Optimal climate policy in the face of tipping points and asset stranding

November 25, 2020

1 Introduction

The accumulation of greenhouse gases in the atmosphere is putting at risk the stability of the climate system (IPCC, 2014). There is already solid empirical evidence that climate fluctuations reduce productivity, damage capital and thereby slow economic growth (e.g. Dell et al., 2012; Burke et al., 2015). There is also significant concern about non-linearities and tipping points in the climate system that may be encountered in the not-too-distant future (Lenton et al., 2008; IPCC, 2014; Lenton et al., 2019). For these reasons and others, the international community has agreed to limit the increase in global temperatures by reducing greenhouse gas emissions to net zero over a period of just a few decades (UNFCCC, 2016). Reducing emissions in this way will require a drastic reduction in the production and consumption of fossil fuels: a transition to a low-carbon economy.

Two main obstacles lie in the way of decarbonisation. First, modern production systems are heavily dependent on fossil fuels as intermediate inputs. Fossil fuels are still the main source of energy for electricity generation globally, they are required in the manufacturing of steel, cement, chemicals and other goods, and they power international transport. While in certain sectors low-carbon technologies are available and relatively attractive (e.g. renewable electricity), in others low-carbon alternatives are still far from being competitive with the incumbent technologies e.g. in heavy-duty road transport, shipping and aviation, and in cement, chemicals and steel manufacturing (Energy Transitions Commission, 2018). Hence, further innovation is needed to offer viable low-carbon technologies to firms, which takes time.

Second, much of the existing stock of capital and infrastructure has been designed to work in tandem with the use of fossil fuels (e.g. coal and gas power plants and accompanying electricity distribution networks, steel mills and road networks). These are typically long-lived physical assets, financed with large initial investments under the expectation of a long and profitable asset life. Once installed, it would be costly, or sometimes impossible, to convert high-carbon stocks to other low-carbon uses. This creates a strong incumbency bias against a rapid low-carbon transition, as: i) firms do not want to suffer a negative revaluation of their assets, or a reduction in capacity utilisation; ii) investors holding financial assets issued by high-carbon firms want to avoid a drop in their market valuation; and iii) governments aim to avoid large-scale investments to decarbonise public infrastructure faster than its natural replacement cycle.

What is the most appropriate course of action, given the potential trade-off between climate stability and the stranding of existing high-carbon physical and financial assets? One possibility is to continue using existing high-carbon capital stocks at full capacity until they reach their natural end-of-life. If there is no new investment in fossil capital, and assuming the availability of low-carbon technological alternatives, this would lead to a smooth process of substitution between high- and low-carbon capital stocks, avoiding losses for the owners of fossil capital assets. However, the long lives of certain high-carbon assets (e.g. coal plants typically operate for around 50 years) is thought to make this strategy incompatible with the achievement of more stringent climate goals such as limiting global warming to 2°C (Davis and Socolow, 2014; Pfeiffer et al., 2016), as well as accentuating the risks of crossing climate tipping points. An alternative strategy would be to 'strand' today excess high-carbon capital stocks by stopping using them ahead of their natural end-of-life. This scenario is implied by the large initial downward jump in emissions that several models find to be optimal to mitigate climate change (e.g. DICE). However, in a context in which converting capital stocks to new (low-carbon) purposes, when possible, takes time and money, this strategy is likely to trigger wider macroeconomic and financial spillovers. The business operations of the firms owning the stranded capital stocks would be disrupted, with negative repercussions for their profitability; lending banks and institutions holding the financial assets of affected firms would suffer losses, which might propagate to others within the financial network; downstream firms would lose a source of intermediate inputs supply (possibly unsubstitutable, in the short-term).

This paper investigates how to strike a balance between avoiding dangerous climate change on the one hand and stranding high-carbon assets on the other hand. To do so, we develop an integrated assessment model (IAM) that simulates socially optimal carbon emissions and corresponding investments in two distinct capital stocks: green and dirty (in other words low- and high-carbon). The model has three distinctive characteristics. First, we introduce rigidities – adjustment costs – associated with the accumulation of either type of capital, with the conversion of high-carbon capital into low-carbon capital, and with the speed of emission abatement. Capital conversion plays a particularly important role. Whenever optimal investment in high-carbon capital is negative, high-carbon assets are stranded and firms spend money converting these assets into low-carbon capital. This incurs resource costs to the firms concerned and, if undertaken at high speed, it incurs wider economic costs. Second, optimal capital investment must be made under multiple climate and economic uncertainties, each of which is captured by a stochastic process. The social planner anticipates these uncertainties and responds to them by making corrections to the optimal emissions/investment path. That is, we solve the optimal path using dynamic programming, and our model belongs to the emerging class of 'recursive IAMs' (Lemoine and Rudik, 2017). In doing so, we employ Epstein-Zin-Weil preferences (Epstein and Zin, 1989; Weil, 1990), which disentangle risk aversion from intertemporal consumption substitution, consistent with empirical data on asset returns. Third, we carefully calibrate the costs of emissions abatement in our model on data from a large number energy systems models contained in the IPCC database.

After calibrating the model, we run numerical simulations from 2020 (or 2030-2040, in the case of delayed policy scenarios) to 2100. We consider a number of different scenarios and perform sensitivity analysis on parameter values EXPLAIN. We find that, across all scenarios considered, it would be optimal to immediately stop investing in high-carbon capital and moreover to strand some of it, in the range of . After a few decades, it becomes optimal to allow for some dirty investments again. We also find higher optimal carbon prices compared to a simpler model with no inertia in capital stocks and no uncertainty. More specifically, optimal carbon prices are Finally, we show how a delay in the implementation of mitigation policies leads to higher asset stranding, in the order of

The remainder of the paper is structured as follows. Section 2 discusses related literature and identifies our contribution to it. Section 3 presents the model. Section 4 explains our calibration strategy. Section 5 presents and discusses our numerical results. Section 6 concludes.

2 Motivation and related literature

Our paper contributes to the literature studying possible stranding of assets along the transition to a low-carbon economy and the wider economic implications of this. Asset stranding takes place whenever the utilisation and/or monetary value of certain assets suffers an unanticipated reduction. The general idea that a technological transition can lead to assets losing their value or having to be reconverted at a cost has been highlighted by innovation scholars (see for instance Perez, 2010) and in recent years it has been applied to the low-carbon transition. The root of the issue in the case of the low-carbon transition is the disparity between, on the one hand, the still substantial amount of fossil fuel reserves and resources, and, on the other hand, the comparatively small atmospheric carbon budget that is available if global warming is to be limited to a tolerable level. Meeting climate goals consequently requires some fossil fuel reserves to remain in the ground (Meinshausen et al., 2009; McGlade and Ekins, 2015).

The idea of 'unburnable carbon' (Carbon Tracker, 2011) has two main ramifications beyond the fossil fuel extraction sector itself. First, physical capital stocks requiring fossil inputs to operate might lose value, because their utilisation rate falls, or because they must be converted to different uses. This should impact the profits of the non-financial firms that operate said assets and it can spill over to cause additional asset stranding upstream and downstream in the value chain (Cahen-Fourot et al., 2019). Second, a reduction in the expected profits of these non-financial firms should impact the value of the financial assets they issue. This in turn could lead to a propagation of losses within the highly interconnected financial network through second-round effects, where a financial institution with little or no exposure to the fossil industry is negatively affected by the revaluation of financial assets in its possession issued by other financial institutions exposed to the fossil industry (Battiston et al., 2017). The combination of these two dynamic effects then poses a risk of wider macroeconomic dynamics characterised by financial instability, unemployment, recession and stress on public finances; an eventuality framed by the former governor of the Bank of England, Mark Carney, as a 'climate Minsky moment' (Carney et al., 2019).

Asset stranding can be studied using different approaches, including conceptual studies (Semieniuk et al., 2020), analysis of emissions embodied in physical capital stocks (Pfeiffer et al., 2018; Cui et al., 2019), analysis of production and financial networks (Cahen-Fourot et al., 2019; Battiston et al., 2017), and empirical analysis of financial asset pricing (Campiglio et al., 2019; Sen and von Schickfus, 2020). We contribute instead to the research effort trying to incorporate asset stranding in dynamic economic models. Within this effort, a variety of modelling traditions has been deployed, ranging from dynamic stochastic general equilibrium (DSGE) models (Annicchiarico and Di Dio, 2016) and capital asset pricing models (CAPM) (Karydas and Xepapadeas, 2019) to stock-flow consistent (SFC) models (Dafermos et al., 2018) and agent-based models (ABMs) (Lamperti et al., 2019). We mainly build upon and contribute to the stream of literature developing IAMs rooted in neoclassical growth theory.

Traditionally, IAMs have abstracted from the issue of asset stranding, both from a physical and financial perspective. However, recent contributions have started to fill this gap in the literature. Some of them (Baldwin et al., 2020; Rozenberg et al., 2020; Coulomb et al., 2019; Vogt-Schilb et al., 2018) develop Ramsey-type growth models where, building on previous contributions introducing irreversible investments (Jorgenson, 1967), they allow for under-utilisation of capital stocks. This is introduced by setting an additional constraint to the optimisation problem, whereby the amount of capital that can be utilised is bounded by the amount of capital installed. The general conclusion of this stream of work is that, in order to respect a certain carbon budget, it would be optimal to leave part of the capital stock un-utilised. Rozenberg et al. (2020) take a step further by assuming an additional political constraint, whereby no stranding is allowed even if it would maximise welfare to do so, and show how non-pricing policies (e.g. mandates, feebates) become optimal in this case. The focus of this line of work is stranding of physical assets; no financial considerations are developed.

A parallel strand of the literature has developed models with a stronger focus on financial asset stranding. For instance, van der Ploeg and Rezai (2020) study stranding in the form of changes in the market valuation of firms in the fossil exploration and extraction sector. Karydas and Xepapadeas (2019) introduce climate change and transition risks in a dynamic asset pricing framework to study how these affect equity premiums. Hambel et al. (2020) also employ a dynamic asset pricing framework to analyse the trade-off between emissions abatement and asset diversification motives (i.e. that there can be a diversification motive to continue investing in dirty assets). To capture inertia in investment dynamics, and building on optimal investment theory (Gould, 1968; Lucas Jr, 1967), some of the contributions mentioned above include adjustment costs as a convex function of the level of investment (also see Coulomb et al., 2019; Vogt-Schilb et al., 2018), or of the investment-capital ratio (Hambel et al., 2020; van der Ploeg and Rezai, 2020). Investment adjustment costs aim to capture the fact that a proportion of the (opportunity) costs that firms bear when installing new capital are not transformed into new capital stock (e.g. the labour employed in the production of new capital, which is thereby not employed in the production of consumption goods). Given the convexity of the adjustment cost function, large capital investment efforts concentrated in one period are assumed to be more expensive than the same capital expansion spread over multiple periods. Coulomb et al. (2019) also differentiate investment adjustment cost functions by technology, assuming that the cost function for renewables has a steeper slope than the cost function for gas because of the additional storage costs associated with renewables.

An alternative approach to including asset stranding is to treat stranding as the costs required to reconvert capital stocks to new (low-carbon) uses. Hambel et al. (2020) introduce quadratic stranding costs by allowing dirty capital to be reconverted to clean capital with proportionally increasing frictions. All types of capital stocks remain fully utilised. While unable to analyse the issue of capacity underutilisation, this approach has the advantage of being able to capture both the directs costs of technological conversion and the wider macroeconomic spillovers that the process might trigger. We also include reconversion costs using a quadratic cost function, but we differentiate ourselves from Hambel et al. (2020) in its interpretation.

Some of the asset stranding models also include stochastic elements. Bretschger and Soretz (2018) and van der Ploeg and Rezai (2020) focus on uncertainty about the introduction of emissions abatement policies using optimal control theory. Hambel et al. (2020) and Karydas and Xepapadeas (2019) introduce stochastic processes governing both climate and macroeconomic variables. In doing so, these latter two papers connect the literature on stranded assets with the literature on so-called 'recursive integrated assessment' (Lemoine and Rudik, 2017), which studies the implications of uncertainty for optimal climate policy in a closed-loop set up, in which uncertainty is gradually resolved and policy-makers can make adjustments mid-course (and anticipate this possibility). Recursive IAMs solve a dynamic programming problem and often apply Epstein-Zin-Weil preferences (Epstein and Zin, 1989; Weil, 1990), which disentangle risk aversion from the elasticity of intertemporal substitution, providing one way to reconcile preferences in the IAM with the asset pricing puzzles.

3 The model

We consider an economy in continuous time with an infinite time horizon. A single type of good exists, used for both consumption and investment purposes. The expectations of agents are rational. Two technologies exist: a dirty capital good (K_d) and a clean one (K_c), with total capital $K = K_d + K_c$. Two main features distinguish dirty and clean capital. First, we assume K_d is more productive than K_c. Second, we assume use of K_d produces CO2 emissions E at a carbon intensity ψ (that is: $E = \psi K_d$), while use of K_c does not produce any emissions.

In our model, dirty capital K_d represents all forms of capital that require fossil fuels to produce goods and services (e.g. electricity from coal and gas, steel, cement, and others), while clean capital K_c comprises capital that does not require fossil fuels for production. While not explicitly representing networks of intermediate exchange, this approach allows us to partly capture the distinction between upstream and downstream sectors. Upstream sectors where fossil fuels enter directly as intermediate inputs require a shift towards low-carbon technological alternatives. Downstream sectors (e.g. technological firms), where fossil fuels are not directly used, can already be considered clean, as they are dirty only to the extent to which they employ intermediate inputs of polluting upstream sectors (Kemp-Benedict, 2018). Therefore, contrary to most of the related literature that assumes clean capital is a small expanding niche in a system dominated by dirty capital, our definition implies most of the capital stock is clean (or at least not dirty *per se*), and we treat as dirty only the capital that directly uses fossil fuels.

3.1 Production

Output Y is produced through the following function:

$$Y = AL^{1-\alpha}(K_d + K_c)^{\alpha} e^{\xi} \tag{1}$$

where A is a parameter representing total factor productivity, increasing exogenously at a rate equal to $(1-\alpha)g$,[1] L is labour growing at rate g_L, and α is the capital share. Appendix 1 provides an overview of all parameter values.

Building on Dietz and Venmans (2019), we also assume that, at each time t, production is perturbed by some 'reward' and 'penalty' factors, aggregated in the term ξ. More specifically, we assume production is affected by:

- A reward for using more productive dirty capital K_d, or, to put it another way, the extra cost of clean capital/emissions abatement. However, the reward for polluting gradually declines with the level of emissions and eventually becomes zero for large enough values of E (thus making the two types of capital equally productive).

- A penalty for having high cumulative emissions S. Building on recent scientific evidence showing that warming is approximately a linear function of cumulative emissions (see Dietz and Venmans, 2019, and references therein), this is equivalent to introducing climate damages to production caused by increasing temperatures. We choose a quadratic damage function, in keeping with much of the literature (e.g. with the DICE model).

- A penalty for reducing emissions too quickly. Given $E = \psi K_d$, we can define emissions change $\dot{E}$ as a direct function of changes in two variables: i) the dirty capital stock, which, as we will explain in the next subsection,

[1] This results in a steady state growth rate of g.

is expanded by investments and reduced by both depreciation and repurposing of capital; and ii) carbon intensity ψ, which we assume decreases at a constant rate $-g_\psi$. This penalty on the speed of abatement allows us to consider the costs of a more abrupt transition even in the absence of stranding costs, as it captures inertia in existing energy systems, the struggle to retrain and reskill workers, and bottlenecks to fast innovation (Ha-Duong et al., 1997).

We can thus write ξ as:

$$\xi = \phi E - \frac{\varphi}{2}E^2 + \beta\dot{E} - \frac{\gamma}{2}S^2, \tag{2}$$

where ϕ, φ, β and γ are constant parameters governing the costs of emissions abatement and warming respectively. With this formulation, the marginal productivity of clean capital Y_{K_c} is equal to $Y\frac{\alpha}{K}$.[2] The marginal productivity of dirty capital Y_{K_d} is instead equal to $Y\left[\frac{\alpha}{K} + \phi\psi - \varphi\psi^2 K_d\right]$, where the first term is equal to the productivity of clean capital Y_{K_c}, the second term represents the extra productivity of K_d when emissions are zero and the third term represents the declining marginal productivity of K_d. This corresponds to the assumption of a marginal abatement cost function that is linear as a proportion of output, as in Dietz and Venmans (2019), which allows us to avoid obtaining unrealistic infinite marginal abatement costs for levels of emissions close to zero. Note that despite the fact that both capital forms enter the Cobb-Douglass function additively, they are far from being perfect substitutes. The ratio of margainal productivities writes $\frac{Y_{K_d}}{Y_{K_c}} = \frac{\frac{\alpha}{K}+\phi\psi-\varphi\psi^2K_d}{\frac{\alpha}{K}}$. At zero emissions, dirty capital is 4.7 times more productive than clean capital. On the contrary, at BAU emissions, both forms of capital have the same productivity.

However, in this paper, emissions abatement is obtained by substituting dirty capital by clean capital. That is, the monetary loss of reducing emissions at the margin is equal to $Y_E = Y(\phi - \varphi E - \beta\delta) = \frac{1}{\psi}(Y_{K_d} - Y_{K_c})$. At an emissions level $E = \frac{\phi-\beta\delta}{\varphi}$, the marginal abatement cost is zero, both forms of capital are equally productive and, in the absence of carbon taxes, it would be optimal to choose a level of K_d equal to $\frac{\phi-\beta\delta}{\varphi\psi}$. At zero emissions, substitution of dirty capital by clean capital is costly and marginal abatement costs, as a proportion of production, are equal to $\phi - \beta\delta$.

Finally, defining effective labour $l = L_0 e^{(g_L+g)t}$ and capital per unit of effective labour $k_i = \frac{K_i}{L_0 e^{(g+g_L)t}}$ allows us to write production per unit of effective labour y as

$$y = A_0\,(k_d + k_c)^\alpha\, e^{\phi E - \frac{\varphi}{2}E^2 + \beta\dot{E} - \frac{\gamma}{2}S^2}. \tag{3}$$

[2]We use the subscript to denote the derivative of a variable with respect to the subscript variable. In this case, $Y_{K_c} \equiv \dfrac{\partial Y}{\partial K_c}$.

3.2 Capital dynamics

Four main factors govern the dynamics of the two capital stocks, k_d and k_g.

First, existing stocks of capital depreciate at a constant rate δ, which we assume to be equal between the two sectors. If left to depreciate, capital stocks would asymptotically converge towards zero.

Second, both types of capital stock can be expanded by investment, i_d and i_c. We assume investment adjustment costs are a positive and convex function of investment: $c(i) = \chi i^2$. For any amount i of disbursed investment expenditure, the value of newly installed capital will only be a proportion of investment expenditure equal to $i - \chi i^2$, where χ is the investment adjustment cost parameter. We assume χ to be the same in the two sectors. This makes large investment proportionally more expensive and creates a first incentive to smooth investment over time.

Third, we allow capital stocks to be converted at a cost from dirty to clean if optimal investment in the dirty sector is negative ($i_d < 0$)[3]. For convenience, we rename negative dirty investments as r. We assume repurposing costs to be a positive and convex function of total capital to be repurposed: $c(r) = \theta_1 r^{\theta_2}$.[4] For low levels of r, it is possible to transform dirty capital into clean capital at relatively low costs; that is, the monetary value of the new clean capital stock is a large proportion of that of the original dirty capital (although still less than 100%). As the capital stock to be repurposed r becomes larger, repurposing costs become more than proportionally larger, taking into account the fact that transforming a large proportion of the operating capital stock all at once would create impediments to the smooth functioning of the sector, as well as to the provision of intermediate inputs to downstream sectors, possibly also causing financial disruption. To offer an example, if only a small proportion of coal plants is repurposed every year to burn wood pellets instead, the value of the capital stock after the conversion will be only marginally lower than before; if all coal plants are repurposed at the same time, the entire sector will simultaneously shut down for a period, which might have consequences for coal firms, their electricity customers and their investors.This means that, for large enough values of stranding, $c(r) > r$.

Fourth, both capital stocks are subject to a geometric Brownian motion with correlation coefficient ρ. This stochastic process represents random changes to the value of capital and its productivity, similar to changes in the market valuation of a firm on stock markets, and allows us to capture uncertainty

[3]We do not consider the $i_c < 0$ case, so no conversion from dirty to clean capital is allowed.

[4]In the level equation, the penalty is $\hat{\theta}_t R^{\theta_2}$, with $\hat{\theta}_t = \theta L_0 e^{(1-\theta_2)(g+g_L)t}$ a time dependent coefficient. This scaling factor corresponds to the assumption that it is the relative disinvestment compared to the total stock of dirty capital that matters for repurposing costs, rather than the absolute level of disinvestment.

concerning future growth prospects.

Hence, defining capital per unit of effective labour as $k = \frac{K}{L_0 e^{(g_L+g)t}}$, the equations of motion for the dirty and clean capital stocks are

$$dk_d = \left(i_d - \chi_d i_d^2 - r - (\delta + g_L + g)k_d\right) dt + \sigma_d k_d dW_d, \tag{4}$$

$$dk_c = \left(i_c - \chi_c i_c^2 + r - \theta_1 r^{\theta_2} - (\delta + g_L + g)k_c\right) dt + \sigma_c k_c dW_c. \tag{5}$$

3.3 Emissions and tipping points

Emissions are proportional to the size of the dirty capital stock

$$E = \psi_t K_d \tag{6}$$

Temperature is a linear function of cumulative emissions $T = \zeta S$, where ζ is the slope coefficient, the transient climate response to cumulative CO2 emissions or TCRE (Collins et al., 2013). Temperature is subject to both a geometric brownian motion and a jump process. We interpret the brownian motion as capturing physical climate and damage uncertainties, while the jump process represents a tipping point in the climate system. The probability of tipping over a short time interval is λdt. The size of the tipping point is proportional to global warming at the time of tipping. We can thus write:

$$dT = \zeta E dt + \sigma_T T dW + T dP \tag{7}$$

ψ_t is the carbon intensity of dirty capital, which decreases at a constant rate g_ψ, therefore $E = \psi_0 L_0 k_d e^{(-g_\psi + g_L + g)t}$. The abatement speed is

$$\dot{E} = E\left[g + g_L - g_\psi + \frac{\dot{k}_d}{k_d}\right] = E\left[-\delta - g_\psi + \frac{i_d - \chi_d i_d^2 - r}{k_d}\right]. \tag{8}$$

Therefore, at zero dirty investment, emissions decrease at rate $\delta + g_\psi$, which is in some sense the 'speed limit' to decarbonization, beyond which dirty capital needs repurposed at a cost.We assume that the parameters of the abatement cost function, ϕ, φ, β decrease at a constant rate.Marginal abatement costs at zero emissions decrease at rate g_ϕ,due to exogenous technological improvement, which we set at 1%. We assume a faster decrease in the slope of the MAC curve, such that BAU emissions ($\frac{\phi}{\varphi}$) increase by rate $g_\phi - g_\varphi$, which we set at -1%. Technological improvement also decreases the penalty for abatement speed over time, again at 1% per year. We choose growth rates so to be able to cancel out time from the equations, except for the equation of motion of cumulative emissions[5]. We use an iterative procedure to approximate the cumulative emissions, explained in appendix 5. For a subset of runs, we show in appendix 8 results for other growth rates of the parameters. This will require time as a 4th state variable of the value function. Time's natural unboundedness is inconvenient in the estimation method. It is helpful to introduce a strictly monotonic transformation that maps t $\in[0,\infty)$ to $\tau = 1 - e^{-\zeta t} \in [0, 1)$. (Traeger 4-stated DICE, 2014)

3.4 Consumption and utility

Consumption per unit of effective labour is

$$c = y - i_d - i_c. \tag{9}$$

In the case of expected utility, relative risk aversion is also the inverse of the intertemporal elasticity of substitution. Therefore, high levels of risk aversion will automatically increase the discount rate and lead to lower climate ambition, all else being equal. In order to disentangle relative risk aversion from the intertemporal elasticity of substitution, we use recursive utility, a.k.a. Epstein-Zin-Weil preferences or Kreps-Porteus utility (Duffie and Epstein, 1992). The representative household maximises utility from per capita consumption, discounted at the pure time preference rate ρ, with standardized aggregator function f and value function V :

$$V(k_d, k_c, S, t) = max_{i_c i_d} \mathbb{E} \int_0^{\infty} -f(k_d, k_c, S, t, V, i_d, i_c) d\tau, \tag{10}$$

$$f(k_d, k_c, S, t, V, i_d, i_c) = \frac{1 - RRA}{1 - \eta} \left(\rho - g_L + (\eta - 1)\, g\right) V \left[\frac{(c)^{1-\eta}}{((1 - RRA)\, V)^{\frac{1-\eta}{1-RRA}}} - 1 \right]. \tag{11}$$

Appendix 2 shows the formulas for expected utility (the special case where $\eta = RRA$) and how we transform the present value function into a current value function. The Hamilton-Jacobi-Bellman equation is

$$max_E \left\{ f + \frac{1}{dt} \mathbb{E}[dV(k_d, k_c, S, t)] \right\} = 0. \tag{12}$$

Applying Ito's Lemma gives[6]

[5] More specifically, this requires $g_\phi = -\left(g + g_L - g_\psi\right); g_\varphi = -2\left(g + g_L - g_\psi\right); g_\beta = -\left(g + g_L - g_\psi\right)$.

[6] In the case of expected utility, a transform of the value function leads to a similar formula, where the factor $\frac{(1-RRA)V}{1-\eta} \left[\frac{c^{1-\eta}}{((1-RRA)V)^{\frac{1-\eta}{1-RRA}}} - 1 \right]$ is replaced by $\frac{c^{1-\eta}}{1-\eta}$.

$$
\begin{aligned}
0 \quad = \quad & max_{i_d,i_c}\{(\rho - g_L + (\eta-1)\,g)\,\frac{(1-RRA)\,V}{1-\eta}\left[\frac{c^{1-\eta}}{((1-RRA)\,V)^{\frac{1-\eta}{1-RRA}}}-1\right] \\
& +V_{k_d}\left(i_d - \chi i_d^2 - (\delta+g_L+g)k_d\right) + V_{k_c}\left(i_c - \chi i_c^2 - (\delta+g_L+g)k_c\right) \\
& +0.5V_{k_dk_d}k_d^2\sigma_d^2 + 0.5V_{k_ck_c}k_c^2\sigma_c^2 + V_{k_ck_d}\rho\sigma_c\sigma_d \\
& +V_S E + 0.5V_{SS}S^2\sigma_S^2 + \lambda_S\left[V(S+\xi S) - V\right] \\
& +V_t\}
\end{aligned}
$$

Applying the envelope theorem to the Bellman equation gives an analytical expression for the shadow price of cumulative emissions, i.e. the social cost of carbon expressed in utils. [7]

$$
\begin{aligned}
V_S \quad = \frac{1}{(\rho-g_L+(\eta-1)g)} \quad & \Phi\{\Psi c^{-\eta}y(-\gamma S) + V_{SS}E \\
& +V_{Sk_d}\left(i_d - \chi i_d^2 - (\delta+g_L+g)k_d\right) + V_{Sk_c}\left(i_c - \chi i_c^2 - (\delta+g_L+g)k_c\right) + 0.5V_{Sk_dk_d}k_d^2\sigma_d^2 \\
& +0.5V_{SSS}S^2\sigma_S^2 + V_{SS}S\sigma_S^2 + \lambda_S\left[V_S(S+\xi S) - V_S\right] + V_{tS}\},
\end{aligned}
$$

with $\Phi(V,c) = \frac{1-\eta}{1-RRA+(RRA-\eta)c^{1-\eta}((1-RRA)V)^{-\frac{1-\eta}{1-RRA}}}$ and $\Psi(V) = \frac{\rho-g_L+(\eta-1)g}{((1-RRA)V)^{\frac{1-\eta}{1-RRA}-1}}$, both of which are equal to one in the case of expected utility. Although we solve the problem stated in this section as a command optimum, the resulting solution corresponds to the outcome in a decentralized market economy provided carbon emissions are priced at an amount equal to the marginal damage cost of CO2 or in other words the social cost of carbon, and that there are no other externalities or market failures. Henceforth we will use the price of carbon and the SCC interchangeably. Figure 2. shows a decomposition of the different terms of the SCC.

The first term $c^{-\eta}y(-\gamma S)$ is the marginal damage from warming, typically modest in 2020 with 1°C of warming and independent of the fact that there is inertia. Inertia leads to two major differences regarding term 2 and 3. Firstly, in a model with inertia, emissions start at the current level of emissions, whereas in a model without inertia, optimal emissions jump from the current level of emissions to an optimal level that is much lower. This will increase the second term$-V_{SS}E$ proportionally. This second term can be considered as a temperature trend effect, because emissions are proportional to the time derivative of temperature. The temperature trend matters because the social cost of carbon depends also on future marginal damages. Note that this effect is especially

[7] The Social Cost of Carbon expressed in units of consumption (dollars) is $SCC = \frac{V_S}{V_{k_c}}L_0e^{(g+g_L)t}$

large for very convex damage functions. We show below that this term represents more than 100% of the carbon price in our model. Next, the third term $V_{Sk_d} * trend(k_d)$ is proportional to the speed of abatement and is absent in the model without inertia. We call this the temperature concavity effect, because the trend of dirty capital is proportional to the second time derivative of temperature. V_{SE} is typically negative, because the social cost of carbon is larger in a larger economy and emissions increase production. Therefore, the third term reduces the SCC, especially for productions functions with a large cross derivative in k_d and S (e.g. when both abatement costs and damages are proportional to production size). The intuition is that repurposing costs increase the temperature trend at the start, but if this temperature path is at the same time very concave, the difference with a model without repurposing costs will be limited. In the extreme case with a negligible penalty for abatement speed, this third term compensates the increased magnitude of the second term. The carbon price converges to the case without emission inertia. In the case of a binding penalty for abatement speed at the start, the effect on the second term will dominate the counteracting effect on the third term and the carbon price will be higher. This implies that the optimal emissions path incurs a nonzero penalty on abatement speed at the start of the optimal path (except if there are interaction terms in the cross derivative V_{SX}). Appendix 2 shows that the above features hold in any model with inertia in emissions.

4 Calibration of the model

4.1 Estimation of abatement costs

We calibrate abatement costs in our model using the results of the scenario ensemble underpinning the IPCC Special Report on Global Warming of 1.5°C (IPCC, 2018). The database aggregates the results of several modelling or inter-model comparison exercises involving twenty-four leading energy systems models[8]. Each model or project has a business-as-usual (BAU) reference scenario (e.g. one in which no mitigation policy is implemented), which may differ depending on underlying assumptions about population growth, GDP growth, the openness of economies etc. (e.g. differences in the SSP scenarios). Mitigation scenarios include various assumptions concerning the strength of mitigation policies and temperature objectives, ranging from below 1.5°C to more than 3°C. After excluding scenarios with warming above 3°C in 2100, we are left with 89 scenarios for which we have values for emissions, carbon prices, GDP and other variables.

[8]The database is available at https://data.ene.iiasa.ac.at/iamc-1.5c-explorer.

As mentioned in section 3.1, emissions when no abatement policy is implemented are equal to $E_{BAU} = \frac{\phi}{\varphi}$. We can thus define abatement $B = \frac{\phi}{\varphi} - E$ and $GDP_{BAU} = AL^{1-\alpha}K^{\alpha}e^{\frac{\phi^2}{2\varphi}-\frac{\gamma}{2}S^2}$.

This leads to the following expression for total abatement costs as a percentage of GDP:

$$ln\frac{GDP_t}{GDP_{BAU_t}} = -\frac{\varphi}{2}\left(E - \frac{\phi}{\varphi}\right)^2 + \beta\dot{E}. \quad (13)$$

[9]

Alternatively, we use carbon prices as a function of abatement in models and fit this to our marginal abatement cost function:

$$MAC = Y(\phi - \varphi E) = \varphi A \quad (14)$$

and $\phi = \varphi E_{2020}$.

Table 1 shows the results of a non-linear regression minimizing the squared deviation between both sides of equation 13.

4.2 Estimation of the depreciation rate δ_d

The depreciation rate of dirty capital δ_d represents the rate at which capital stocks reach their 'natural' end of life. This plays a particularly important role in our model as, together with the decline rate of emission intensity ψ (see next subsection), it defines the maximum speed at which the system can abate emissions without having to repurpose existing capital to low-carbon uses (hence incurring our non-linear repurposing costs). Given the heterogeneity of k_d, it is not straightforward to establish the correct value of δ_d. Ideally, one would have, for each type of dirty capital considered, data regarding the vintages of existing capital and the expected asset lifetime at the plant level, in order to be able to construct a timeline of future expected retirements, at least for currently operating stocks. However, while data are available for certain types of assets (see for instance Cui et al. (2019) for coal plants), for other activities data are much more scattered. For simplicity, we thus assume k_d to be internally homogenous and depreciating at a constant rate δ_d, which we assume to be equal to the inverse of average asset lifetime. The assets we consider here tend to have

[9] Defining investment net of stranding costs as $\hat{i_c} = i_c + r - \theta_1 r^{\theta_2}$ gives the following consumption equation $c = y - \hat{i_c} - \hat{i_d} - \theta_1 r^{\theta_2}$. Therefore, a model where repurposing costs create a loss in consumption or production rather than in clean investment results in the same optimization problem, provided that $\hat{i_d}$ is interpreted as net clean investments. Assuming that a relative loss in production is a good proxy for a loss in consumption we also fit the following equation wich includes stranding costs $GDP_t = GDP_{BAU_t}e^{-\frac{\varphi}{2}A^2+\beta\dot{E}} - \theta_1 r^{\theta_2} L_0 e^{(g+g_L)t}$, with $r = \frac{-\dot{E}-(g_\psi-\delta)E}{\psi_0 L_0 e^{\left(-g_\psi+g_L+g\right)t}}$ from equation 8

Table 1: Nonlinear regression estimating equation 13 for total abatement costs and equation **??** for marginal abatement costs. Only scenarios with temperatures below 3°C in 2100 are included. Stranding is set to zero for emissions below 5GtCO2. Carbon prices are winsorized at the 5% level (p5=2150$/tCO2); beyond that they are likely to be influenced by nonlinear spikes for a small amount of the most expensive abatement. Total abatement costs are winsorized at 1% (p1=14% of GDP_BAU). A weighting scheme using a disount rate of 3.3% is used to take into account that data far in the future are estimated with lower precision. We exogenously set $\theta_2 = 2.1; \delta_d = 0.04; \psi = 2.5$.

	E_BAU_endog	E_BAU_endog__GDP	E_BAU_endog2	E_BAU_55 E_BAU_55_incl
varphi	5.5e-06***	6.8e-06***	9.2e-06***	2.0e-05*** 2.0e-05***
phi	5.5e-04***	6.1e-04***	7.3e-04***	
beta	-.0015	.0011	9.1e-04	.0042*** .0033***
theta1	34.23**		22.05***	9.559 16.88***
obs	1012	1012	1033	1033 2090
ll	-3671	-3673	2337	2324 5173
bic	7369	7367	-4647	-4627 -1.0e+04
aic	7349	7352	-4667	-4642 -1.0e+04
				legend: * p<.1; ** p<.05; *** p

a long lifetime. Coal plants, for instance, have an average lifetime of 46 years (Cui et al., 2019), with many of them lasting up to 60 or 70 years. Similarly, IEA (2019) uses a reference lifetime of coal plants of 50 years. Other carbon-related assets like transport infrastructure and buildings have even longer life-spans, reaching 100 or 200 years, while manufacturing equipment is estimated to last between 10 and 40 years (Hallegatte, 2009; IEA, 2002). We choose a central parameter value for δ_d equal to 4%, corresponding to an average asset mean lifetime of 25 years. This is roughly in line with depreciation rates adopted by models including long-lived physical assets (see for instance van der Ploeg and Rezai, 2020; Vogt-Schilb et al., 2018; Baldwin et al., 2020; Coulomb et al., 2019)[10]. For simplicity and to facilitate the dynamics of the model, we also set $\delta_c = \delta_d$.

4.3 Estimation of the carbon intensity of dirty assets ψ

We define ψ as the carbon intensity of dirty assets. To calibrate ψ, we start by considering starting 2020 values for two variables: 50 gigatonnes for CO2

[10]For instance,van der Ploeg and Rezai (2020) use a depreciation rate of 5% for exploration capital; Coulomb et al. (2019) use 3.33% for electricity generation plants; Baldwin et al. (2020) use 5% for general capital stock and 2.5% for fossil energy capital; Vogt-Schilb et al. (2018) use 4% for industry and 2.5% for energy capital.

emissions and 335 trillion \$ for the global stock of capital. We then need to estimate the proportion of the capital stock that can be considered 'dirty'. To obtain this value, we use the World Input-Output Database (WIOD), containing sectoral capital stock data for 43 countries at a NACE level 2 disaggregation (Timmer et al., 2015). Certain sectors are labelled dirty because of their fossil-dependent technological basis (e.g. mining, coke, chemicals, plastics, metals, electricity, transportation). Where possible, we employ data from another multi-regional input-output dataset called Exiobase (Stadler et al., 2018) to further disaggregate stocks and isolate those directly related to fossil fuels (e.g. the capital stock used for the exploration and extraction of coal, gas and oil) from those with less of a connection to fossil fuels (e.g. mining of non-ferrous metals and other materials)[11]. With this method, we estimate that approximately 8% of the capital stock can be considered dirty, i.e. $K_D \approx 25$ trillion \$. We thus estimate $\psi = \frac{50}{25000} = 0.002$ GtCO2/billion\$. .

This is in line with the emission intensity of coal plants, one of the dirty assets that are most at risk of being stranded. 25% of world emissions are related to coal powerplants. IEA estimates that 5GtCO2 can be abated in 2040 by stranding 1500GW. The IEA 2019 estimates a cost (at the beginning of lifetime) of 0.7 to 2.5 billion \$/GW depending on the region (World Energy Outlook 2019, p 237, Lazard IEA estimates that 5GtCO2 can be avoided in 2040 by stranding 1500GW.). Using the middle of the range value of 1.66 billion \$/GW, we have $\psi = 5GtCO_2/1500GW * 1GW/1.66billion\$ = 0.002\ GtCO_2/\$Billion$.

4.4 Costs of converting dirty capital

Decommissioning old coal-fired power plants is not as costly as decommissioning new coal plants. Therefore we fit θ_1 and θ_2 on the age distribution of coal plants in the IEA and Cui et al. 2019. The lifetime of stranded coal plants in the IEA is 25 years and older. This gives 1/4 of the new value for all stranded plants if the simple rule of stranding after 25 years is applied. However retrofitting is a more interesting option, also captured by our variables θ_1 and theta θ_2. IEA mentions at least 1 billion \$ per GW retrofitted with CCS. only millions per GW for retrofitting to biomass.

5 Results

In this section we will present and discuss the results of our numerical simulations. We start by comparing our baseline scenario with a scenario using the

[11]To perform this disaggregation we assume that the share of fossil-related capital stock on the total of a specific sector is the same as the fossil-related output on the total sectoral output.

same set of central parameter values but assuming no inertia in capital repurposing. This is crucial to show how introducing stranding costs and related frictions modifies the optimal abatement plan. We then focus on the consequences of delaying policy action to 2030 or 2040 in section 5.2 and explore the implications of imposing a constraint on capital reconversion in section 5.3. Finally, we compare scenarios with different assumptions regarding the rate of relative risk aversion in section 5.4. Since our model is stochastic, future values depend on realisations of the stochastics. Therefore, in all figures, we report means of 1000 Monte Carlo runs. A probability distribution of the baseline scenario can be found in appendix 6.

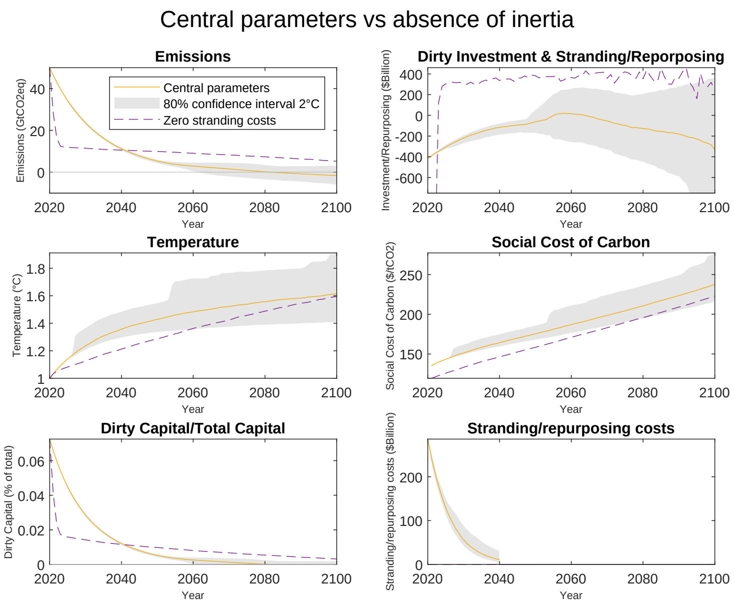
5.1 The effect of inertia in emissions due to stranding cost

The purpose of this section is to compare the optimal investment and emission paths resulting from two scenarios. The first scenario is based on the set of central parameter values presented in Table 2. The second scenario is identical to the first, except for the absence of frictions in capital stock repurposing, i.e. zero stranding costs. K_d can be immediately converted into K_c without losing any of its value (i.e. $c(r) = 0$).

The results of the comparison are striking, and shown in figure 1. In the scenario without stranding, emissions start at 50 GtCO2-eq in 2020, but immediately jump to 15 GtCO2-eq and decrease very slowly thereafter. This rapid drop in emissions is possible thanks to the immediate repurposing of a large proportion (around70%) of the outstanding stock of dirty assets. The remaining part of K_d, since its emissions are increasingly costly to abate, is gradually replaced by new K_c as it reaches the end of its natural life. Temperature increases to reach a steady state warming of 2.5°C only after a few centuries. The social cost of carbon moves from just below 100$ in 2020 to around twice this value in 2100.

Introducing real-world frictions in repurposing capital stocks leads to a starkly different optimal emission path. An immediate jump to a lower level of emissions is no longer optimal, because it would incur large stranding costs. As a matter of fact, any rate of decrease of K_d higher than $\delta + g_\psi$ (around 5.5% according to our calibration) will create increasingly relevant repurposing costs. As a result, optimal emission paths are smoother and reach the unconstrained path only around 2040. After 2040, the constrained emission path is consistently lower than the unconstrained one. This is because, since the steady state of the constrained model is the same as the unconstrained one, and we assume temperature to be proportional to cumulative emissions, both trajectories have the same cumulative emissions. However, a larger proportion of the carbon

Figure 1: Results for the model with central parameters compared to a model without stranding costs ($\theta_1 = 0$)



budget has been consumed by the constrained model before 2040, thus forcing emissions to be lower after that. By the end of the century, the optimal path with stranding costs is close to zero emissions, whereas the unconstrained path has positive emissions for another century. Less abrupt dynamics are also observed in optimal investments/repurposing of dirty capital. Repurposing starts immediatelyat around \$300 billion of assets per year and gradually decreases until it ends around 2045, with dirty capital representing approximately 3% of the total. After 2040, the optimal abatement speed is slower than 5.5% year, which in turn corresponds to slightly positive levels of dirty investment. One can think of the remaining dirty capital as gas turbines required for the stability of the electricity network, or airplanes. Since its emissions are costly to abate, dirty capital is replaced by more dirty capital when it reaches its end of life.

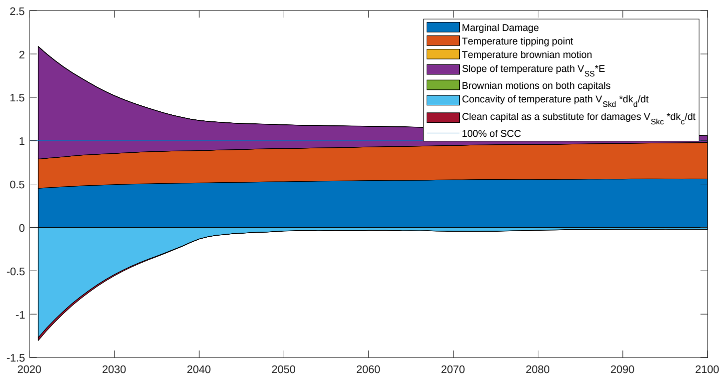
Since stranding costs lead to larger emissions at the start, the temperature trajectory is higher compared to the unconstrained scenario. After 2040, when stranding costs lead to lower emissions, both temperature paths slowly converge to the same steady state. In 2100 temperature is still 0.05°C higher. The optimal carbon tax/social cost of carbon is 14% higher in the case of non-zero stranding costs, in line with our analytical results above. From a damages perspective, this is logical, because temperatures (and therefore marginal damages) are higher over the entire trajectory. Since temperatures converge only after centuries, the difference between carbon prices is fairly stable over time. The higher carbon price can also be understood from an abatement cost perspective. Stranding/repurposing costs are added on top of marginal abatement costs for greenfield investments. A higher price is required to incentivize companies not only to switch to green investments, but also to incur costs for repurposing or stranding the existing dirty capital stock.

. As shown above, the optimal path for the model with stranding/repurposing costs must start by repurposing dirty capital.

Figure 1.d shows total repurposing/stranding costs. On an optimal emissions trajectory starting in 2020, stranding costs are fairly limited, at \$25 billion. This contextualises the \$300 billion repurposed/stranded assets in 2020. Since in 2020 only 2% of dirty capital is repurposed/stranded, only those assets that can be repurposed and keep 90% of their value are repurposed.

Figure 2 shows the decomposition of the SCC. As the temperautre path of the model with stranding costs has higher emissions in 2020, the temperature path has a much steeper slope in 2020 (see Figure 1.c). Therefore, the SCC has a large term related to the temperature trend, more than 100% of the SCC. On the other hand, the temperature path in 2020 is concave, wich limits its deviation from the model without stranding costs. This leads to a large negative term related to the concavity of the temperautre path. Our damage function is quadratic in

Figure 2: Decomposition of the social cost of carbon. Different components correspond to the relative size of the terms in equation **??**, for central parameters and optimal policy starting in 2020.

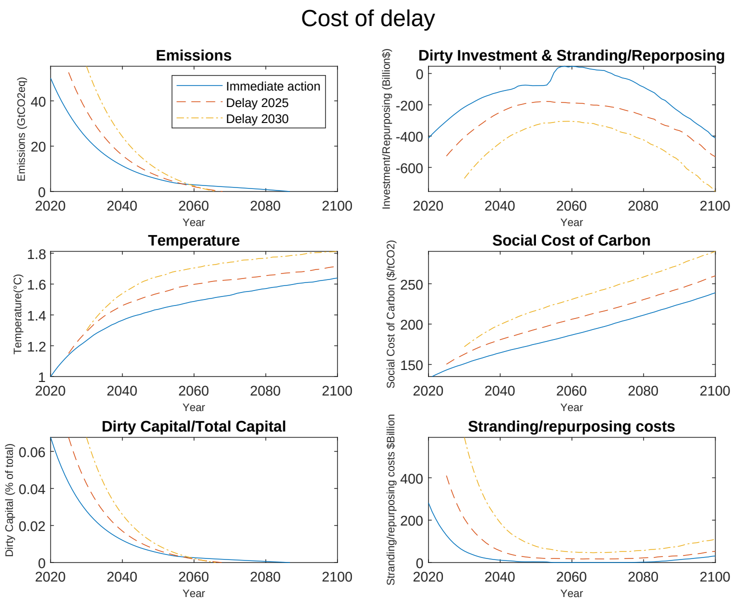


temperature, in other words the third derivative of consumption with respect to S is zero, leading to a negligible risk premium for gradual uncertainty on temperature (the brownian motion on S). Note that the risk primium would be larger if the policy would not optimally and instantaneously adapt to new information on temperature. By contrast, the tipping point on temperature (jump process) represents 45% of the value of the SCC. Note that the jump proces increases both the expected value of damages as its uncertainty, which explains the large contrast with the negligible effect of the brownian motion.

5.2 The cost of delay

Figure 3 shows the effect of delaying the introduction of a carbon price by a decade to 2030 or two decades to 2040. Recall that we define optimality as maximizing welfare including both damages and abatement costs, not minimizing abatement costs to stay within a given level of warming this century. As a result, temperatures in 2100 are higher for delayed policies, although they result in the same steady state warming of 2.5°C in the very long run. Delayed policies lead to a much steeper decrease of emissions, with an abatement speed beyond $\delta_d + g_\psi$ (the dirty capital depreciation rate plus the rate of reduction of the carbon intensity of dirty capital). Therefore, dirty investment is negative over

Figure 3: The cost of delay. Optimal policy starting in 2020, 2030 or 2040. Before the optimal trajectory, dirty capital is assumed to be constant per unit of effective labor. .



the entire path. If the optimal trajectory starts in 2030, \$500 billion per year of dirty capital is stranded in real terms, resulting in a yearly cost of \$45 billion. If the optimal trajectory starts in 2040, \$800 billion per year is stranded, resulting in a yearly cost of \$82 billion. The sum of all stranding cost in the first decade is \$551 billion. [12]

Delayed scenario, have two effects on temperature. Firstly, the absence of effort leads to a higher temperature at the time when the optimal emissions path starts, in 2030 and 2040 respectively. This is the only effect in the absence of emissions inertia. Secondly, delay locks in more dirty capital, leading to a steeper increase of temperature starting in 2030 and 2040 respectively. In other words the shape of the temperature path for a given intial temperature has changed. This is an effect that only exists in the presence of emissions inertia. Technically, in a model without inertia, the value function only depends on cumulative emissions (and other state variables) but is independent of the level of emissions, because emissions is a decision variable. As a result, a temperature path is only determined by the initial temperature, not by its history. By contrast, in a model with inertia, emissions is a state variable, affecting the value function and therefore affecting the future path. The temperature path is not only affected by its initial value, but also by its history, i.e. we see hysteresis.

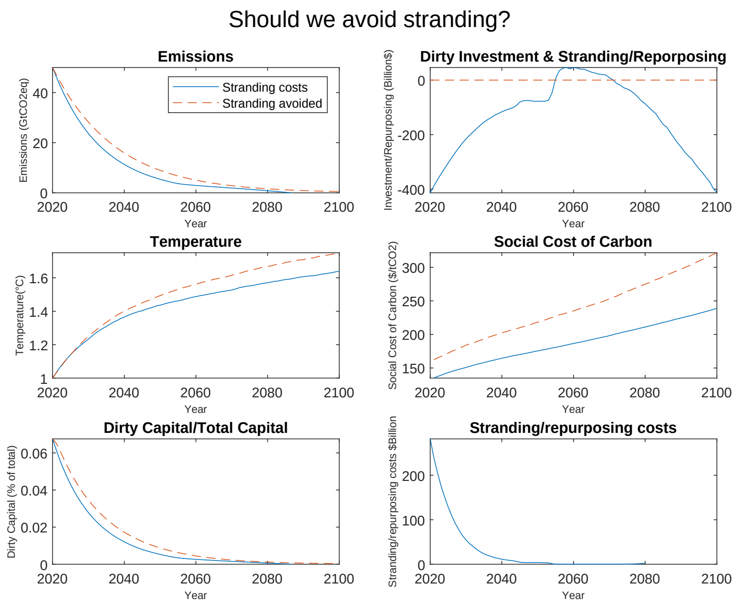
The different temperature trajectories also lead to different carbon price scenarios. For a model where the temperature path is independent of delay (only depends on initial temperature), there is only one carbon price path (and the initial carbon price depends on initial temperature). By contrast, we show that the carbon price path is not only shifted to the left, it has also a different shape, slightly steeper at the start, to compensate for the extra cost of stranding/repurposing. Postponing action until 2040 leads to a loss of the welfare functional of 0.7%, despite arriving at the same steady state temperautre.

5.3 Should we avoid stranding? Feebates versus a carbon tax

Figure 4 compares an optimal policy with a policy imposing $i_d \geq 0$. Stranding can for example be avoided by the use of feebates, i.e. policies which tax dirty investment and subsidize clean investment instead of taxing emissions. As a result, emissions decrease at rate $\delta_d + g_\psi$, i.e. 5.5% per year until 2055, instead of

[12] Our MAC function allows for negative emissions and assumes the same linear increasing MAC for abatement beyond zero emissions. This implies that total capital (which is 100% clean by then) equals $k_d + k_c$ which are now both abstract concepts. The total investment for total capital writes $i_c - \theta_1(r)^{\theta_2}$, where the second term is a penalty for negative emissions. As a conclusion, the term $\theta_1(r)^{\theta_2}$ represents stranding costs in the positive domain of emissions, and a penalty for abatement speed in the negative domain of emissions. Note that maintaining k_d at a constant negative level also requires a constant level of r due to the term $(\delta + g + g_L)k_d$ in equation 4.

Figure 4: Optimal policy versus a policy where stranding is avoided.



a rate beyond 5.5% in the standard model. This leads to a temperature trajectory that is more or less 2% higher from 2040 onwards. This higher temperature logically leads to a higher carbon price. The carbon price in the model is not the carbon tax, but the marginal cost of the most expensive abatement technology. The carbon price is on average 8% higher over the period 2020-2100. This leads to an increase in total abatement costs that exceeds the stranding/repurposing costs in the standard model. Expressed in terms of the present value of welfare, avoiding stranding/repurposing reduces the welfare by $25 Trillion (0.2%).

5.4 The effect of risk aversion

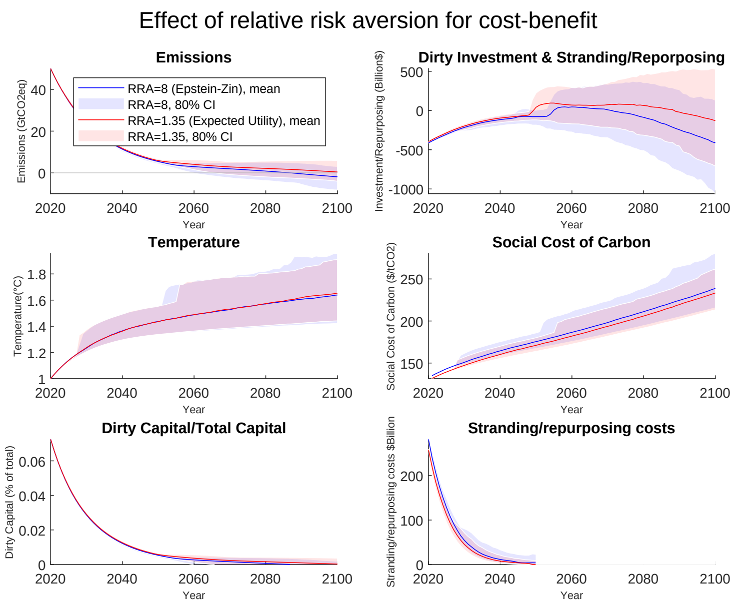
Figure 5 shows the different optimal paths under relative risk aversion of 1.35 and 8, keeping the intertemporal elasticity of substitution constant at 1/1.35). In other words the figure compares the optimal paths under expected utility and Epstein-Zin-Weil utility. It shows that the initial emissions path is not much affected by the uncertainty. It is optimal in both cases to decrease emissions as fast as possible, while limiting the costs of stranding/repurposing. Even in the event of tipping points, emissions paths are similar to paths where tipping points come later, due to fastly rising repurposing costs. However, once repurposing costs are zero, confidence intervals widen due the stochastics of the model and the risk-averse model has lower emissions by 2 or 3GtCO2-eq per year. This emissions path requires a carbon price that is 5% higher on average over the next century, slightly less so in 2020. Also, in the more risk avers scenario, the uncertainty range of the future carbon price is larger.

5.5 Cost-minimization to stay within 2°C

Many integrated assesment models apply a cost-effectiveness analysis, minimizing costs to stay within a maximum warming of 2°C (cfr Paris Agreement). In a deterministic setting, cost-minimzation can be obtained by using a damage function that is zero until 2°C and infinite thereafter. In a probabilistic setting however, the maximum warming is often interpreted as a probabilistic constraint.[13] We model this by solving the Bellman equation using a damage function that is negligible until 2°C and raises fast beyond 2°C (at the 8th power), reaching 4% GDP loss at 2.5°C. As a result, our model has a mean steady state warming of 2°C, although damages are zero until 2°C. It is not optimal to increase temperature beyond 2°C, because the mean steady state

[13] If our model were to stay below 2°C in any circumstance, mean warming would exceed 1.5°C to ensure that even in the event of atipping point, the constraint is not exceeded. Moreover, this steady state warming would be very sensitive to precision with which the stochastic processes are modelled (even a modest brownian motion has a non-zero chance of giving an extra 0,5°C).

Figure 5: Optimal policy under constant relative risk aversion of 1.35 and 8. Both models have the same intertemporal elasticity of substitution of 1/1.35. As a result, the relative risk aversion of 1.35 corresponds to a model with expected utility. The figure is made as a Monte Carlo simulation with 1000 scenarios. Each scenarios is a unique draw from the distribution of brownian motions and jumps. The 80% confidence intervals correspond to internal variation, where policy optimally adapts to shock in the economy and in the climate system.



warming of 2°C includes also scenarios with more warming due to the jump proces and brownian motion on temperature. The high damages beyond the 2°C is in the spirit of the Paris Agreement, assuming that warming beyond 2°C is dangerous.

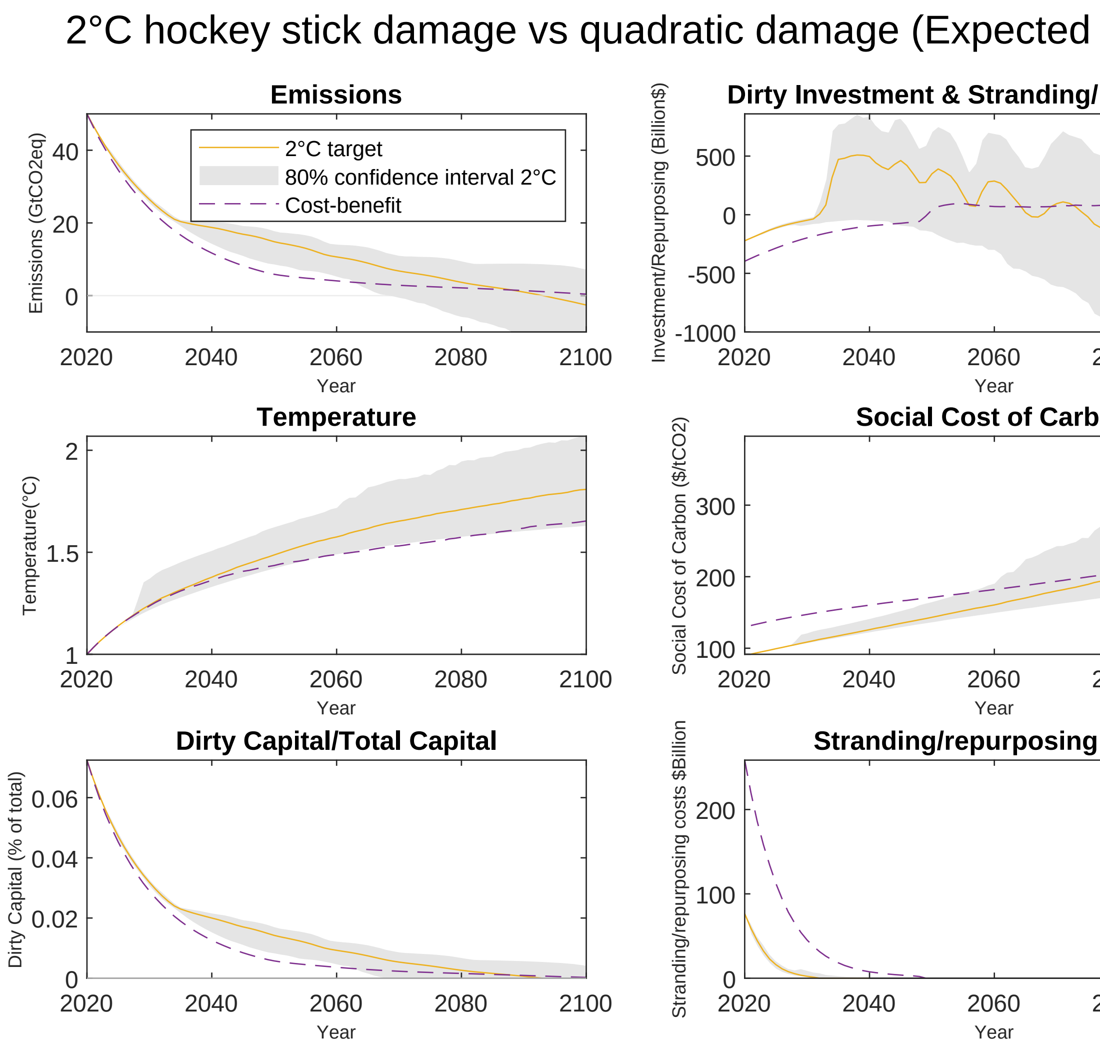
Figure "2degHockyStick_n945" shows the results for both the cost-effectiveness and cost-benefit analysis. Since the cost-effectiveness model is insensitive to the timing of the damages, the model has more emissions during the first decades compared to a cost-benefit analysis.[14] For example, in 2050, the cost-effectiveness analysis has more than double the emissions of the cost-benefit analysis. The larger early emissions of cost-effectiveness analysis are compensated by lower emissions at the end of the century. The tendency for more emissions at the start also results in 50% less stranding/repurposing in the cost-effectiveness model.

6 Conclusions

In this paper, we have studied the optimal transition to a low-carbon economy, recognizing that the transition is not going to be seamless. Many IAMs and studies into optimal carbon emissions paths do assume a seamless transition, because they assume way adjustment costs and frictions that constitute obstacles to rapidly decumulating dirty capital on the one hand and accumulating clean capital on the other hand. By contrast, our model includes (i) adjustment costs pertaining to investment in clean (and dirty) capital, (ii) costs associated with converting/repurposing dirty capital as clean capital, and (iii) abatement costs that are an increasing function of the speed of abatement, capturing bottlenecks in clean technology innovation.

We find that introducing costs to repurposing dirty capital as clean leads to a slower decline of emissions initially, which, given temperature is a linear function of cumulative CO2 emissions, leads to the need for greater emissions reductions later. These frictions thus optimally push abatement effort further into the future, with consequences for 21st century temperature, but not steady-state temperature. However, even though emissions decline more slowly given these frictions, carbon prices are higher, because dirty capital owners need an incentive to incur those extra costs of repurposing that are optimal from society's point of view. The share of genuinely dirty capital in the economy – capital that requires fossil fuels to produce goods and services – is actually quite small, a point that can be substantiated using data from multi-regional input-output tables, which likely overstate the share of dirty capital, even. Therefore the

[14]Our cost-benefit analysis has a similar long term effect reaching 2.1°C in the year 2300.


2°C hockey stick damage vs quadratic damage (Expected
Emissions
Emissions (GtCO2eq)
2°C target
80% confidence interval 2°C
Cost-benefit
Year
Dirty Investment & Stranding/
Investment/Repurposing (Billion$)
Temperature
Temperature(°C)
Social Cost of Carb
Social Cost of Carbon ($/tCO2)
Dirty Capital/Total Capital
Dirty Capital (% of total)
Stranding/repurposing
Stranding/repurposing costs $Billion

value of assets repurposed or 'stranded' in this way is rather small.

Delaying the introduction of the globally optimal carbon tax by a decade or two, which we can think of as an approximation of continued difficulties in negotiating a sufficiently strong set of national climate commitments and implementing them, increases the value of stranded assets in real terms, even in a set-up such as ours where there is no temperature constraint per se, rather the planner maximizes social welfare taking into account both damages and abatement costs. Delaying climate action by 2 decades leads to a decrease in welfare of 0.71%, a cost of $108.5 trillion dollar at current prices.

We could seek to avoid stranding assets. Instead of imposing a Pigouvian tax on CO2 emissions, governments could institute a combination of taxes on dirty capital investment and subsidies on clean capital investment. We quantify the welfare cost of this second-best policy and we find that welfare is reduced by 0.2%, i.e. $25 Trillion at current prices.

Appendix 1 : Parameters

Table 2: Parameter values for the central scenario

Symbol	Value	Remark
A	4.092	Match 2019 GDP from the World Bank (PPP, current international$)
L	7.403	DICE
K_{d_0}	25	8% of total capital
K_{c_0}	310	Total capital per unit of effective labour should be close to steady state.
α	0.3	As in DICE
ϕ	0.0016	Our fit to the IPCC database for the report on 1.5°C
φ	0.000032	Our fit to the IPCC database for the report on 1,5°C
γ_T	0.014	By assumption. 2°C warming reduces GDP by 3%. Steady state of 2.5°C.
ζ	0.0006	γ in text is $\zeta^2\gamma_T$. IPCC 5th assessment report. Matthews et al. 2009.
S_0	2083	1°C warming in 2020, IPCC report on 1.5°C.
χ_1	**0.001**	
χ_2	**0.001**	
θ_1	**1.8**	Our fit to the IPCC database for the report on 1.5°C
θ_2	**2.1**	By assumption.
δ	**0.4**	See text
ψ	2	
g_ψ	**0.015**	1.5% is Mean for period 1995-2018 according to BP energy outlook 2020. Also used in DICE.
g_L	0.005	United Nations, 2017.
g	0.02	By assumption
ρ	0.011	Drupp, et al. 2018
η	1.35	Drupp, et al. 2018
RRA	**8**	
σ_d^2	**4e-4**	Hambel, Kraft, van der Ploeg 2019
σ_c^2	**4e-4**	
ρ_{dc}	**0.5**	
σ_S^2	**1e-6**	10% chance of having more than 5GtCO2 extra in 2020.
λ	**0.15**	Jump increases temperature by 15%. A larger jump is technically very challenging to model, it brings S outside its range with Chebychev nodes.
$jumpProb$	**0.015**	By assumption. On average 1 tipping point per 66 years.

References

Annicchiarico, B., Di Dio, F., Feb. 2016. GHG Emissions Control and Monetary Policy. Environmental and Resource Economics, 1–29.
URL https://link.springer.com/article/10.1007/s10640-016-0007-5

Baldwin, E., Cai, Y., Kuralbayeva, K., Mar. 2020. To build or not to build? Capital stocks and climate policy. Journal of Environmental Economics and

Management 100, 102235.
URL http://www.sciencedirect.com/science/article/pii/S0095069617308173

Battiston, S., Mandel, A., Monasterolo, I., Schütze, F., Visentin, G., Mar. 2017. A climate stress-test of the financial system. Nature Climate Change 7, 283–288.
URL http://www.nature.com/nclimate/journal/vaop/ncurrent/full/nclimate3255.html

Bretschger, L., Soretz, S., Mar. 2018. Stranded Assets: How Policy Uncertainty Affects Capital, Growth, and the Environment. SSRN Scholarly Paper ID 3157079, Social Science Research Network, Rochester, NY.
URL https://papers.ssrn.com/abstract=3157079

Burke, M., Hsiang, S. M., Miguel, E., 2015. Global non-linear effect of temperature on economic production. Nature 527, 235–239.

Cahen-Fourot, L., Campiglio, E., Dawkins, E., Godin, A., Kemp-Benedict, E., 2019. Capital stranding cascades: The impact of decarbonisation on productive asset utilisation. Working Paper 18, WU Institute for Ecological Economics, Vienna.

Campiglio, E., Monnin, P., von Jagow, A., 2019. Climate risks in financial assets. Discussion Note 2019/2, Council on Economic Policies, Zurich.

Carbon Tracker, 2011. Unburnable carbon: Are the world's financial markets carrying a carbon bubble? Tech. rep., Carbon Tracker, London.

Carney, M., Villeroy de Galhau, F., Elderson, F., 2019. Open letter on climate-related financial risks. Tech. rep., Bank of England, London.
URL https://www.bankofengland.co.uk/news/2019/april/open-letter-on-climate-related-finan

Collins, M., Knutti, R., Arblaster, J., Dufresne, J.-L., Fichefet, T., Friedlingstein, P., Gao, X., Gutowski, W., Johns, T., Krinner, G., Shongwe, M., Tebaldi, C., A.J., W., Wehner, M., 2013. Long-term climate change: projections, commitments and irreversibility. In: Stocker, T., Qin, C., Plattner, G.-K., Tignor, M., Allen, S., Boschung, J., Nauels, A., Xia, Y., Bex, V., Midgeley, P. (Eds.), Climate Change 2013: The Physical Science Basis. Contribution of Working Group I to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change. Cambridge University Press, Cambridge, UK and New York, NY, USA.

Coulomb, R., Lecuyer, O., Vogt-Schilb, A., Jun. 2019. Optimal Transition from Coal to Gas and Renewable Power Under Capacity Constraints and Adjustment Costs. Environmental and Resource Economics 73 (2), 557–590.
URL https://doi.org/10.1007/s10640-018-0274-4

Cui, R. Y., Hultman, N., Edwards, M. R., He, L., Sen, A., Surana, K., McJeon, H., Iyer, G., Patel, P., Yu, S., Nace, T., Shearer, C., Oct. 2019. Quantifying operational lifetimes for coal power plants under the Paris goals. Nature Communications 10 (1), 1–9.
URL https://www.nature.com/articles/s41467-019-12618-3

Dafermos, Y., Nikolaidi, M., Galanis, G., Oct. 2018. Climate Change, Financial Stability and Monetary Policy. Ecological Economics 152, 219–234.
URL http://www.sciencedirect.com/science/article/pii/S0921800917315161

Davis, S. J., Socolow, R. H., 2014. Commitment accounting of co2 emissions. Environmental Research Letters 9 (8), 084018.

Dell, M., Jones, B. F., Olken, B. A., 2012. Temperature shocks and economic growth: Evidence from the last half century. American Economic Journal: Macroeconomics 4 (3), 66–95.

Dietz, S., Venmans, F., Jul. 2019. Cumulative carbon emissions and economic policy: In search of general principles. Journal of Environmental Economics and Management 96, 108–129.
URL http://www.sciencedirect.com/science/article/pii/S0095069618302122

Duffie, D., Epstein, L. G., Jul. 1992. Asset Pricing with Stochastic Differential Utility. The Review of Financial Studies 5 (3), 411–436, publisher: Oxford Academic.
URL https://academic.oup.com/rfs/article/5/3/411/1576347

Energy Transitions Commission, 2018. Mission possible: Reaching net-zero carbon emissions from harder-to-abate sectors by mid-century. Tech. rep.

Epstein, L. G., Zin, S. E., 1989. Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework. Econometrica 57 (4), 937–969, publisher: [Wiley, Econometric Society].
URL https://www.jstor.org/stable/1913778

Gould, J. P., 1968. Adjustment Costs in the Theory of Investment of the Firm. The Review of Economic Studies 35 (1), 47–55, publisher: JSTOR.

Ha-Duong, M., Grubb, M. J., Hourcade, J.-C., 1997. Influence of socioeconomic inertia and uncertainty on optimal CO 2-emission abatement. Nature 390 (6657), 270–273, publisher: Nature Publishing Group.

Hallegatte, S., May 2009. Strategies to adapt to an uncertain climate change. Global Environmental Change 19 (2), 240–247.
URL http://www.sciencedirect.com/science/article/pii/S0959378008001192

Hambel, C., Kraft, H., van der Ploeg, F., 2020. Asset Pricing and Decarbonization: Diversification versus Climate Action. Discussion Paper Series 901, Department of Economics, University of Oxford.

IEA, 2002. Beyond Kyoto. Energy Dynamics and Climate Stabilisation. Tech. rep., International Energy Agency, Paris.

IEA, 2019. World Energy Outlook 2019. Tech. rep., International Energy Agency, Paris.

IPCC, 2014. Climate Change 2014: Synthesis Report. Contribution of Working Groups I, II and III to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change. Tech. rep., Intergovernmental Panel on Climate Change.

IPCC, 2018. Global warming of 1.5 °C. Tech. rep., Intergovernmental Panel on Climate Change, Geneva.

Jorgenson, D., 1967. The theory of investment behavior. In: Determinants of investment behavior. NBER, pp. 129–175.

Karydas, C., Xepapadeas, A., 2019. Pricing climate change risks: CAPM with rare disasters and stochastic probabilities. CER-ETH Working Paper Series Working Paper 19, 311.

Kemp-Benedict, E., Nov. 2018. Investing in a Green Transition. Ecological Economics 153, 218–236.
URL http://www.sciencedirect.com/science/article/pii/S092180091731399X

Lamperti, F., Bosetti, V., Roventini, A., Tavoni, M., Nov. 2019. The public costs of climate-induced financial instability. Nature Climate Change 9 (11), 829–833.
URL https://www.nature.com/articles/s41558-019-0607-5

Lemoine, D., Rudik, I., 2017. Managing climate change under uncertainty: Recursive integrated assessment at an inflection point. Annual Review of Resource Economics 9, 117–142.

Lenton, T. M., Held, H., Kriegler, E., Hall, J. W., Lucht, W., Rahmstorf, S., Schellnhuber, H. J., 2008. Tipping elements in the earth's climate system. Proceedings of the National Academy of Sciences 105 (6), 1786–1793.

Lenton, T. M., Rockström, J., Gaffney, O., Rahmstorf, S., Richardson, K., Steffen, W., Schellnhuber, H. J., 2019. Climate tipping points - too risky to bet against. Nature 575.

Lucas Jr, R. E., 1967. Adjustment costs and the theory of supply. Journal of political economy 75 (4, Part 1), 321–334, publisher: The University of Chicago Press.

McGlade, C., Ekins, P., Jan. 2015. The geographical distribution of fossil fuels unused when limiting global warming to 2C. Nature 517 (7533), 187–190.
URL http://www.nature.com/nature/journal/v517/n7533/abs/nature14016.html

Meinshausen, M., Meinshausen, N., Hare, W., Raper, S. C. B., Frieler, K., Knutti, R., Frame, D. J., Allen, M. R., Apr. 2009. Greenhouse-gas emission targets for limiting global warming to 2C. Nature 458 (7242), 1158–1162.

Perez, C., Jan. 2010. Technological revolutions and techno-economic paradigms. Cambridge Journal of Economics 34 (1), 185–202.
URL https://academic.oup.com/cje/article/34/1/185/1699623/Technological-revolutions-and-

Pfeiffer, A., Hepburn, C., Vogt-Schilb, A., Caldecott, B., 2018. Committed emissions from existing and planned power plants and asset stranding required to meet the paris agreement. Environmental Research Letters 13 (5), 054019.

Pfeiffer, A., Millar, R., Hepburn, C., Beinhocker, E., 2016. The '2 C capital stock' for electricity generation: Committed cumulative carbon emissions from the electricity generation sector and the transition to a green economy. Applied Energy 179, 1395–1408.
URL http://www.sciencedirect.com/science/article/pii/S0306261916302495

Rozenberg, J., Vogt-Schilb, A., Hallegatte, S., Mar. 2020. Instrument choice and stranded assets in the transition to clean capital. Journal of Environmental Economics and Management 100, 102183.
URL http://www.sciencedirect.com/science/article/pii/S009506961730623X

Semieniuk, G., Campiglio, E., Mercure, J.-F., Volz, U., Edwards, N. R., 2020. Low-carbon transition risks for finance. WIREs Climate Change Forthcoming.

Sen, S., von Schickfus, M.-T., 2020. Climate policy, stranded assets, and investors' expectations. Journal of Environmental Economics and Management 100, 102277.

Stadler, K., Wood, R., Bulavskaya, T., Södersten, C.-J., Simas, M., Schmidt, S., Usubiaga, A., Acosta-Fernández, J., Kuenen, J., Bruckner, M., Giljum, S., Lutter, S., Merciai, S., Schmidt, J. H., Theurl, M. C., Plutzar, C., Kastner, T., Eisenmenger, N., Erb, K.-H., Koning, A. d., Tukker, A., 2018. EXIOBASE 3: Developing a Time Series of Detailed Environmentally Extended Multi-Regional Input-Output Tables. Journal of Industrial Ecology 22 (3), 502–515,

_eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1111/jiec.12715.
URL `https://onlinelibrary.wiley.com/doi/abs/10.1111/jiec.12715`

Timmer, M. P., Dietzenbacher, E., Los, B., Stehrer, R., Vries, G. J. d., 2015. An Illustrated User Guide to the World Input–Output Database: the Case of Global Automotive Production. Review of International Economics 23 (3), 575–605, _eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1111/roie.12178.
URL `https://onlinelibrary.wiley.com/doi/abs/10.1111/roie.12178`

UNFCCC, Jan. 2016. Report of the Conference of the Parties on its twenty-first session, held in Paris from 30 November to 13 December 2015. Addendum. Part two: Action taken by the Conference of the Parties at its twenty-first session. Tech. Rep. FCCC/CP/2015/10/Add.1, United Nations Framework Convention on Climate Change, Paris.

van der Ploeg, F., Rezai, A., Mar. 2020. The risk of policy tipping and stranded carbon assets. Journal of Environmental Economics and Management 100, 102258.
URL `http://www.sciencedirect.com/science/article/pii/S0095069618302481`

Vogt-Schilb, A., Meunier, G., Hallegatte, S., Mar. 2018. When starting with the most expensive option makes sense: Optimal timing, cost and sectoral allocation of abatement investment. Journal of Environmental Economics and Management 88, 210–233.
URL `http://www.sciencedirect.com/science/article/pii/S0095069617308392`

Weil, P., 1990. Nonexpected utility in macroeconomics. The Quarterly Journal of Economics 105 (1), 29–42.

Appendix 2: Value functions and Bellman equations

Expected Utility

Consider the following value function as the discounted value of future utility:

$$J(K_d, K_c, S, t) = max_{I_d,I_c} \int_{\tau=0}^{\infty} e^{-\rho(t+\tau)} L_{t+\tau} u \left(\frac{C_{t+\tau}}{L_{t+\tau}} \right) d\tau. \tag{15}$$

This function increases with the starting time t, because utility is higher in the future. J does not converge to a steady state. Using $C = cL_0 e^{(g+g_L)(t+\tau)}$, we have

$$\frac{C_{t+\tau}}{L_{t+\tau}} = e^{g(t+\tau)}\left(c\right). \tag{16}$$

For a CES utility function, the value function can be written as

$$J(k_d, k_c, S, t) = max_{i_d, i_c} L_0 e^{-(\rho - g_L + (\eta - 1)g)t} \int_{\tau=0}^{\infty} e^{-(\rho - g_L + (\eta-1)g)\tau} \frac{c^{1-\eta}}{1-\eta} d\tau. \tag{17}$$

Define a current value value function $V = \int_{\tau=0}^{\infty} e^{-(\rho-g_L+(\eta-1)g)\tau} \frac{c^{1-\eta}}{1-\eta} d\tau$, allowing us to write the differential of J as $J = L_0 e^{-(\rho-g_L+(\eta-1)g)t} V \Leftrightarrow dJ/dt = -\left(\rho - g_L + (\eta-1)\,g\right) L_0 e^{-(\rho-g_L+(\eta-1)g)t} V + L_0 e^{-(\rho-g_L+(\eta-1)g)t} dV/dt$.

The standard Bellman equation is $0 = max_{i_d,i_c} \left\{ u\left(\frac{C_t}{L_t}\right) L_0 e^{-(\rho-g_L)t} + \frac{d}{dt}\mathbb{E}J(k_d,k_c,S,t)\right\}$. Substitute for consumption per unit of effective labor and value function V:

$$0 = max_{i_d,i_c} \left\{ L_0 e^{-(\rho-g_L+(\eta-1)g)t} \frac{c^{1-\eta}}{1-\eta} - \left(\rho - g_L + (\eta-1)\,g\right) L_0 e^{-(\rho-g_L+(\eta-1)g)t} V + L_0 e^{-(\rho-g_L+(\eta-1)g)t} \frac{d}{dt}\mathbb{E} \right. \tag{18}$$

$$\Leftrightarrow \left(\rho - g_L + (\eta-1)\,g\right) V(k_d,k_c,S,t) = max_{i_d,i_c} \left\{ \frac{c^{1-\eta}}{1-\eta} + \frac{d}{dt}\mathbb{E}V(k_d,k_c,S,t) \right\}. \tag{19}$$

In case consumption per unit of effective labour is not an explicit function of t, we have reduced the number of states from 4 to 3. If c is still a function of t, we have a value function that converges to a steady state. Also, working with a discount rate that is larger than ρ is also beneficial for computational convergence.

Apply Ito's Lemma to the value function V:

$$\begin{aligned} \left(\rho - g_L + (\eta-1)\,g\right) V(k_d,k_c,S,t) \quad = \quad & max_{i_d,i_c}\{ \frac{c^{1-\eta}}{1-\eta} \\ & + V_{k_d}\left(i_d - \chi i_d^2 - (\delta + g_L + g)k_d\right) + V_{k_c}\left(i_c - \chi i_c^2 - (\delta + g_L + g)k_c\right) \\ & + 0.5 V_{k_d k_d} k_d^2 \sigma_d^2 + 0.5 V_{k_c k_c} k_c^2 \sigma_c^2 + V_{k_c k_d} \rho \sigma_c \sigma_d \\ & + V_S E + 0.5 V_{SS} S^2 \sigma_S^2 + \lambda_S \left[V(S + \xi S) - V\right] \\ & + V_t\}. \end{aligned}$$

The first order conditions in the maximand are:

$$V_{k_d} = \frac{-c^{-\eta} c_{i_d}}{1 - 2\chi i_d}, \tag{20}$$

$$V_{k_c} = \frac{c^{-\eta}}{1 - 2\chi i_c}. \tag{21}$$

Applying the envelope theorem with respect to S gives an expression for the shadow price of cumulative emissions:

$$
\begin{aligned}
V_S \quad = \quad & \frac{1}{\rho - g_L + (\eta - 1)\, g} \{ c^{-\eta} y(-\gamma S) + V_{SS} E \\
& + 0.5 V_{SSS} S^2 \sigma_S^2 + V_{SS} S \sigma_S^2 + \lambda_S \left[V_S(S + \xi S) - V_S \right] + V_{tS} \\
& + V_{Sk_d} \left(i_d - \chi i_d^2 - (\delta + g_L + g) k_d \right) + V_{Sk_c} \left(i_c - \chi i_c^2 - (\delta + g_L + g) k_c \right) + 0.5 V_{Sk_d k_d} k_d^2 \sigma_d^2 + 0.5 V_{Sk_c k_c} k
\end{aligned}
$$

The first line is the same as in the model without uncertainty and emissions inertia in Dietz & Venmans 2019. In this model, $V_{SS} E = \dot{V_S} = -\dot{MAC}$. In other words, the shadow price of cumulative emissions contains the speed of its increase. By introducing inertia, this speed is much larger. Technically, the term $V_{SS} E$ is much larger in our model, because initial emissions are much larger in any model with inertia.

The formula of the social cost of carbon is

$$
SCC = \frac{-V_S}{V_{k_d}} L_0 e^{(g+g_L)t} \tag{22}
$$

The intuition of this formula is as follows. V_S is the shadow price of cumulative emissions, expressed in utils per unit of effective labor. $V_{k_c}$is the shadow price of capital, also expressed in utils per unit of effective labor. The quotient of both gives a result in units of capital per unit of effective labor k_c per GtC. By multiplying by $L_0 e^{(g+g_L)t}$ we obtain a result in dollars per tonne of carbon. Equation 21 shows that $V_{k_c}$is marginal utility, slightly reduced by investment inertia effects. Note however V_{k_c} (and therefore consumption) is also lowered by the risk in the model (this can be seen by applying the envelope theorem to the Bellman equation with respect to clean capital).

Epstein-Zin-Weil preferences

The present value value function is:

$$
J(K_d, K_c, S, t) = max_{I_d, I_c} \mathbb{E} \int_t^{\infty} -\hat{f}(K_d, K_c, S, t, J, I_d, I_c) d\tau
$$

s.t. $\dot{s} = g$, with

$$
\hat{f}(K_d, K_c, S, t, J, I_d, I_c) = \frac{1 - RRA}{1 - \eta} (\rho - g_L)\, J \left[\frac{\left(\frac{C}{L}\right)^{1-\eta}}{\left((1 - RRA)\, J\right)^{\frac{1-\eta}{1-RRA}}} - 1 \right] \tag{23}
$$

Defining a current value value function $V = e^{-g(1-\eta)t} J$, a current value aggregator function $f = \hat{f} e^{-(1-\eta)gt}$ and standardized consumption (per unit of utility-corrected effective labor) $c = \frac{C}{L_0 e^{(g+g_L)t}}$ allows us to obtain an optimization problem without time as an explicit argument (or at least a stable steady state):

$$V(k_d, k_c, S, t) = max_E \mathbb{E} \int_0^\infty -f(k_d, k_c, S, t, V, i_d, i_c) d\tau \tag{24}$$

$$f = \frac{1 - RRA}{1 - \eta} \left(\rho - g_L + (\eta - 1)\, g\right) V \left[\frac{(c)^{1-\eta}}{\left((1 - RRA)\, V\right)^{\frac{1-\eta}{1-RRA}}} - 1 \right] \tag{25}$$

The Bellman equation is

$$max_E \left\{ f + \frac{1}{dt} \mathbb{E}[dV(k_d, k_c, S, t)] \right\} = 0 \tag{26}$$

$$\begin{aligned}
0 \quad = \quad & max_{i_d, i_c} \{ \frac{1 - RRA}{1 - \eta} \left(\rho - g_L + (\eta - 1)\, g\right) V \left[\frac{(c)^{1-\eta}}{\left((1 - RRA)\, V\right)^{\frac{1-\eta}{1-RRA}}} - 1 \right] \\
& + V_{k_d} \left(i_d - \chi i_d^2 - (\delta + g_L + g) k_d \right) + V_{k_c} \left(i_c - \chi i_c^2 - (\delta + g_L + g) k_c \right) \\
& + 0.5 V_{k_d k_d} k_d^2 \sigma_d^2 + 0.5 V_{k_c k_c} k_c^2 \sigma_c^2 + V_{k_c k_d} \rho \sigma_c \sigma_d \\
& + V_S E + 0.5 V_{SS} S^2 \sigma_S^2 + \lambda_S \left[V(S + \xi S) - V \right] \\
& + V_t \}
\end{aligned}$$

Applying the envelope theorem with respect to S gives an expression for the shadow price of cumulative emissions.

$$\begin{aligned}
V_S \quad = \frac{1}{(\rho - g_L + (\eta - 1) g)} \quad & \Phi(V, c) \{ \Psi(V) c^{-\eta} y(-\gamma S) + V_{SS} E \\
& + V_{Sk_d} \left(i_d - \chi i_d^2 - (\delta + g_L + g) k_d \right) + V_{Sk_c} \left(i_c - \chi i_c^2 - (\delta + g_L + g) k_c \right) + 0.5 V_{Sk_d k_d} k_d^2 \sigma_d^2 \\
& + 0.5 V_{SSS} S^2 \sigma_S^2 + V_{SS} S \sigma_S^2 + \lambda_S \left[V_S(S + \xi S) - V_S \right] + V_{tS} \}
\end{aligned}$$

with $\Phi = \frac{1-\eta}{1 - RRA + (RRA - \eta) c^{1-\eta} ((1-RRA)V)^{-\frac{1-\eta}{1-RRA}}}$ and $\Psi = \frac{\rho - g_L + (\eta - 1) g}{((1-RRA)V)^{\frac{1-\eta}{1-RRA} - 1}}$.

Appendix 3: The general effect of inertia on the carbon price

In this section we make the general case that a model with emissions inertia, i.e. a MAC that is larger at high abatement speed, will increase the carbon price.

Define E as emissions, S as cumulative emissions (proportional to temperature), X as a vector of other state variables (possibly different types of capital, fossil resources etc.) with trend $\mu_X((X,t,y))$ and brownian motion with variance $\sigma_X(X,t,y)$, and y as a vector of other decision variables. V is the current-value value function, discounted at rate r. In a model with emissions inertia, emissions is a state variable, not a decision variable. The Hamilton-Jacobi-Bellman equation is [15]

$$rV = max_y \left\{ u + V_S E + V_E \dot{E} + V_X \mu_X + tr0.5 V_{XX} \sigma_X^2 + V_t \right\}. \tag{29}$$

The shadow price of cumulative emissions corresponds to the social cost of carbon (expressed in utils) and is obtained by applying the envelope theorem to the Bellman equation[16] [17]

$$SCC(utils) = -V_S = \frac{1}{r}\left(-u_S - V_{SS}E - V_{SE}\dot{E} - \frac{\partial}{\partial S}\left(V_X \dot{X} + tr0.5 V_{XX}\sigma_X^2 + V_t \right) \right). \tag{31}$$

The first term is the marginal damage from warming, typically modest in 2020 with 1°C of warming and independent of the fact that there is inertia. Inertia leads to two major differences in terms 2 and 3 respectively. Firstly, in a model with inertia, emissions start at the current level of emissions, whereas in a model without inertia, optimal emissions jump from the current level of emissions to an optimal level that is much lower. This will increase the second term $-V_{SS}E$ proportionally, especially for very convex damage functions. We show below that this term represents 70% of the carbon price in our model. Secondly, the term $-V_{SE}\dot{E}$ is absent in the model without inertia. V_{SE} is typically negative, because emissions increase production whereas cumulative emissions decrease production via temperature damages. Therefore, the term $-V_{SE}\dot{E}$ reduces the SCC, especially for production functions with a large cross derivative in E and S (e.g. when both abatement costs and damages are proportional to production size). In the extreme case with a negligible penalty for abatement speed, $\dot{E}$ is

[15] We could simplify to

$$rV = max_y \left\{ u + V_S E + V_E \dot{E} + V_X \dot{\dot{X}} + V_t \right\} \tag{27}$$

Or work with our JEEM model and add any penalty for $\dot{E}$ in the production function (possibly combined with brownian motion and jump on temperature):

$$rV = max_{\dot{E}} \left\{ u + V_S E + V_E \dot{E} \right\} \tag{28}$$

[16] This equation corresponds to the equation of motion of the shadow price of cumulative emissions in optimal control theory: $\dot{\lambda^S} = r\lambda^S - H_S$

[17] The explicit form is

$$SCC(utils) = -V_S = \frac{1}{r}\left(-u_S - V_{SS}E - V_{SE}\dot{E} - V_{SX}\dot{X} + V_X \frac{\partial}{\partial S}\dot{X} + tr0.5 V_{SXX}\sigma_X^2 + tr0.5 V_{XX}\frac{\partial}{\partial S}\sigma_X^2 + V_{St} \right) \tag{30}$$

largely negative and compensates the increased magnitude of the second term. The carbon price converges to the case without emission inertia. In the case of a binding penalty for abatement speed at the start, the effect on the second term will dominate the counteracting effect on the third term and the carbon price will be higher. This implies that the optimal emissions path incurs a non-zero penalty on abatement speed at the start of the optimal path (except if there are interaction terms in the cross derivative V_{SX}).

Appendix 4: Equations used in the Compecon Toolbox

The compecon toolbox for dynamic programming requires the gradient and hessian of both the objective function f and the difference function g. Capital letters are aggregate values, small letters are per unit of effective labour.

Call $s = [k_d, k_c, S, \tau]$ and $x = [i_d, i_c]$. Since it is never optimal to strand and invest in dirty capital at the same time, they are both captured by the same variable, where stranding is $-i_d$.

The conversion from time to synthetic time, in its different variants, reads

$$\tau = 1 - e^{-\zeta t} \Leftrightarrow t = -\frac{1}{\zeta} ln(1-\tau) \Leftrightarrow e^t = (1-\tau)^{-\frac{1}{\zeta}} . \tag{32}$$

Emissions are

$$E = \psi K_d = \psi_0 L_0 k_d e^{(g+g_L-g_\psi)t} = \psi_0 L_0 k_d \left(1-\tau\right)^{\frac{\left(-g-g_L+g_\psi\right)}{\zeta}} . \tag{33}$$

Emissions change (middle expression gives 0/0 for k_d=0) is

$$\dot{E} = E\left(g + g_L - g_\psi + \frac{\dot{k_d}}{k_d}\right) = E\left(\frac{i_d - \chi_d i_d^2}{k_d} - g_\psi - \delta\right) = \psi_0 L_0 \left(1-\tau\right)^{\frac{\left(-g-g_L+g_\psi\right)}{\zeta}} \left(i_d - \chi_d i_d^2 - \left(g_\psi + \delta\right) k_d\right) . \tag{34}$$

Stranding costs and derivatives are

$$strcost = 1_{I_d<0} \frac{\theta_1 \left(-I_d\right)^{\theta_2}}{L_0 e^{(g+g_L)t}} = \theta_1 \left(-i_d\right)^{\theta_2} L_0^{\theta_2 - 1} e^{(\theta_2-1)(g+g_L)t} = \theta_1 \left(-i_d\right)^{\theta_2} L_0^{\theta_2-1} \left(1-\tau\right)^{\frac{(1-\theta_2)(g+g_L)}{\zeta}} , \tag{35}$$

$$strcost_{i_d} = -1_{I_d<0} \theta_1 \theta_2 \left(-i_d\right)^{\theta_2 - 1} L_0^{\theta_2-1} \left(1-\tau\right)^{\frac{(1-\theta_2)(g+g_L)}{\zeta}} , \tag{36}$$

$$strcost_{i_d i_d} = 1_{I_d<0}\theta_1\theta_2\left(\theta_2-1\right)\left(-i_d\right)^{\theta_2-2}L_0^{\theta_2-1}\left(1-\tau\right)^{\frac{(1-\theta_2)\left(g+g_L\right)}{\zeta}}. \tag{37}$$

Consumption and derivatives are

$$c = A_0\left(k_d+k_c\right)^{\alpha}e^{\phi E-\frac{\varphi}{2}E^2+\beta\dot{E}-\frac{\gamma}{2}S^2}-i_d-i_c-strcost, \tag{38}$$

$$c_{i_d} = -1+y\beta\psi_0L_0\left(1-\tau\right)^{\frac{\left(-g-g_L+g_\psi\right)}{\zeta}}\left(1-2\chi_d i_d\right)-strcost_{i_d}, \tag{39}$$

$$c_{i_d i_d} = y\left(\beta\psi_0L_0\left(1-\tau\right)^{\frac{\left(-g-g_L+g_\psi\right)}{\zeta}}\left(1-2\chi_d i_d\right)\right)^2+y\beta\psi_0L_0\left(1-\tau\right)^{\frac{\left(-g-g_L+g_\psi\right)}{\zeta}}(-2\chi_d)-strcost_{i_d i_d}. \tag{40}$$

The objective function is

$$f = \frac{1}{1-\eta}c^{1-\eta}. \tag{41}$$

The gradient of the objective function with respect to the two decision variables is

$$f_{i_d} = c^{-\eta}c_{id}, \tag{42}$$

$$f_{i_c} = -c^{-\eta}. \tag{43}$$

The hessian is

$$f_{i_d i_d} = -\eta c^{-\eta-1}\left(c_{i_d}\right)^2+c^{-\eta}c_{i_d i_d}, \tag{44}$$

$$f_{i_c i_c} = -\eta c^{-\eta-1}, \tag{45}$$

$$f_{i_d i_c} = \eta c^{-\eta-1}c_{i_d}. \tag{46}$$

For the difference function g we have

$$
\begin{aligned}
k_{d_{t+1}} &= i_d - \chi_d i_d^2 + (1 - \delta - g_L - g + \sigma_d \Delta W_d) k_d, \\
k_{c_{t+1}} &= i_c - \chi_c i_c^2 + (1 - \delta - g_L - g + \sigma_c \Delta W_c) k_c, \\
S_{t+1} &= E + (1 + \sigma_S \Delta W + \lambda \Delta P) S_t, \\
\tau_{t+1} &= 1 - e^{-\zeta(t+1)} = 1 - (1-\tau)\, e^{-\zeta}.
\end{aligned}
$$

The gradient reads

$$
dg/dx = \begin{bmatrix} 1 - 2\chi i_d & 0 \\ 0 & 1 - 2\chi i_c \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \tag{47}
$$

with hessian

$$
d^2 g/dx^2 = \begin{bmatrix} -2\chi & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \cdots \begin{bmatrix} 0 & 0 \\ 0 & -2\chi \\ 0 & 0 \\ 0 & 0 \end{bmatrix}. \tag{48}
$$

Appendix 5: Cumulative emissions in the model with three states

For cumulative emissions, we have:

$$
S_t = S_0 + \int_0^t E_\tau d\tau + \int_0^t S_\tau \sigma_S dW + \int_0^t \xi S_\tau dP. \tag{49}
$$

The second term is

$$
\int_0^t \psi_0 L_0 k_d e^{(g+g_L-g_\psi)t} dt. \tag{50}
$$

We omit the factor $e^{(g+g_L-g_\psi)t}$, underestimating cumulative emissions and therefore damages in the long run. This has a limited impact, however, because optimal emissions decrease rapidly and are about zero at the end of the century. We use an iterative approach where we adapt the damage coefficient as follows:

$$
\hat{\gamma} = \gamma (S/\hat{S})^2 = \gamma \left[\frac{\int_{2020}^{2100} \psi_0 L_0 k_d e^{(g+g_L-g_\psi)t} dt}{\int_{2020}^{2100} \psi_0 L_0 k_d dt} \right]^2 \tag{51}
$$

If emissions are not at zero in 2100, we extend the estimation horizon. This gives the correct damage factor $e^{-\gamma S^2}$ in the steady state. We show the difference between the full model and this approximation for a set of parameters below.

Appendix 6: Use of total capital and proportion of dirty capital as state variables

The model with states k_d and k_c has difficulties converging in the 4 states setting. One of the difficulties is that both states are very interdependent. Easier convergence is obtained by using total capital and the proportion of dirty capital in total capital (μ) as states. The latter is defined as

$$\mu = \frac{k_d}{k} \tag{52}$$

$$E = \psi K_d = \psi_0 L_0 \mu k \left(1-\tau\right)^{\frac{\left(-g-g_L+g_\psi\right)}{\zeta}} \tag{53}$$

$$\dot{E} = \psi_0 L_0 \left(1-\tau\right)^{\frac{\left(-g-g_L+g_\psi\right)}{\zeta}} \left(i_d - \chi_d i_d^2 - \left(g_\psi + \delta\right)\mu k\right) \tag{54}$$

Applying Ito's Lemma gives

$$d\mu = \left[\left(1-\mu\right)\frac{i_d - \chi i_d^2}{k} - \mu\frac{(i_c - \chi i_c^2)}{k} + \left(\frac{1}{2} + 2\mu\right)\mu^2\sigma_d^2 - \left(\frac{1}{2} - 2\mu\right)\left(1-\mu\right)^2\sigma_c^2 - 2\mu^2(1-\mu)\rho_{cd}\sigma_d\sigma_c\right]dt \tag{55}$$

$$+\left(1-\mu\right)\mu\left(\sigma_d dW_d - \sigma_c dW_c\right) \tag{56}$$

$$dk = dk_d + dk_c \tag{57}$$

Difference equation g becomes

$$\begin{aligned}
\mu_{t+1} &= \mu + \left(1-\mu\right)\frac{i_d - \chi i_d^2}{k} - \mu\frac{(i_c - \chi i_c^2)}{k} \\
&\quad + \left(\frac{1}{2} + 2\mu\right)\mu^2\sigma_d^2 - \left(\frac{1}{2} - 2\mu\right)\left(1-\mu\right)^2\sigma_c^2 - 2\mu^2(1-\mu)\rho_{cd}\sigma_d\sigma_c \\
&\quad + \mu\left(1-\mu\right)\left(\sigma_d\Delta W_d - \sigma_c\Delta W_c\right) \\
k_{t+1} &= i_d + i_c - \chi_d i_d^2 - \chi_c i_c^2 + (1 - \delta - g_L - g + \mu\sigma_d\Delta W_d + (1-\mu)\sigma_c\Delta W_c)k \\
S_{t+1} &= E + (1 + \sigma_S\Delta W + \lambda\Delta P)S_t \\
\tau_{t+1} &= 1 - e^{-\zeta(t+1)} = 1 - \left(1-\tau\right)e^{-\zeta}
\end{aligned}$$

The gradient reads

$$dg/dx = \begin{bmatrix} (1-\mu)\frac{1-2\chi i_d}{k} & -\mu\frac{1-2\chi i_c}{k} \\ 1-2\chi i_d & 1-2\chi i_c \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{58}$$

with hessian

$$d^2g/dx^2 = \begin{bmatrix} -(1-\mu)\frac{2\chi}{k} & 0 \\ -2\chi & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} ... \begin{bmatrix} 0 & \mu\frac{2\chi}{k} \\ 0 & -2\chi \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{59}$$

Appendix 7: The probability distribution of stranded assets

Figure 7: The probability distribution of stranded assets at different time horizons. Brownian motions have a rather limited effect on the risk of stranding because optimal policy reacts relatively fast to small changes in damages. However, the jump process leads to a set of scenarios with much higher stranding than the mean scenario.