	44	20	0.83
$u4$ (CCGT)	39	30	0.37
$u5$ (gas)	53	30	0.50
$u6$ (oil)	91	70	0.72
$u7$ (hydro)	0	30	0

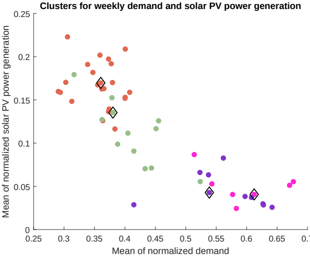
Installed Generation Capacity (GW) and Used Availability Percentages for Conventional Technology $u1$–$u7$, Solar, and Wind

Node	Producer	$u1$	$u2$	$u3$	$u4$	$u5$	$u6$	$u7$	S	W
$n1$	Uniper	-	0.9	3.2	2.7	0.5	1.2	-	-	0.3
	RWE	2.6	9.1	2.8	2.5	1.7	-	0.3	-	0.3
	EnBW	2.7	0.9	3.0	0.4	-	0.4	0.2	-	0.3
	Vattenfall	-	-	2.9	0.6	0.9	0.1	-	-	0.6
	FringeD	4.2	7.4	9.3	10.9	2.2	0.4	1.3	40.1	54.6
$n2$	EDF	63.1	-	4.0	1.4	-	7.0	15.0	0.3	1.5
	FringeF	-	-	-	3.8	2.4	-	3.6	6.5	12.3
$n3$,	Electrabel	5.9	-	-	1.7	1.4	-	-	-	0.5
$n6$	EDF (Luminus)	-	-	-	0.4	0.4	-	-	-	0.2
	FringeB	-	-	-	1.0	-	-	-	3.3	2.2
$n4$,	Electrabel	-	-	-	2.8	0.1	-	-	-	-
$n5$,	Essent/RWE	-	-	1.3	1.9	0.6	-	-	-	-
$n7$	Nuon/Vattenfall	-	-	0.9	3.2	1.1	-	-	-	-
	FringeN	0.5	-	2.9	3.2	0.7	-	-	2.0	4.3
%	Available	80	85	84	89	86	86	30	Fig.	Fig.

Representative Weeks and Demand/VRE Clusters

Week, m (1-52)	Weight, W_m	Demand profile	Wind profile	Solar profile
6	7/52	high	low	low
18	12/52	low	high	high
20	23/52	low	low	high
47	10/52	high	high	low





Welfare Effects of Storage Investment on Social Welfare (SW), Investor Surplus (IS), Producer Surplus (PS), Consumer Surplus (CS), and Merchandising Surplus (MS) under PC

Cost (€/MWh)	Model	SW (k€)	IS (k€)	PS (k€)	CS (k€)	MS (k€)	Capacity (GWh)
–	No inv. PC	2 201 628.15	–	438 683.35	1 749 290.80	13 654.01	–
65	1. CP / 2. SW-PC	+2.542	+1.95	-86.53	+88.02	-0.90	0.3
	3. M-PC	+2.541	+2.23	-82.50	+81.90	+0.92	0.2
50	1. CP / 2. SW-PC	+7.04	+6.45	-86.53	+88.02	-0.90	0.3
	3. M-PC	+7.04	+6.45	-86.53	+88.02	-0.90	0.3
35	1. CP / 2. SW-PC	+13.06	+11.95	-101.63	+103.54	-0.80	0.6
	3. M-PC	+12.83	+11.98	-90.93	+93.13	-1.34	0.5

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–	No inv. CO	1 923 769.99	–	989 457.19	917 432.27	16 880.53	–
55	4. SW-CO	+0.16	-0.40	+0.56	+0.60	-0.60	0.1
	5. M-CO	-	-	-	-	-	-
50	4. SW-CO	+0.66	+0.10	+0.56	+0.60	-0.60	0.1
	5. M-CO	+0.66	+0.10	+0.56	+0.60	-0.60	0.1
25	4. SW-CO	+4.88	+1.78	+0.88	+0.10	+2.13	0.4
	5. M-CO	+3.95	+2.87	-0.25	+1.33	-	0.2
15	4. SW-CO	+9.15	+6.34	+0.87	+0.45	+1.50	0.5
	5. M-CO	+8.43	+6.92	-1.05	+3.27	- 0.71	0.6

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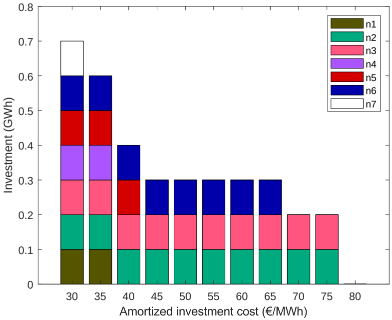
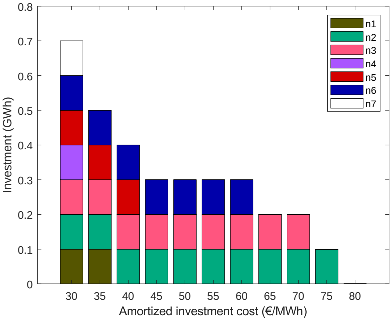
Optimal Storage Investment Size and Location under PC

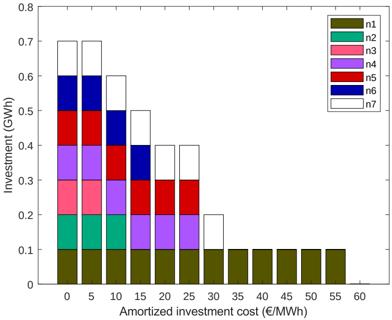


Welfare maximiser

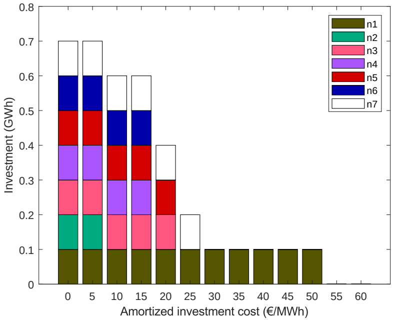


Merchant

Optimal Storage Investment Size and Location under CO



Welfare maximiser



Merchant

Conclusions

Summary

- **Directly compare the impact of market structure and investor type on storage adoption**
  - **Market power affects investment more than the investor type**
  - **PC: higher investment capacity because of higher temporal price differentials, especially in nuclear-dominated Belgium and France**
  - **CO: higher but smoother prices, which results in storage arbitrage to be sought in Germany due to its high VRE capacity**
  - **Welfare maximiser generally invests in at least as much capacity as the merchant**
  - **Exception: low storage-investment cost spurs a merchant to adopt more capacity, i.e., to assume a volumetric strategy, under CO**
- **Future work: enhance solution methods for large-scale MIQCQP problem instances, represent uncertain VRE output, transmission expansion**

Mathematical Appendix

ISO

$$\max_{\Omega^{\text{ISO}}} \sum_{m \in \mathcal{M}} \sum_{t \in \mathcal{T}} \sum_{n \in \mathcal{N}} W_m \left(D^{\text{int}}_{m,t,n} q_{m,t,n} - \frac{1}{2} D^{\text{slp}}_{m,t,n} q^2_{m,t,n} \right) \tag{1}$$

s.t.

$$\sum_{n \in \mathcal{N}} T_t H_{\ell,n} v_{m,t,n} - T_t K_\ell \leq 0 \ (\overline{\mu}_{m,t,\ell}), \ \forall m, t, \ell \tag{2}$$

$$-\sum_{n \in \mathcal{N}} T_t H_{\ell,n} v_{m,t,n} - T_t K_\ell \leq 0 \ (\underline{\mu}_{m,t,\ell}), \ \forall m, t, \ell \tag{3}$$

$$q_{m,t,n} - \sum_{i' \in \mathcal{I}'} \sum_{u \in \mathcal{U}_{n,i'}} g^{\text{conv}}_{m,t,n,i',u} - \sum_{i' \in \mathcal{I}'} \sum_{e \in \mathcal{E}} g^{e}_{m,t,n,i'} - \sum_{i \in \mathcal{I}} r^{\text{out}}_{m,t,n,i} + \sum_{i \in \mathcal{I}} r^{\text{in}}_{m,t,n,i} - \sum_{n' \in \mathcal{N}} T_t B_{n,n'} v_{m,t,n'} = 0 \ (\theta_{m,t,n}), \ \forall m, t, n \tag{4}$$

where $\Omega^{\text{ISO}} \equiv \{q_{m,t,n} \geq 0, v_{m,t,n} \text{ u.r.s.}\}$

ISO

$$\max_{\Omega^{\text{ISO}}} \sum_{m\in\mathcal{M}} \sum_{t\in\mathcal{T}} \sum_{n\in\mathcal{N}} W_m \left(D^{\text{int}}_{m,t,n} q_{m,t,n} - \frac{1}{2} D^{\text{slp}}_{m,t,n} q^2_{m,t,n} \right) \quad (1)$$

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ISO

$$\max_{\Omega^{\text{ISO}}} \sum_{m\in\mathcal{M}} \sum_{t\in\mathcal{T}} \sum_{n\in\mathcal{N}} W_m \left(D^{\text{int}}_{m,t,n} q_{m,t,n} - \frac{1}{2} D^{\text{slp}}_{m,t,n} q^2_{m,t,n} \right) \quad (1)$$

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Storage Operator j

$$\max_{\Omega^j} \sum_{m\in\mathcal{M}} \sum_{t\in\mathcal{T}} \sum_{n\in\mathcal{N}} W_m \left[\frac{\theta_{m,t,n}}{W_m} (r^{\text{out}}_{m,t,n,j} - r^{\text{in}}_{m,t,n,j}) - C^{\text{sto}} r^{\text{out}}_{m,t,n,j} \right] \quad (5)$$

s.t.

$$r^{\text{sto}}_{m,t,n,j} - (1 - E^{\text{sto}}_j)^{T_t}\, r^{\text{sto}}_{m,t-1,n,j} - E^{\text{in}}_j\, r^{\text{in}}_{m,t,n,j} + r^{\text{out}}_{m,t,n,j} = 0 \ (\lambda^{\text{bal}}_{m,t,n,j}),\ \forall m,t,n \quad (6)$$

$$r^{\text{in}}_{m,t,n,j} - T_t\, R^{\text{in}}_j \sum_{y\in\mathcal{Y}} z_{n,j,y} \overline{R}^d_y \le 0 \ (\lambda^{\text{in,p}}_{m,t,n,j}),\ \forall m,t,n \quad (7)$$

$$r^{\text{out}}_{m,t,n,j} - T_t\, R^{\text{out}}_j \sum_{y\in\mathcal{Y}} z_{n,j,y} \overline{R}^d_y \le 0 \ (\lambda^{\text{out,p}}_{m,t,n,j}),\ \forall m,t,n \quad (8)$$

$$r^{\text{sto}}_{m,t,n,j} - \sum_{y\in\mathcal{Y}} z_{n,j,y} \overline{R}^d_y \le 0 \ (\lambda^{\text{ub,p}}_{m,t,n,j}),\ \forall m,t,n \quad (9)$$

$$\underline{R}_{n,j} \sum_{y\in\mathcal{Y}} z_{n,j,y} \overline{R}^d_y - r^{\text{sto}}_{m,t,n,j} \le 0 \ (\lambda^{\text{lb,p}}_{m,t,n,j}),\ \forall m,t,n \quad (10)$$

where $\Omega^j \equiv \{r^{\text{out}}_{m,t,n,j} \ge 0,\ r^{\text{in}}_{m,t,n,j} \ge 0,\ r^{\text{sto}}_{m,t,n,j}\}$

Storage Operator j

$$\max_{\Omega^j} \sum_{m\in\mathcal{M}} \sum_{t\in\mathcal{T}} \sum_{n\in\mathcal{N}} W_m \left[\frac{\theta_{m,t,n}}{W_m} (r^{\text{out}}_{m,t,n,j} - r^{\text{in}}_{m,t,n,j}) - C^{\text{sto}} r^{\text{out}}_{m,t,n,j} \right] \tag{5}$$

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where $\Omega^j \equiv \{ r^{\text{out}}_{m,t,n,j} \geq 0,\ r^{\text{in}}_{m,t,n,j} \geq 0,\ r^{\text{sto}}_{m,t,n,j} \}$

Storage Operator j

$$\max_{\Omega^j} \sum_{m\in\mathcal{M}} \sum_{t\in\mathcal{T}} \sum_{n\in\mathcal{N}} W_m \left[\frac{\theta_{m,t,n}}{W_m} (r^{\text{out}}_{m,t,n,j} - r^{\text{in}}_{m,t,n,j}) - C^{\text{sto}} r^{\text{out}}_{m,t,n,j} \right] \tag{5}$$

s.t.

$$r^{\text{sto}}_{m,t,n,j} - (1 - E^{\text{sto}}_j)^{T_t} r^{\text{sto}}_{m,t-1,n,j} - E^{\text{in}}_j r^{\text{in}}_{m,t,n,j} + r^{\text{out}}_{m,t,n,j} = 0 \ (\lambda^{\text{bal}}_{m,t,n,j}), \ \forall m,t,n \tag{6}$$

$$r^{\text{in}}_{m,t,n,j} - T_t R^{\text{in}}_j \sum_{y\in\mathcal{Y}} z_{n,j,y} \overline{R}^d_y \leq 0 \ (\lambda^{\text{in,p}}_{m,t,n,j}), \ \forall m,t,n \tag{7}$$

$$r^{\text{out}}_{m,t,n,j} - T_t R^{\text{out}}_j \sum_{y\in\mathcal{Y}} z_{n,j,y} \overline{R}^d_y \leq 0 \ (\lambda^{\text{out,p}}_{m,t,n,j}), \ \forall m,t,n \tag{8}$$

$$r^{\text{sto}}_{m,t,n,j} - \sum_{y\in\mathcal{Y}} z_{n,j,y} \overline{R}^d_y \leq 0 \ (\lambda^{\text{ub,p}}_{m,t,n,j}), \ \forall m,t,n \tag{9}$$

$$\underline{R}_{n,j} \sum_{y\in\mathcal{Y}} z_{n,j,y} \overline{R}^d_y - r^{\text{sto}}_{m,t,n,j} \leq 0 \ (\lambda^{\text{lb,p}}_{m,t,n,j}), \ \forall m,t,n \tag{10}$$

where $\Omega^j \equiv \{ r^{\text{out}}_{m,t,n,j} \geq 0, r^{\text{in}}_{m,t,n,j} \geq 0, r^{\text{sto}}_{m,t,n,j} \}$

Storage Operator j

$$\max_{\Omega^j} \sum_{m\in\mathcal{M}} \sum_{t\in\mathcal{T}} \sum_{n\in\mathcal{N}} W_m \left[\frac{\theta_{m,t,n}}{W_m} (r^{\text{out}}_{m,t,n,j} - r^{\text{in}}_{m,t,n,j}) - C^{\text{sto}} r^{\text{out}}_{m,t,n,j} \right] \tag{5}$$

s.t.

$$r^{\text{sto}}_{m,t,n,j} - (1 - E^{\text{sto}}_j)^{T_t} r^{\text{sto}}_{m,t-1,n,j} - E^{\text{in}}_j r^{\text{in}}_{m,t,n,j} + r^{\text{out}}_{m,t,n,j} = 0 \ (\lambda^{\text{bal}}_{m,t,n,j}), \ \forall m,t,n \tag{6}$$

$$r^{\text{in}}_{m,t,n,j} - T_t R^{\text{in}}_j \sum_{y\in\mathcal{Y}} z_{n,j,y} \overline{R}^d_y \le 0 \ (\lambda^{\text{in,p}}_{m,t,n,j}), \ \forall m,t,n \tag{7}$$

$$r^{\text{out}}_{m,t,n,j} - T_t R^{\text{out}}_j \sum_{y\in\mathcal{Y}} z_{n,j,y} \overline{R}^d_y \le 0 \ (\lambda^{\text{out,p}}_{m,t,n,j}), \ \forall m,t,n \tag{8}$$

$$r^{\text{sto}}_{m,t,n,j} - \sum_{y\in\mathcal{Y}} z_{n,j,y} \overline{R}^d_y \le 0 \ (\lambda^{\text{ub,p}}_{m,t,n,j}), \ \forall m,t,n \tag{9}$$

$$\underline{R}_{n,j} \sum_{y\in\mathcal{Y}} z_{n,j,y} \overline{R}^d_y - r^{\text{sto}}_{m,t,n,j} \le 0 \ (\lambda^{\text{lb,p}}_{m,t,n,j}), \ \forall m,t,n \tag{10}$$

where $\Omega^j \equiv \{ r^{\text{out}}_{m,t,n,j} \ge 0, r^{\text{in}}_{m,t,n,j} \ge 0, r^{\text{sto}}_{m,t,n,j} \}$

Storage Operator j

$$\max_{\Omega^j} \sum_{m\in\mathcal{M}} \sum_{t\in\mathcal{T}} \sum_{n\in\mathcal{N}} W_m \left[\frac{\theta_{m,t,n}}{W_m} (r^{\text{out}}_{m,t,n,j} - r^{\text{in}}_{m,t,n,j}) - C^{\text{sto}} r^{\text{out}}_{m,t,n,j} \right] \quad (5)$$

s.t.

$$r^{\text{sto}}_{m,t,n,j} - (1 - E^{\text{sto}}_j)^{T_t} r^{\text{sto}}_{m,t-1,n,j} - E^{\text{in}}_j r^{\text{in}}_{m,t,n,j} + r^{\text{out}}_{m,t,n,j} = 0 \ (\lambda^{\text{bal}}_{m,t,n,j}), \ \forall m,t,n \quad (6)$$

$$r^{\text{in}}_{m,t,n,j} - T_t R^{\text{in}}_j \sum_{y\in\mathcal{Y}} z_{n,j,y} \overline{R}^d_y \leq 0 \ (\lambda^{\text{in,p}}_{m,t,n,j}), \ \forall m,t,n \quad (7)$$

$$r^{\text{out}}_{m,t,n,j} - T_t R^{\text{out}}_j \sum_{y\in\mathcal{Y}} z_{n,j,y} \overline{R}^d_y \leq 0 \ (\lambda^{\text{out,p}}_{m,t,n,j}), \ \forall m,t,n \quad (8)$$

$$r^{\text{sto}}_{m,t,n,j} - \sum_{y\in\mathcal{Y}} z_{n,j,y} \overline{R}^d_y \leq 0 \ (\lambda^{\text{ub,p}}_{m,t,n,j}), \ \forall m,t,n \quad (9)$$

$$\underline{R}_{n,j} \sum_{y\in\mathcal{Y}} z_{n,j,y} \overline{R}^d_y - r^{\text{sto}}_{m,t,n,j} \leq 0 \ (\lambda^{\text{lb,p}}_{m,t,n,j}), \ \forall m,t,n \quad (10)$$

where $\Omega^j \equiv \{r^{\text{out}}_{m,t,n,j} \geq 0, r^{\text{in}}_{m,t,n,j} \geq 0, r^{\text{sto}}_{m,t,n,j}\}$

Storage Operator j

$$\max_{\Omega^j} \sum_{m\in\mathcal{M}} \sum_{t\in\mathcal{T}} \sum_{n\in\mathcal{N}} W_m \left[\frac{\theta_{m,t,n}}{W_m} (r^{\text{out}}_{m,t,n,j} - r^{\text{in}}_{m,t,n,j}) - C^{\text{sto}} r^{\text{out}}_{m,t,n,j} \right] \quad (5)$$

s.t.

$$r^{\text{sto}}_{m,t,n,j} - (1 - E^{\text{sto}}_j)^{T_t} r^{\text{sto}}_{m,t-1,n,j} - E^{\text{in}}_j r^{\text{in}}_{m,t,n,j} + r^{\text{out}}_{m,t,n,j} = 0 \ (\lambda^{\text{bal}}_{m,t,n,j}), \ \forall m,t,n \quad (6)$$

$$r^{\text{in}}_{m,t,n,j} - T_t R^{\text{in}}_j \sum_{y\in\mathcal{Y}} z_{n,j,y} \overline{R}^d_y \le 0 \ (\lambda^{\text{in,p}}_{m,t,n,j}), \ \forall m,t,n \quad (7)$$

$$r^{\text{out}}_{m,t,n,j} - T_t R^{\text{out}}_j \sum_{y\in\mathcal{Y}} z_{n,j,y} \overline{R}^d_y \le 0 \ (\lambda^{\text{out,p}}_{m,t,n,j}), \ \forall m,t,n \quad (8)$$

$$r^{\text{sto}}_{m,t,n,j} - \sum_{y\in\mathcal{Y}} z_{n,j,y} \overline{R}^d_y \le 0 \ (\lambda^{\text{ub,p}}_{m,t,n,j}), \ \forall m,t,n \quad (9)$$

$$\underline{R}_{n,j} \sum_{y\in\mathcal{Y}} z_{n,j,y} \overline{R}^d_y - r^{\text{sto}}_{m,t,n,j} \le 0 \ (\lambda^{\text{lb,p}}_{m,t,n,j}), \ \forall m,t,n \quad (10)$$

where $\Omega^j \equiv \{r^{\text{out}}_{m,t,n,j} \ge 0, r^{\text{in}}_{m,t,n,j} \ge 0, r^{\text{sto}}_{m,t,n,j}\}$

Firm i'

$$\max_{\Omega^{i'}} \sum_{m\in\mathcal{M}} \sum_{t\in\mathcal{T}} \sum_{n\in\mathcal{N}} W_m \Big[(g^{\text{conv}}_{m,t,n,i',u} + g^{e}_{m,t,n,i'} + r^{\text{out}}_{m,t,n,i'} - r^{\text{in}}_{m,t,n,i'}) \times (D^{\text{int}}_{m,t,n} - D^{\text{slp}}_{m,t,n} q_{m,t,n}) - \sum_{u\in\mathcal{U}_{n,i'}} C^{\text{conv}}_u g^{\text{conv}}_{m,t,n,i',u} - C^{\text{sto}} r^{\text{out}}_{m,t,n,i'} \Big] \quad (11)$$

s.t.

$$g^{\text{conv}}_{m,t,n,i',u} - T_t\, \overline{G}^{\text{conv}}_{n,i',u} \le 0 \; (\beta^{\text{conv}}_{m,t,n,i',u}),\ \forall m,t,n,u \in \mathcal{U}_{n,i'} \quad (12)$$

$$g^{\text{conv}}_{m,t,n,i',u} - g^{\text{conv}}_{m,t-1,n,i',u} - T_t\, R^{\text{up}}_u\, \overline{G}^{\text{conv}}_{n,i',u} \le 0 \; (\beta^{\text{up}}_{m,t,n,i',u}),\ \forall m,t,n,u \in \mathcal{U}_{n,i'} \quad (13)$$

$$g^{\text{conv}}_{m,t-1,n,i',u} - g^{\text{conv}}_{m,t,n,i',u} - T_t\, R^{\text{down}}_u\, \overline{G}^{\text{conv}}_{n,i',u} \le 0 \; (\beta^{\text{down}}_{m,t,n,i',u}),\ \forall m,t,n,u \in \mathcal{U}_{n,i'} \quad (14)$$

$$g^{e}_{m,t,n,i'} - T_t\, A^{e}_{m,t,n}\, \overline{G}^{e}_{n,i'} = 0 \; (\beta^{e}_{m,t,n,i'}),\ \forall m,t,n,e \quad (15)$$

$$r^{\text{sto}}_{m,t,n,i'} - (1 - E^{\text{sto}}_{i})^{T_t}\, r^{\text{sto}}_{m,t-1,n,i'} - E^{\text{in}}_{i}\, r^{\text{in}}_{m,t,n,i'} + r^{\text{out}}_{m,t,n,i'} = 0 \; (\lambda^{\text{bal}}_{m,t,n,i'}),\ \forall m,t,n \quad (16)$$

$$r^{\text{in}}_{m,t,n,i'} - T_t\, R^{\text{in}}_{i'}\, \overline{R}_{n,i'} \le 0 \; (\lambda^{\text{in,p}}_{m,t,n,i'}),\ \forall m,t,n \quad (17)$$

$$r^{\text{out}}_{m,t,n,i'} - T_t\, R^{\text{out}}_{i'}\, \overline{R}_{n,i'} \le 0 \; (\lambda^{\text{out,p}}_{m,t,n,i'}),\ \forall m,t,n \quad (18)$$

$$r^{\text{sto}}_{m,t,n,i'} - \overline{R}_{n,i'} \le 0 \; (\lambda^{\text{ub,p}}_{m,t,n,i'}),\ \forall m,t,n \quad (19)$$

$$\underline{R}_{n,i'}\overline{R}_{n,i'} - r^{\text{sto}}_{m,t,n,i'} \le 0 \; (\lambda^{\text{lb,p}}_{m,t,n,i'}),\ \forall m,t,n \quad (20)$$

where $\Omega^{i'} \equiv \{g^{\text{conv}}_{m,t,n,i',u} \ge 0, g^{e}_{m,t,n,i'} \ge 0, r^{\text{out}}_{m,t,n,i'} \ge 0, r^{\text{in}}_{m,t,n,i'} \ge 0, r^{\text{sto}}_{m,t,n,i'} \ge 0\}$

Firm i'

$$\max_{\Omega^{i'}} \sum_{m\in\mathcal{M}} \sum_{t\in\mathcal{T}} \sum_{n\in\mathcal{N}} W_m \Big[(g^{\text{conv}}_{m,t,n,i',u} + g^{e}_{m,t,n,i'} + r^{\text{out}}_{m,t,n,i'} - r^{\text{in}}_{m,t,n,i'}) \times (D^{\text{int}}_{m,t,n} - D^{\text{slp}}_{m,t,n} q_{m,t,n}) - \sum_{u\in\mathcal{U}_{n,i'}} C^{\text{conv}}_{u} g^{\text{conv}}_{m,t,n,i',u} - C^{\text{sto}} r^{\text{out}}_{m,t,n,i'} \Big] \quad (11)$$

s.t.

$$g^{\text{conv}}_{m,t,n,i',u} - T_t\, \overline{G}^{\text{conv}}_{n,i',u} \leq 0 \; (\beta^{\text{conv}}_{m,t,n,i',u}),\; \forall m,t,n,u \in \mathcal{U}_{n,i'} \quad (12)$$

$$g^{\text{conv}}_{m,t,n,i',u} - g^{\text{conv}}_{m,t-1,n,i',u} - T_t\, R^{\text{up}}_{u}\, \overline{G}^{\text{conv}}_{n,i',u} \leq 0 \; (\beta^{\text{up}}_{m,t,n,i',u}),\; \forall m,t,n,u \in \mathcal{U}_{n,i'} \quad (13)$$

$$g^{\text{conv}}_{m,t-1,n,i',u} - g^{\text{conv}}_{m,t,n,i',u} - T_t\, R^{\text{down}}_{u}\, \overline{G}^{\text{conv}}_{n,i',u} \leq 0 \; (\beta^{\text{down}}_{m,t,n,i',u}),\; \forall m,t,n,u \in \mathcal{U}_{n,i'} \quad (14)$$

$$g^{e}_{m,t,n,i'} - T_t\, A^{e}_{m,t,n}\, \overline{G}^{e}_{n,i'} = 0 \; (\beta^{e}_{m,t,n,i'}),\; \forall m,t,n,e \quad (15)$$

$$r^{\text{sto}}_{m,t,n,i'} - (1 - E^{\text{sto}}_{i})^{T_t}\, r^{\text{sto}}_{m,t-1,n,i'} - E^{\text{in}}_{i}\, r^{\text{in}}_{m,t,n,i'} + r^{\text{out}}_{m,t,n,i'} = 0 \; (\lambda^{\text{bal}}_{m,t,n,i'}),\; \forall m,t,n \quad (16)$$

$$r^{\text{in}}_{m,t,n,i'} - T_t\, R^{\text{in}}_{i'}\, \overline{R}_{n,i'} \leq 0 \; (\lambda^{\text{in,p}}_{m,t,n,i'}),\; \forall m,t,n \quad (17)$$

$$r^{\text{out}}_{m,t,n,i'} - T_t\, R^{\text{out}}_{i'}\, \overline{R}_{n,i'} \leq 0 \; (\lambda^{\text{out,p}}_{m,t,n,i'}),\; \forall m,t,n \quad (18)$$

$$r^{\text{sto}}_{m,t,n,i'} - \overline{R}_{n,i'} \leq 0 \; (\lambda^{\text{ub,p}}_{m,t,n,i'}),\; \forall m,t,n \quad (19)$$

$$\underline{R}_{n,i'}\overline{R}_{n,i'} - r^{\text{sto}}_{m,t,n,i'} \leq 0 \; (\lambda^{\text{lb,p}}_{m,t,n,i'}),\; \forall m,t,n \quad (20)$$

where $\Omega^{i'} \equiv \{ g^{\text{conv}}_{m,t,n,i',u} \geq 0, g^{e}_{m,t,n,i'} \geq 0, r^{\text{out}}_{m,t,n,i'} \geq 0, r^{\text{in}}_{m,t,n,i'} \geq 0, r^{\text{sto}}_{m,t,n,i'} \geq 0 \}$

Firm i'

$$\max_{\Omega^{i'}} \sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{N}} W_m\Big[(g^{\text{conv}}_{m,t,n,i',u}+g^{e}_{m,t,n,i'}+r^{\text{out}}_{m,t,n,i'}-r^{\text{in}}_{m,t,n,i'})$$

$$\times (D^{\text{int}}_{m,t,n}-D^{\text{slp}}_{m,t,n}q_{m,t,n}) - \sum_{u\in\mathcal{U}_{n,i'}} C^{\text{conv}}_u g^{\text{conv}}_{m,t,n,i',u} - C^{\text{sto}} r^{\text{out}}_{m,t,n,i'}\Big] \quad (11)$$

s.t.

$$g^{\text{conv}}_{m,t,n,i',u} - T_t\,\overline{G}^{\text{conv}}_{n,i',u} \le 0\ (\beta^{\text{conv}}_{m,t,n,i',u}),\ \forall m,t,n,u\in\mathcal{U}_{n,i'} \quad (12)$$

$$g^{\text{conv}}_{m,t,n,i',u} - g^{\text{conv}}_{m,t-1,n,i',u} - T_t\,R^{\text{up}}_u\,\overline{G}^{\text{conv}}_{n,i',u} \le 0\ (\beta^{\text{up}}_{m,t,n,i',u}),\ \forall m,t,n,u\in\mathcal{U}_{n,i'} \quad (13)$$

$$g^{\text{conv}}_{m,t-1,n,i',u} - g^{\text{conv}}_{m,t,n,i',u} - T_t\,R^{\text{down}}_u\,\overline{G}^{\text{conv}}_{n,i',u} \le 0\ (\beta^{\text{down}}_{m,t,n,i',u}),\ \forall m,t,n,u\in\mathcal{U}_{n,i'} \quad (14)$$

$$g^{e}_{m,t,n,i'} - T_t\,A^{e}_{m,t,n}\,\overline{G}^{e}_{n,i'} = 0\ (\beta^{e}_{m,t,n,i'}),\ \forall m,t,n,e \quad (15)$$

$$r^{\text{sto}}_{m,t,n,i'} - (1-E^{\text{sto}}_{i})^{T_t}\,r^{\text{sto}}_{m,t-1,n,i'} - E^{\text{in}}_{i}\,r^{\text{in}}_{m,t,n,i'} + r^{\text{out}}_{m,t,n,i'} = 0\ (\lambda^{\text{bal}}_{m,t,n,i'}),\ \forall m,t,n \quad (16)$$

$$r^{\text{in}}_{m,t,n,i'} - T_t\,R^{\text{in}}_{i'}\,\overline{R}_{n,i'} \le 0\ (\lambda^{\text{in,p}}_{m,t,n,i'}),\ \forall m,t,n \quad (17)$$

$$r^{\text{out}}_{m,t,n,i'} - T_t\,R^{\text{out}}_{i'}\,\overline{R}_{n,i'} \le 0\ (\lambda^{\text{out,p}}_{m,t,n,i'}),\ \forall m,t,n \quad (18)$$

$$r^{\text{sto}}_{m,t,n,i'} - \overline{R}_{n,i'} \le 0\ (\lambda^{\text{ub,p}}_{m,t,n,i'}),\ \forall m,t,n \quad (19)$$

$$\underline{R}_{n,i'}\overline{R}_{n,i'} - r^{\text{sto}}_{m,t,n,i'} \le 0\ (\lambda^{\text{lb,p}}_{m,t,n,i'}),\ \forall m,t,n \quad (20)$$

where $\Omega^{i'} \equiv \{g^{\text{conv}}_{m,t,n,i',u}\ge 0, g^{e}_{m,t,n,i'}\ge 0, r^{\text{out}}_{m,t,n,i'}\ge 0, r^{\text{in}}_{m,t,n,i'}\ge 0, r^{\text{sto}}_{m,t,n,i'}\ge 0\}$

Firm i'

$$\max_{\Omega^{i'}} \sum_{m\in\mathcal{M}} \sum_{t\in\mathcal{T}} \sum_{n\in\mathcal{N}} W_m \Big[(g^{\text{conv}}_{m,t,n,i',u} + g^{e}_{m,t,n,i'} + r^{\text{out}}_{m,t,n,i'} - r^{\text{in}}_{m,t,n,i'}) \times (D^{\text{int}}_{m,t,n} - D^{\text{slp}}_{m,t,n} q_{m,t,n}) - \sum_{u\in\mathcal{U}_{n,i'}} C^{\text{conv}}_u g^{\text{conv}}_{m,t,n,i',u} - C^{\text{sto}} r^{\text{out}}_{m,t,n,i'} \Big] \quad (11)$$

s.t.

$$g^{\text{conv}}_{m,t,n,i',u} - T_t \, \overline{G}^{\text{conv}}_{n,i',u} \leq 0 \ (\beta^{\text{conv}}_{m,t,n,i',u}), \ \forall m,t,n,u \in \mathcal{U}_{n,i'} \quad (12)$$

$$g^{\text{conv}}_{m,t,n,i',u} - g^{\text{conv}}_{m,t-1,n,i',u} - T_t \, R^{\text{up}}_u \, \overline{G}^{\text{conv}}_{n,i',u} \leq 0 \ (\beta^{\text{up}}_{m,t,n,i',u}), \ \forall m,t,n,u \in \mathcal{U}_{n,i'} \quad (13)$$

$$g^{\text{conv}}_{m,t-1,n,i',u} - g^{\text{conv}}_{m,t,n,i',u} - T_t \, R^{\text{down}}_u \, \overline{G}^{\text{conv}}_{n,i',u} \leq 0 \ (\beta^{\text{down}}_{m,t,n,i',u}), \ \forall m,t,n,u \in \mathcal{U}_{n,i'} \quad (14)$$

$$g^{e}_{m,t,n,i'} - T_t \, A^{e}_{m,t,n} \, \overline{G}^{e}_{n,i'} = 0 \ (\beta^{e}_{m,t,n,i'}), \ \forall m,t,n,e \quad (15)$$

$$r^{\text{sto}}_{m,t,n,i'} - (1 - E^{\text{sto}}_i)^{T_t} \, r^{\text{sto}}_{m,t-1,n,i'} - E^{\text{in}}_i \, r^{\text{in}}_{m,t,n,i'} + r^{\text{out}}_{m,t,n,i'} = 0 \ (\lambda^{\text{bal}}_{m,t,n,i'}), \ \forall m,t,n \quad (16)$$

$$r^{\text{in}}_{m,t,n,i'} - T_t \, R^{\text{in}}_{i'} \, \overline{R}_{n,i'} \leq 0 \ (\lambda^{\text{in,p}}_{m,t,n,i'}), \ \forall m,t,n \quad (17)$$

$$r^{\text{out}}_{m,t,n,i'} - T_t \, R^{\text{out}}_{i'} \, \overline{R}_{n,i'} \leq 0 \ (\lambda^{\text{out,p}}_{m,t,n,i'}), \ \forall m,t,n \quad (18)$$

$$r^{\text{sto}}_{m,t,n,i'} - \overline{R}_{n,i'} \leq 0 \ (\lambda^{\text{ub,p}}_{m,t,n,i'}), \ \forall m,t,n \quad (19)$$

$$\underline{R}_{n,i'} \overline{R}_{n,i'} - r^{\text{sto}}_{m,t,n,i'} \leq 0 \ (\lambda^{\text{lb,p}}_{m,t,n,i'}), \ \forall m,t,n \quad (20)$$

where $\Omega^{i'} \equiv \{ g^{\text{conv}}_{m,t,n,i',u} \geq 0, g^{e}_{m,t,n,i'} \geq 0, r^{\text{out}}_{m,t,n,i'} \geq 0, r^{\text{in}}_{m,t,n,i'} \geq 0, r^{\text{sto}}_{m,t,n,i'} \geq 0 \}$

Firm i'

$$\max_{\Omega^{i'}} \sum_{m\in\mathcal{M}} \sum_{t\in\mathcal{T}} \sum_{n\in\mathcal{N}} W_m \Big[(g^{\text{conv}}_{m,t,n,i',u} + g^{e}_{m,t,n,i'} + r^{\text{out}}_{m,t,n,i'} - r^{\text{in}}_{m,t,n,i'}) \times (D^{\text{int}}_{m,t,n} - D^{\text{slp}}_{m,t,n} q_{m,t,n}) - \sum_{u\in\mathcal{U}_{n,i'}} C^{\text{conv}}_{u} g^{\text{conv}}_{m,t,n,i',u} - C^{\text{sto}} r^{\text{out}}_{m,t,n,i'} \Big] \quad (11)$$

s.t.

$$g^{\text{conv}}_{m,t,n,i',u} - T_t\, \overline{G}^{\text{conv}}_{n,i',u} \leq 0\ (\beta^{\text{conv}}_{m,t,n,i',u}),\ \forall m,t,n,u \in \mathcal{U}_{n,i'} \quad (12)$$

$$g^{\text{conv}}_{m,t,n,i',u} - g^{\text{conv}}_{m,t-1,n,i',u} - T_t\, R^{\text{up}}_{u}\, \overline{G}^{\text{conv}}_{n,i',u} \leq 0\ (\beta^{\text{up}}_{m,t,n,i',u}),\ \forall m,t,n,u \in \mathcal{U}_{n,i'} \quad (13)$$

$$g^{\text{conv}}_{m,t-1,n,i',u} - g^{\text{conv}}_{m,t,n,i',u} - T_t\, R^{\text{down}}_{u}\, \overline{G}^{\text{conv}}_{n,i',u} \leq 0\ (\beta^{\text{down}}_{m,t,n,i',u}),\ \forall m,t,n,u \in \mathcal{U}_{n,i'} \quad (14)$$

$$g^{e}_{m,t,n,i'} - T_t\, A^{e}_{m,t,n}\, \overline{G}^{e}_{n,i'} = 0\ (\beta^{e}_{m,t,n,i'}),\ \forall m,t,n,e \quad (15)$$

$$r^{\text{sto}}_{m,t,n,i'} - (1 - E^{\text{sto}}_{i})^{T_t}\, r^{\text{sto}}_{m,t-1,n,i'} - E^{\text{in}}_{i}\, r^{\text{in}}_{m,t,n,i'} + r^{\text{out}}_{m,t,n,i'} = 0\ (\lambda^{\text{bal}}_{m,t,n,i'}),\ \forall m,t,n \quad (16)$$

$$r^{\text{in}}_{m,t,n,i'} - T_t\, R^{\text{in}}_{i'}\, \overline{R}_{n,i'} \leq 0\ (\lambda^{\text{in,p}}_{m,t,n,i'}),\ \forall m,t,n \quad (17)$$

$$r^{\text{out}}_{m,t,n,i'} - T_t\, R^{\text{out}}_{i'}\, \overline{R}_{n,i'} \leq 0\ (\lambda^{\text{out,p}}_{m,t,n,i'}),\ \forall m,t,n \quad (18)$$

$$r^{\text{sto}}_{m,t,n,i'} - \overline{R}_{n,i'} \leq 0\ (\lambda^{\text{ub,p}}_{m,t,n,i'}),\ \forall m,t,n \quad (19)$$

$$\underline{R}_{n,i'}\overline{R}_{n,i'} - r^{\text{sto}}_{m,t,n,i'} \leq 0\ (\lambda^{\text{lb,p}}_{m,t,n,i'}),\ \forall m,t,n \quad (20)$$

where $\Omega^{i'} \equiv \{g^{\text{conv}}_{m,t,n,i',u} \geq 0, g^{e}_{m,t,n,i'} \geq 0, r^{\text{out}}_{m,t,n,i'} \geq 0, r^{\text{in}}_{m,t,n,i'} \geq 0, r^{\text{sto}}_{m,t,n,i'} \geq 0\}$

Upper-Level Objective Function

$$\max_{z_{n,j,y}} \sum_{m\in\mathcal{M}} \sum_{t\in\mathcal{T}} \sum_{n\in\mathcal{N}} W_m \Bigg[\left(D^{\text{int}}_{m,t,n} q_{m,t,n} - \frac{1}{2} D^{\text{slp}}_{m,t,n} q^2_{m,t,n} \right) - \sum_{i'\in\mathcal{I}'} \sum_{u\in\mathcal{U}_{n,i'}} C^{\text{conv}}_u g^{\text{conv}}_{m,t,n,i',u} - \sum_{i\in\mathcal{I}} C^{\text{sto}} r^{\text{out}}_{m,t,n,i} \Bigg] - \sum_{n\in\mathcal{N}} \sum_{y\in\mathcal{Y}} z_{n,j,y} I\overline{R}^d_y \tag{21}$$

$$\text{s.t.} \quad \sum_{y\in\mathcal{Y}} z_{n,j,y} = 1, \forall n, \quad z_{n,j,y} \in \{0,1\}, \forall n, y \tag{22}$$

$$\max_{z_{n,j,y}} \sum_{m\in\mathcal{M}} \sum_{t\in\mathcal{T}} \sum_{n\in\mathcal{N}} W_m \left[\frac{\theta_{m,t,n}}{W_m} (r^{\text{out}}_{m,t,n,j} - r^{\text{in}}_{m,t,n,j}) - C^{\text{sto}} r^{\text{out}}_{m,t,n,j} \right] - \sum_{n\in\mathcal{N}} \sum_{y\in\mathcal{Y}} z_{n,j,y} I\overline{R}^d_y \tag{23}$$

s.t. (22)

Upper-Level Objective Function

$$\max_{z_{n,j,y}} \sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{N}} W_m \Bigg[\left(D^{\text{int}}_{m,t,n} q_{m,t,n} - \frac{1}{2} D^{\text{slp}}_{m,t,n} q^2_{m,t,n}\right) - \sum_{i'\in\mathcal{I}'}\sum_{u\in\mathcal{U}_{n,i'}} C^{\text{conv}}_u g^{\text{conv}}_{m,t,n,i',u} - \sum_{i\in\mathcal{I}} C^{\text{sto}} r^{\text{out}}_{m,t,n,i}\Bigg] - \sum_{n\in\mathcal{N}}\sum_{y\in\mathcal{Y}} z_{n,j,y} \overline{IR}^d_y \tag{21}$$

s.t.

$$\sum_{y\in\mathcal{Y}} z_{n,j,y} = 1, \forall n, \quad z_{n,j,y} \in \{0,1\}, \forall n, y \tag{22}$$

$$\max_{z_{n,j,y}} \sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{N}} W_m \left[\frac{\theta_{m,t,n}}{W_m}\left(r^{\text{out}}_{m,t,n,j} - r^{\text{in}}_{m,t,n,j}\right) - C^{\text{sto}} r^{\text{out}}_{m,t,n,j}\right] - \sum_{n\in\mathcal{N}}\sum_{y\in\mathcal{Y}} z_{n,j,y} \overline{IR}^d_y \tag{23}$$

s.t. (22)

Upper-Level Objective Function

$$\max_{z_{n,j,y}} \sum_{m\in\mathcal{M}} \sum_{t\in\mathcal{T}} \sum_{n\in\mathcal{N}} W_m \Bigg[\Big(D^{\text{int}}_{m,t,n} q_{m,t,n} - \frac{1}{2} D^{\text{slp}}_{m,t,n} q^2_{m,t,n} \Big) - \sum_{i'\in\mathcal{I}'} \sum_{u\in\mathcal{U}_{n,i'}} C^{\text{conv}}_u g^{\text{conv}}_{m,t,n,i',u} - \sum_{i\in\mathcal{I}} C^{\text{sto}} r^{\text{out}}_{m,t,n,i} \Bigg] - \sum_{n\in\mathcal{N}} \sum_{y\in\mathcal{Y}} z_{n,j,y} \overline{IR}^d_y \quad (21)$$

$$\text{s.t.} \quad \sum_{y\in\mathcal{Y}} z_{n,j,y} = 1, \forall n, \quad z_{n,j,y} \in \{0,1\}, \forall n, y \quad (22)$$

$$\max_{z_{n,j,y}} \sum_{m\in\mathcal{M}} \sum_{t\in\mathcal{T}} \sum_{n\in\mathcal{N}} W_m \Bigg[\frac{\theta_{m,t,n}}{W_m} \big(r^{\text{out}}_{m,t,n,j} - r^{\text{in}}_{m,t,n,j} \big) - C^{\text{sto}} r^{\text{out}}_{m,t,n,j} \Bigg] - \sum_{n\in\mathcal{N}} \sum_{y\in\mathcal{Y}} z_{n,j,y} \overline{IR}^d_y \quad (23)$$

s.t. (22)

Upper-Level Objective Function

$$\max_{z_{n,j,y}} \sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{N}} W_m \Bigg[\left(D^{\text{int}}_{m,t,n} q_{m,t,n} - \frac{1}{2} D^{\text{slp}}_{m,t,n} q^2_{m,t,n}\right)$$
$$- \sum_{i'\in\mathcal{I}'}\sum_{u\in\mathcal{U}_{n,i'}} C^{\text{conv}}_u g^{\text{conv}}_{m,t,n,i',u} - \sum_{i\in\mathcal{I}} C^{\text{sto}} r^{\text{out}}_{m,t,n,i}\Bigg]$$
$$- \sum_{n\in\mathcal{N}}\sum_{y\in\mathcal{Y}} z_{n,j,y} I\overline{R}^d_y \qquad (21)$$

$$\text{s.t.} \quad \sum_{y\in\mathcal{Y}} z_{n,j,y} = 1, \forall n, \quad z_{n,j,y} \in \{0,1\}, \forall n, y \qquad (22)$$

$$\max_{z_{n,j,y}} \sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{N}} W_m \left[\frac{\theta_{m,t,n}}{W_m}\left(r^{\text{out}}_{m,t,n,j} - r^{\text{in}}_{m,t,n,j}\right) - C^{\text{sto}} r^{\text{out}}_{m,t,n,j}\right]$$
$$- \sum_{n\in\mathcal{N}}\sum_{y\in\mathcal{Y}} z_{n,j,y} I\overline{R}^d_y \qquad (23)$$

s.t. (22)