How to design reserve markets? The case of the demand function in capacity markets

Leopold Monjoie[1]

[1] Université Paris Dauphine, PSL Research University, LEDa, Place du Marechal de Lattre de Tassigny, 75016, Paris, France

[2] Chair European Electricity Markets

leopold.monjoie@dauphine.eu

Abstract. This paper studies reservation markets' design in the context of the provision of essential goods with time-varying and uncertain stochastic demand, which are typically under-procured by private agents and lead to under-investment to meet peak demand compared to the social optimal. In these industries, a regulator typically determines the capacity required to met peak demand and organizes the procurement of the capacity deemed necessary through a reservation market. The paper contributes to the literature by developing a novel approach to study the design of the demand function in the reservation market and the interdependencies with the subsequent production and retail markets. We provide a complete framework using a sequential analytical model of the three markets and demonstrate how the reservation market demand curve specification affects the demand and the equilibria in the subsequent production and retail markets. Our analysis focuses on comparing two regimes: (i) a centralized design, where a regulated entity buys the capacity availability and then allocates the cost onto the retailers and the consumers; and (ii) a decentralized regime where retailers and consumers must buy the capacity availability to cover their demand. We describe different market design regimes, their process through which those markets are impacted, and their outcome in terms of investment level, prices, and welfare. The model results are discussed first using a general framework and then with a closed-form solution in reference to the example of electricity markets where capacity reservation is often used to ensure adequate investment to ensure the security of supplies.

Keywords: market design · investment decisions · imperfect competition · regulation.

1 Introduction

For some essential goods with demand varying over time, such as electricity or medical supplies, wholesale markets' private incentives are not sufficient to ensure that producers make enough investments to meet peak demand in advance of the time when the peak demand materializes. In such industries, due to their critical importance of these goods, policymakers tend to intervene and implement price caps or other types of regulation that distort the price signal at peak times and undermine investment incentives. At the same time, these goods typically encompass some public good characteristics, especially during peak demand periods, for instance, a cold wave with peak electricity demand or a pandemic with peak demand for medicine or medical equipment. In such circumstances, the absence of adequacy between the capacity and the peak demand, combined with the difficulty of implementing efficient rationing, can lead to high costs for society.

One solution to restore the optimal level of investment is the implementation of a reservation market in which producers commit to having capacity available to meet the expected peak demand collectively. On the supply side, each participating producer makes a price-quantity offer for a capacity. If a producer sells capacity in this reservation market, it commits to be available to produce over a specific period in the future.

While the supply function emerges naturally from producers on those markets, the demand function requires a regulatory intervention. Indeed, the public-good nature of investment during high-demand periods implies that consumers are unwilling to buy capacities in reservation markets. Hence, the regulator must create the demand function administratively, so the market clears and provides producers' capacity prices. To overcome this issue, regulators have proposed different options to create an ad hoc demand in the reservation market. One of the ongoing central debates amongst economists and policymakers is whether those capacities should be centrally bought by a single entity (the centralized design) or decentrally bought by each retailer (the decentralized design).

This paper provides discussions on the economic impacts those two designs for reservation markets can have and their policy implications. We stress that the centralized design is, above all, a question of cost passthrough between the single entity, which estimates the total consumption of final consumers, and ultimately the retailers and consumers. On the other hand, in the decentralized design, each retailer estimates its client portfolio's consumption, makes an individual demand function in the reservation market, and passes the cost onto its clients. Therefore, a key question under the decentralized design is to assess the marginal value of a capacity for retailers and which drivers can impact it.

To our knowledge, there has been no formal comparison between each reservation market design option, the incentive properties of these two alternative design approaches, and their ability to restore the socially optimal level of investment. In this paper, we develop a model that sheds light on the interactions between the reservation market design and the incentives of producers and retailers. The model provides some new and somewhat non-intuitive effects, which underline that the design of such a reservation market is not straightforward. When choosing the reservation market's demand function, policymakers indeed need to be careful about the indirect effects. We demonstrate that each design option has distinct economic implications for the demand side, which affects not only the outcomes in terms of prices and quantities traded on the economic system but also redistribution effects for consumers, producers, and retailers. More generally, our model shows that there is an endogeneity effect between the first-best solution considered and the economic instrument implemented to reach it. A type of reservation market design (e.g., the centralized approach) can be seen as optimal given a set of inefficiencies. However, other types of designs (e.g., the decentralized approach) may approach the first best but still bring more welfare than the initial first-best solution by reducing the inefficiencies effect.

We start our analysis using the canonical benchmark model for non-storable goods characterized by time-varying demand, which describes the relationship between the short-term production decisions and the long-term investment decisions. Producers make long-run investments in a single technology in the upstream market in order to be able to subsequently produce a homogeneous good, given an uncertain future demand. Then, the downstream retailers aggregate and resell the goods at no cost to the final consumers. The first best solution is given by maximizing social welfare, which boils down to the equality between the long-term marginal cost and the expected marginal revenue under the assumption of no inefficiencies. In addition, to characterize the essential nature of the good studied in this paper, we introduce political intervention through a price cap regulation. It can be interpreted as representing the different types of price distortions induced by a range of market failures and policy interventions, which are common for essential goods and can take the form of price caps or market regulations with a similar effect on price dynamics in practice. Such a price cap reduces expected revenues of producers (particularly during peak periods) and undermines investment compared to the level that would be needed to reach the socially optimal level of installed capacity. Moreover, when the price cap is reached, the investment availability becomes a public good as the demand becomes inelastic. Due to the impossibility of efficiently rationing consumers, they incur a significant welfare loss that simulates the system cost.

The paper's main contribution lies in the analytical framework representing the different market design options for the demand specification in reservation markets, which allows drawing new insights on the incentive properties of the alternative design approaches and their ability to restore the socially optimal level of investment. We describe the rules associated with the centralized and decentralized model, enabling us to model the effect of each regime on the retailer's strategies and final consumer's demand and study the regulatory parameters used to implement the reservation market's demand function. We untangle the interdependencies between the three markets (reservation, wholesale, and retail) and their impact on investment decisions. More specifically, our model shows how the design of the reservation market impacts producers' and retailers' profits in subsequent wholesale and retail markets. We analyze how each reservation market design and its corresponding rules affect retailers' demand on the wholesale market.

In the case of a centralized reservation market, we build on the previous literature and start with the canonical design found in Léautier (2016) and Holmberg und Ritz (2020). In this model, the key assumption is that the reservation market does not affect the demand. It is similar to assume that the capacity price is passed onto the consumers via a lump-sum tax from a practical view. Such market design allows the first-best level of investment to be attained, given the inefficiencies in the system.

We then investigate the case in which the reservation price impacts the consumers at the margin. In this case, the centralized design is similar to a Pigouvian unitary tax. We show that the indirect effect of the reservation market is ambiguous for social welfare by bringing more or less welfare than the first best of the canonical design. More precisely, passing the cost as a unitary tax can have a positive effect only when the surplus loss due to inefficient rationing is significant. As the two previous market design focus on an ex-ante temporality with no link with the realized demand, we extend our analysis to one of the current implementations of a centralized regime where the regulator chooses to allocate the cost based on actual retailers' market shares. This allocation creates an intermediary outcome between the unitary tax and the lump-sum tax while having significant redistributional properties.

For the case of a decentralized reservation market, we analyze how individual strategies can form an aggregated demand function in the reservation market when retailers are forced to cover their quantity sold on the retail market given a penalty system. To do so, we analyze the optimal capacity asked by retailers in the reservation market. We find that decentralizing the demand function can approach the optimal level of investment under specific conditions. Indeed, the marginal value a capacity brings retailer profit depends on the market structure in the retail market, the consumers' demand function, and the penalty system.

To conclude, our model establishes a framework that allows us to analyze the effect of the different market designs of reservation markets and to derive some policy implications which are relevant to the current debate in several industries with essential goods characterized by time-varying demand, such as electricity markets or markets for medical equipment/ drugs. More specifically, we provide a closed-form solution of the general framework and we calibrate the model using data from the French power system. It allows to illustrate some of the findings described in the paper. We finally discuss future potential extensions of our work. For instance, a more detailed representation of agents and information and risk issues should give relevant insight into the paper's results.

2 Literature review

We contribute to the literature in multiple ways. We deepen the analysis of investment decisions in multiple periods where producers face an uncertain demand when choosing the investment level. We use the benchmark model that was first developed in a regulated context by Boiteux (1949) for the electricity sector, it was then extrapolated to a market with private producer by Crew und Kleindorfer (1976). This model is widely used primarily to highlight the risk of underinvestment in capacities. In his seminal paper, Joskow und Tirole (2007) demonstrates that wholesale markets with a price cap cannot lead to the first-best solution, as it exists a *Missing Money* issues that prevent sufficient revenue from being collected to cover fixed costs. Following this approach, which also serves as a reference model in our paper, Zöttl (2011) developed a theoretical result on investments under *Cournot* oligopoly with discrete investment and a price cap. Using the same model, Léautier (2016) showed that market power from producers could also be a significant cause of underinvestment. he also introduces a reservation market in the benchmark model where producers can exercise market power. This paper serves as our reference for our implementation of the reservation market. In the second stream of literature, some authors have stated a *Missing Market* issue that causes underinvestment. Risk, market incompleteness, externalities, and the public-good nature of essential goods could lead producers to consider their revenue insufficient to sufficiently invest (Newbery, 2016). Using the same benchmark model, Holmberg und Ritz (2020) showed that an additional capacity payment is necessary as soon as a a negative externality arises. Indeed, they create system costs, which are a negative public good. Hence, producers over procure them.

We also relate to the literature on the design of reservation markets, which to our best knowledge have only been applied to the electricity sector under the denomination of capacity markets. Although, some authors have highlighted the importance of the demand function in the design of those markets. For instance, Hobbs u. a. (2007); Bushnell u. a. (2017) stressed the importance of regulatory errors when designing the demand function. A notable stream of papers has found that the demand function's slope is crucial to control for possible strategic behavior, which can exacerbate some producers' market power. Fabra (2018) found that if the capacity demand function is more elastic than the wholesale demand function, then the reservation market helps dilute the producers's market power. Brown (2018) also established the optimal parameters for the capacity demand function, such as the slope and a price cap. However, those papers still only consider the effect of the reservation market directly on the supply side, while our paper underlines the indirect effect of this instrument on retailers, which then impact producers. To our best knowledge, Scouflaire (2019) has been the first paper to represent retailers' strategies in the reservation market. They develop a theoretical model to analyze the preferences in terms of information precision for the uncertain future demand. Contrary to our approach, they model heterogeneous price taker producers and homogeneous buyers competing for *à la Cournot* under uncertainty on their level of capacity obligation.

We provide in Section 3 a reminder of the benchmark model that describes investment decisions in generation capacity. We also introduce imperfect competition in the retail market, the price cap, and inefficient rationing, and we describe their effect on investment decisions. In the same section, we implement the reservation market and recall the theoretical supply function. Then, we model the different market designs for the demand function in 4 for the centralized regime and 5 for the decentralized regime. The closed-form solution and the numerical application are presented in 6. To conclude, we discuss possible extensions of the model in section 7.

3 Benchmark model

3.1 Assumptions

We consider an initial economic system with three agents: producers, retailers, and final consumers. Producers invest in capacities to produce a homogeneous good. They sell the goods on a wholesale upstream market to retailers. Then, retailers resell the goods on a downstream retail market to consumers.

Producers. We assume perfect competition on the supply side. Producers use a single technology to produce the good. It is characterized by a variable unitary cost c, and a fixed unity investment cost r. We normalized the capacity level, so one unit of capacity allows us to produce one unit of the good. The level of capacity installed in the market is k.

Retailers. Retailers compete *à la Cournout* to resell the goods to final consumers but do not behave as an oligopsony in the wholesale market. The imperfect competition is modeled using a finite number of retailers n. We assume that retailers incur no cost when reselling from the wholesale market to the retail market apart from the wholesale price [3].

Demand. On the retail market, final consumers, are characterized by the following assumptions from Léautier (2016):

- They have the same individual uncertain demand with an aggregate demand $D(p, t)$, t being the state of the world. The demand uncertainty is characterized by a distribution function $f(t)$ and a cumulative distribution function $F(t)$. The inverse demand function is $p(q, t)$,

[3] Therefore, perfect competition implies that prices are strictly equal in the wholesale and the retail market

with q the quantity sold on the retail market [4], such as $D(p(q,t),t) = q$. For convenience, we assume that $p^s(q,t)$ is the price on the wholesale market, and $p(q,t)$ is the price on the retail market.

- The demand function have the following properties[5]: $\forall t \in [0, +\infty)$ (i) $p_t(q,t) > 0$ (*states of the world are ordered*), (ii) $p_q(q,t) < 0$ (*decreasing price with respect to. q*) (iii) $p_q(q,t) + qp_{qq} < 0$ (*decreasing marginal revenue with respect to. q*) (iv) $p_t(q,t) + qp_{qt}(q,t) > 0$ (*increasing marginal revenue with respect to. t*) and (v) $\lim\limits_{q \to +\infty} p(q,t) < c$ (*prices can be below the marginal cost for some t*).

To insure producers invest in capacities we need additional conditions: $p(0,t) > c + r \quad \forall t$ and $\lim\limits_{q \to 0} p(q,t) < c$.

Model stages. The model has three stages. First, producers choose the level of investment. Second, the wholesale market clears. Third, the retail market clears. We assume the final consumers' demand is uncertain for all agents when making investment decisions. On the other hand, when the producers and retailers sell the good, the demand is known. Those two stages can be interpreted as a repetition of multiple states of the world over a given period (for example, one year), drawn from the distribution F(t) (Leautier, 2018).

Uncertain demand — Certain demand

Investment decision —— Wholesale market —— Retail market

3.2 Optimal investment decision

We now describe the three stages in reverse order. We define the equilibrium in each stage and the final optimal level of investment.

Third stage - Retail market. We assume that symmetric retailers can act strategically in the retail market and they take the wholesale price as given. The retailer's profit made on the

[4] We assume the quantity sold on the retail market is strictly the same quantity asked on the wholesale market as storage is not available

[5] For most of the functions $f(x,y)$, $f_x(x,y) = \frac{\partial f}{\partial x}(x,y)$, $f_{xx}(x,y) = \frac{\partial^2 f}{\partial x^2}(x,y)$,$f_{xy}(x,y) = \frac{\partial^2 f}{\partial x \partial y}(x,y)$

retail market is: $\pi_i^r(t) = q_i(p(q,t) - p^s(q,t))$. The first order condition gives the equality between the marginal revenue and the marginal cost. Thus, the inverse demand function of retailers on the wholesale market is a downward rotation at the intercept of the final consumer demand function :

$$p^s(q,t) = p(q,t) + \frac{q}{n} p_q(q,t)$$

Second stage - Wholesale market. Producers know the final consumer demand at this stage, so the retailers' inverse demand function is certain. The price is determined by the investment level k chosen during the first stage. We assume perfectly competitive producers, so when k is not binding, the price is equal to the marginal cost c (*off-peak periods*). When k is binding, the price has to rise above marginal to ensure that supply equals demand (*on-peak periods*). We denote $t_0(k)$ the first state of the world when capacity is binding, that is, when the price at the capacity level is equal to the marginal cost: $p^s(k, t_0(k)) = c$. We also define $q_0(t)$ as the quantity bought by final consumers when the retail price is equal to the marginal cost, such as $p^s(q_0(t), t) = c$. During off-peak periods, when $t_0(k) \geq t$, the price on the wholesale market is the marginal cost c and the price on the retail market is equal to $p(q_0(t))$. During peak periods, when $t_0(k) < t$, the demand function determines the price with $p(k,t) + \frac{k}{n} p_q(k,t)$ the price on the wholesale market, and $p(k,t)$ the price on the retail market.

First stage - Investment decisions. At this stage, final consumer demand is unknown, so is the wholesale and retail price. We find the optimal investment level k^* by maximizing the social welfare given in the following equation:

$$W(k) = \int_0^{t_0(k)} \int_0^{q_0(t)} (p(q,t) - c) dq \, f(t) dt + \int_{t_0(k)}^{+\infty} \int_0^k (p(q,t) - c) dq \, f(t) dt - rk \tag{1}$$

3.3 Market equilibrium for the essential good

Essential goods are characterized by a set of inefficiencies that prevent the market investment level from reaching the first-best solution. Two main reasons explain why private investors do not provide sufficient capacities: (1) the revenue collected on the market is insufficient to cover their production costs, (2) prices do not consider the positive externalities implied by their availability during high demand periods. For the first rational, we derived two inefficiencies that characterized essential goods: a historically concentrated retail market which is represented via retailers' market power, and the suboptimality of the wholesale price modeled via a price cap[6]. For the second rationale, because any inadequacy between the available production capacities and the demand for the good can lead to significant system loss, we introduce inefficient rationing when the price cap is reached. For each cause, we define the welfare function, the market equilibrium, and the optimal payment a producer needs to receive to provide the optimal investment level.

The market outcome in terms of investments is found by estimating the expected inframarginal rent $\phi(k)$. This rent is the net marginal revenue made on the wholesale market, which is the difference between the wholesale price and the marginal production cost. The following equation gives the expected unitary inframarginal rent:

$$\phi(k) = \int_{t_0(k)}^{+\infty} (p^s(k,t) - c)\ f(t)dt \tag{2}$$

The market investment level $\bar{k}$ of investment under imperfect competition framework is found by solving: $\phi(k) = r$. Under perfect competition on the retail market we have $k^* = \bar{k}$.

Retailer market power. The market outcome under imperfect competition is the same as previously described: $\bar{k}$. The following lemma states that market power in the retail market lowers the investment level beyond market power's direct effect. The market investment level is different from the optimal investment level even when we maximize the welfare function given the market power in the retail market.

Lemma 1. *Imperfect competition in the retail market leads to a lower installed capacity compared with the optimal investment level given by the social welfare maximization:* $\bar{k} \leq k^* \quad \forall n \in [2, +\infty)$. *The optimal capacity payment* $\bar{z}$ *is equal to the expected markup of retailers in the retail market:*

[6]This modeling approach can represent both an explicit price cap or an implicit one. In the latter case, political interventions due to the essential nature of the good can artificially alter the price. For instance, when the power system operator needs to carry out technical interventions to avoid system failures

$$\bar{z}(k) = \int_{t_0(k)}^{+\infty} \frac{-k}{n} p_q(k,t) f(t) dt \tag{3}$$

Price cap We implement a price cap denoted p^w. In order to create inefficiencies, the price cap must be binding for some states of the world, so it needs to be below the highest price during the highest demand period; $p^w < \lim_{t \to \infty} p^s(0,t)$. However, to allow for the investment, we also need that the price cap to be above the total unitary cost: $p^w > r + c$ (Leautier, 2018). Following the previous analysis, we introduce a second threshold $t_0^w(k)$. It the first state of the world when the price cap is biding, that is, when the price at the capacity level is equal to the price cap: $p^s(k, t_0^w(k)) = p^w$. We also define $q_0^w(t)$ the quantity bought by retailers (or consumers under perfect competition) when the price is equal to the marginal cost, such as $p^s(q_0^w(t), t) = p^w$. The price cap does not change the social welfare function equal to $W(k)$ as it only redistributes surpluses between consumers, producers, and retailers. We find the investment level quantity by estimating the expected inframarginal rent. The following equation defines this rent. It is shared between the states of the world when prices are above the marginal cost and below the price cap and the states of the world when prices are above the price cap. The conditions on p^w relatively to the marginal cost c ensure that $t_0^w(k) \geq t_0(k)$.

$$\phi^w(k) = \int_{t_0(k)}^{t_0^w(k)} \left(p^s(k,t) - c\right) f(t) dt + \int_{t_0^w(k)}^{+\infty} \left(p^w - c\right) f(t) dt \tag{4}$$

We find the level of capacity installed in the system given the price cap k^w by solving: $\phi^w(k) = r$. The following lemma shows that a price cap in the wholesale market lowers the investment level and increases inefficiency.

Lemma 2. *A binding price cap leads to a lower installed capacity compared with the optimal investment level given by the social welfare maximization:* $k^w \leq k^* \quad \forall p^w \in [c + r, \lim_{n \to \infty} p(0,t)[$. *The optimal capacity payment* $z^w(k)$ *is equal to the expected difference between what should have been the wholesale price and the price cap, when it is binding :*

$$z^w(k) = \int_{t_0^w(k)}^{+\infty} (p^s(k,t) - p^w) f(t) dt \tag{5}$$

Inefficient rationing. The price cap creates a second inefficiency when it is binding. Indeed, at the price cap level, the price-elastic demand becomes inelastic [7]. Therefore, we face the same rationing problem as in the traditional literature with limited production capacities and inelastic consumers (see for instance Joskow und Tirole (2007)). The absence of efficient discrimination between consumers with heterogeneous willingness to pay implies that investment availability is a public good when the price cap is binding. Therefore, it is underprovided by producers when they make their investment decisions. There is various way to describes the cost of involuntary rationing in the literature.[8] Joskow und Tirole (2007) shows that it depends if the rationing is anticipated or not. Leautier (2018) finds that the effect of involuntary rationing can be different if it has an impact on the expected demand level. From a modeling perspective, Holmberg und Ritz (2020) uses a general function $J(.)$ to represent this negative externality. The function depends on the delta between the quantity bought at a price equal to the price cap and the investment level. We note this cost $M(k)$, namely :

$$M(k) = \int_{t_0^w(k)}^{+\infty} J(\Delta_0 k) f(t) dt \tag{6}$$

with $\Delta_0 k$ a function of the difference between the installed capacity k and the quantity bought by retailers at the price cap q_0^w.[9] For instance, we can model rationing using a ratio $(1-h)$, which represents the share of consumers selected indifferently that is forced to stop consuming (Léautier, 2014). When rationing occurs, an optimal ratio h should be endogenously chosen such as we have $(1 - h(t))q_0^w(t) = k$. In this case, consumers sustain an additional loss proportional to their initial surplus with efficient rationing, namely :

$$M(k) = \int_{t_0^w(k)}^{+\infty} (1 - h(t)) \int_0^k (p(q,t) - p^w) dq f(t) dt \tag{7}$$

[7] The introduction of retailers into the model does not change the intuition. At a price p^w, the *Cournot* competition between the retailers pushes them to ask a quantity equal to $q_0^w(t)$.

[8] Note this is an additional cost compared to the loss of the surplus that we described previously with the price cap

[9] Regarding the sign of the cost and its derivatives: $M_k(.) \leq 0 < M_{kk}(.)$. It implies the same sign for the derivatives of J(.). Finally, note also that $\frac{\partial \Delta_0 k}{\partial k} \leq 0$ as an increase of capacity lower the difference for a given value of q_0^w.

Following Holmberg und Ritz (2020), we use the general notation $M(k) = \int_{t_0^w(k)}^{+\infty} J(\Delta_0 k) f(t) dt$. The following equation describes the new social welfare function :

$$W^{bo}(k) \quad = \quad W(k) - \int_{t_0^w(k)}^{+\infty} J(\Delta_0 k) f(t) dt \tag{8}$$

We denote k^{bo} the optimal level of investment when we maximize social welfare. Contrary to the imperfect competition in the retail market and the price cap, the social cost of rationing directly affects the social welfare function. The expected rent collected on the wholesale market by producers remains unchanged when we include inefficient rationing, which only affects consumers' welfare.[10]The following lemma shows that when we cannot efficiently ration final consumers when the price cap is binding it implies a higher inefficiency.

Lemma 3. *When the price cap induces involuntary rationing, the inefficiency is greater than with voluntary rationing. In other words, the delta between the optimal level of investment and the market outcome is greater with the first than with the later:* $k^* - k^w < k^{bo} - k^w$*. The optimal capacity payment* $z^{bo}(k)$ *depends on the representation of the involuntary rationing social cost. Under our assumption, it is equal to the marginal value of an additional capacity for the system, which decreases the cost of involuntary rationing:*

$$z^{bo}(k) = M_k(k) = - \int_{t_0^w(k)}^{+\infty} J_k(\Delta_0 k) + J_k(\Delta_0 k) \frac{\partial \Delta_0 k}{\partial k} \tag{9}$$

Note the two opposite parts of the optimal payment. The first negative part of the equation stands for the initial reduction in rationing: a higher investment level reduces the need to implement inefficient rationing. The second positive part is relative to the hypothesis behind the cost representation: as the rationing cost is proportional to the consumer surplus, a higher surplus generated by the investment indirectly leads to a higher cost of inefficient rationing.

[10]Some authors do include those costs in the producer profit, using a fixed reputational cost (Llobet und Padilla, 2018) or a market shutdown during which producers also lose profit (Fabra, 2018)

3.4 The supply curve on reservation markets

We set in place a reservation market to encourage producers to increase their investment. On the demand side, we assume in this subsection that the demand function in the market is unspecified (neither decentralized nor centralized). We denote its inverse demand function $p^c(k)$ with k the level of capacity offered in the reservation market, the demand function is D^c such as $D^c(p^c) = k$. $p^c(k)$ should be decreasing in k and defined as $p^c(.)$ is twice derivable.

Léautier (2016) defines the equilibrium conditions for the reservation market and the supply side: first, there are no short sells, meaning that producers cannot sell more capacity that they own: $k \leq k$. Second, it is optimal for producers to offer all their capacities if the first condition holds all their capacities: $k = k$. Finally, decision timing does not matter given our current setting: results still hold if the reservation market is set before or after the investment decision as soon as it is before final consumers' demand is known. Therefore, we keep the notation k even for the reservation market outcome.

We build the supply function based on the assumption that producers offer their marginal profit loss associated with the reservation market's participation. It is the common approach in the literature as it represents the cost of investing beyond the optimal capacity level. However, to our best knowledge, this is the first time a supply function in a reservation market is directly modeled using the benchmark framework. As we assume perfect competition in the wholesale market compared to Léautier (2016), and Zöttl (2011), there is no marginal effect of capacity choices on the inframarginal rent. [11] The full profit with a reservation market for a producer is: $\pi_i^s(k) = \phi(k)k_i - rk_i + p^c(k)k_i$. Under perfect competition, the first-order condition gives: $\phi(k) - r + p^c(k) = 0$. Therefore, the reservation market's supply function is equal to the marginal cost associated with the deviation from the market investment level k^0, which would have been made without the reservation market.

Proposition 1. *We denotes the supply function $X(k)$ and the inverse supply function $X^{-1}(p^c)$ such as $X^{-1}(X(k)) = k$. Following a marginalist approach, the supply function on the reservation market is defined as follow :*

[11] Indeed, under perfect competition, the inframarginal rent appears only when total capacity is constraining. Under imperfect competition on the supply side, the inframarginal rent also exists due to market power and can appear before the total capacity is binding.

$$X(k) = \begin{cases} 0 \quad if \quad k \leq k^0 \\ r - \phi(k) \quad k^0 < k \end{cases} \tag{10}$$

Below k^0, the marginal cost is positive, and the supply is null. Indeed, as the wholesale market's profit function is concave, any marginal revenues on the left side of the optimum are above the marginal cost of r. On the right side of the optimal investment level, the marginal revenue is below the marginal cost. Therefore any deviation to the right creates a positive opportunity cost.

4 Centralized demand

4.1 An agnostic demand function

We start our analysis of the demand function specification by assuming a centralized demand in which a single regulated entity determines the whole demand of capacity in the reservation market. To do so, she needs to forecast the future expected demand of final consumers first, and then she builds the demand function in the reservation market. Finally, she transfers the purchasing cost to the retailers using an exogenous ratio or directly to consumers via a lump-sum tax. This assumption corresponds, in fact, to the traditional approach used in the literature on the reservation market. We call this market design the centralized ex-ante option because it does not depend on retailers' realized strategy but rather on exogenous factors such as their past market share. In other words, this section describes only the direct effect of the reservation markets via the incentive to invest by the price of the capacity. There is no effect on the demand because the price of the capacity is simply a surplus transfer from consumers to producers. This approach's result is that the price of capacity when the vertical demand function is calibrated to k^* is equal to the optimal payment allowing to restore an optimal level of capacity whatever the type of inefficiency considered. This result is described in the following proposition and implies that the cost of a reservation market is strictly equal to the transfer necessary to restore the optimal level of capacity.

Proposition 2. *Assuming that*

1. *Producers do offer the marginal opportunity cost on a reservation market (see eq. 10)*

2. *The demand function is designed such that the clearing quantity is equal to the optimal level of investment*

3. *Tthe underinvestment is caused either by the market power in the retail market, a price cap, or inefficient rationing*

Then the clearing price is always equal to the optimal payment needed to restore efficiency.

This result highlights the discussion between implementing a price or a quantity instrument to resolve the market inefficiencies or constraints (Weitzman, 1974; Holmberg und Ritz, 2020). We show in this lemma that the quantity output of the reservation market is strictly equivalent to a capacity price set by the regulator. Under this regime, the centralized approach is optimal because it gives the right investment level given the inefficiencies.

We provide now some comparative statics on the capacity price given the model specification. We show that when only the price cap is considered in the model, the results are intuitive and in line with previous works, with the capacity price being always positively impacted by an increase of the demand intercept or by the product costs (variable and fixed), while the price cap has a negative effect. However, their effect can be ambiguous when we consider inefficient rationing. We start with the optimal payment to the producer defined in 9 which is equal to the absolute value of the marginal surplus loss associated with the inefficient rationing. Recall that it is composed of two distinct parts (i) a positive value for the direct effect of the rationing on consumer welfare and (ii) a negative value for the indirect effect because the loss is based on consumer welfare. We find that the price cap and the demand intercept always have opposite signs between parts (i) and (ii). For instance, an increase of the price cap continuously decreases the positive value associated with the rationing, but because it also decreases the consumer welfare when the price cap binds, then it decreases the negative value, hence the ambiguous effect. To overcome this issue, we derive the necessary condition on the price cap, such as the variables always have a clear-cut effect on the optimal payment. If the threshold is respected, then the price cap always lowers the optimal payment, and an increase of the demand intercept always increases the optimal payment. Then, we derive the comparative statistics of the capacity price given the inefficient rationing, and we find the conditions for which the price cap has a positive effect on the capacity price, and the demand intercepts a negative effect.

4.2 An endogenous centralized demand function with ex-ante requirements

4.2.1 Without inefficient rationing

We now introduce an indirect effect of the reservation market demand function. It implies that the capacity prices marginally impact the final consumer demand via the centralized entity's allocation cost. To compare with the previous setting without this indirect effect, the first previous case can be understood as an increase of the fixed part in a two-part tariff (or a lump sum tax), while the second case in this subsection can be understood as an increase of the variable part (or a unitary tax). The central idea is that we enhance our understanding of an optimal investment level given a market design by highlighting this effect. The first-best solution is endogenous to the market design implemented to reach this first-best. We use a similar approach of the impact of a tax on a partial equilibrium model to illustrate this endogenous effect on investment decisions. We demonstrate the existence of the indirect effect by repeating the steps of the previous model.

Fourth stage - Retail market. Lets $p^c(k)$ be the capacity price adder for final consumers, identical to a unitary consumption tax. k is the quantity bought on the reservation market by the central authority at a price $p^c(k)$

Third stage - Wholesale market The final consumers demand function shifts to the left with its new value equal to: $\tilde{p_1}(q,t) = p^s(q,t) - p^c(k)$. While the demand is always lower or equal to the initial demand function, the impact on the whole system's equilibrium is not trivial. The following proposition summarizes the main insight and states that the new welfare function is always lower or equal to the exogenous case.

Proposition 3. *Allocating the capacity price as a unitary tax only affects the share between on-peak and off-peak periods and the surplus's size during off-peak periods. Namely, only the occurrence of the two periods $t_0(k)$ and the intersection between the demand function and the marginal cost q_0 change, the welfare function becomes:*

$$W_1(k) = \int_0^{t_1(k)} \int_0^{q_1(t)} (p(q,t) - c)dq\ f(t)dt + \int_{t_1(k)}^{+\infty} \int_0^{k} (p(q,t) - c)dq\ f(t)dt - rk$$

With $t_1(k)$ and $q_1(k)$ the new thresholds for respectively the states of the world between on-peak/off-peak periods, and the quantity such as prices are equal to the marginal cost. We can rewrite the equation by showing the initial welfare function without endogeneity: $W(k) - W_1(k) = \Delta W_1(k)$. With

$$\Delta W_1(k) = \int_0^{t_0(k)} \int_{q_1(t)}^{q_0(t)} (p(q,t) - c)dq\ f(t)dt + \int_{t_0(k)}^{t_1(k)} \int_{q_1(t)}^{k} (p(q,t) - c)dq\ f(t)dt$$

The first part of $\Delta W_1(k)$ represents the loss when it is off-peak periods for both cases (indeed we have $t_0(k) \leq t_1(k)$ as lower demand always means a higher chance of being off-peak): the consumers fully support the loss as producers receive the marginal cost. The second part represents the loss when the capacity level is such that it is an off-peak period with the endogenous case and an on-peak for the other case. Therefore, the loss is shared between consumers and producers, the former sustaining a higher price and the latter receiving a lower margin. Note that there is no loss when both cases are in peak periods, as the quantity on the market is strictly equal to the capacity installed. This last remark is particularly interesting because recovering the capacity cost allocation only during peak periods does not generate a deadweight loss. Considering the price cap does not change the previous overall proposition, as the price cap is binding only when both periods are on-peak. That is when no loss is generated. The following lemma concludes on the new optimal investment level given this endogenous regime. It has a strong implication as we state that this regime is always worse than the exogenous regime regarding social welfare.

Lemma 4. *The new first-best solution in terms of investment level given under the endogenous regime is always lower or equal to the first-best solution under the exogenous level. In terms of welfare analysis, the social welfare at the optimal investment level is always lower or equal to the social welfare at the optimal investment level under the exogenous regime.*

The result stems from the analysis of the derivative of $\Delta W_1(k)$ with respect to the level of investment k, which is always positive. We provide now some comparative statistics on the difference between the two welfare function $\Delta W_1(k)$ for the model specification. By construction, the comparative statistic for the difference between the welfare has the same effect on the new first-best solution in terms of investment level. We demonstrate that the price cap and the demand intercept always have a negative effect on $\Delta W_1(k)$. An increase of the price cap reduces the need for the capacity payment through the negative value of the derivative of $q_1(t)$, with respect to the price cap, hence the endogenous effect off the cost allocation. At first sight, a higher demand through the demand intercept has an ambiguous effect on the difference. An increase in its value increases the need for capacity, which also increases the payment. On the other hand, it has a decreasing effect that materializes through the derivative of $q_1(t)$. We find that the second effect always dominates the first, hence the net decreasing effect of the demand intercept on the difference. Finally, an increase in the producer's cost (fixed and variable) always increases the delta, making the investment less profitable. We continue the analysis of the endogenous regime by defining the main equilibrium variables of the economic system.

Second stage - Investment decisions Producers make their investment decision based on the expected net revenue, which is composed of the expected inframarginal rent and the capacity revenue. The net revenue is similar to the exogenous case, except for the new state of the world thresholds and the wholesale price.

$$\tilde{\phi}(k) = \int_{t_1(k)}^{t_1^w(k)} (\tilde{p_1}(k,t) - c)\, f(t)dt + \int_{t_1^w(k)}^{+\infty} (p^w - c)\, f(t)dt + p^c(k) \tag{11}$$

With $t_1^w(k)$ the first state of the world when the price cap is binding with the demand function $\tilde{p_1}(k,t)$. The market equilibrium is found by solving: $\tilde{\phi}(k) = r$. Recall that $\tilde{p_1}(q,t) = p^s(q,t) - p^c(k)$.

First stage - Reservation market When a producer participates in the reservation market, he bids its marginal opportunity cost without the capacity revenue equal to $r - \phi - p^c(k)$. Therefore, following the previous stage, the equilibrium is defined with the following equality $X(k) : p^c(k) =$

$r - \tilde{\phi}(k) - p^c(k)$. The following proposition states how the remaining equilibria are found in the economic system.

Proposition 4. *When the capacity price enters the final consumers demand as a marginal cost, solving the following equation allows to find the market investment equilibrium:*

$$p^c(k) = r - \left(\int_{t_1(k)}^{t_1^w(k)} (p^s(k,t) - c - p^c(k))\, f(t)dt + \int_{t_1^w(k)}^{+\infty} (p^w - c)\, f(t)dt \right) \tag{12}$$

To find the system equilibrium we proceed similarly to the backward induction approach with

1. *Define the wholesale equilibrium with $\tilde{p_1}(k,t)$, $q_1(t)$, $t_1(k)$ and $t_1^w(k)$ using $p^c(k)$.*
2. *Solve the expected inframarginal rent with $p^c(k)$ from the previous condition.*
3. *Determine the reservation market equilibrium with the actual value for $\tilde{p_1}(k,t)$, $q_1(t)$, $t_1(k)$ and $t_1^w(k)$.*

Regarding the reservation market, as the demand is lower under this regime, the opportunity cost associated with providing another capacity is higher. By extension, the supply function on the reservation market is also higher. Therefore, this regime has an ambiguous effect on the reservation market equilibrium: capacity prices can be higher or lower than with exogenous capacity prices, even though the quantity is always lower. This implication can be summarized by defining the optimal payment to restore the optimal level with an endogenous price. Recall that with only a binding price cap, the optimal payment is the expected difference between what should have been the wholesale price and the price cap when the price cap is binding. The following proposition defines the new optimal payment, and we compare it with the previous with the exogenous regime.

Lemma 5. *The optimal payment to restore efficiency when a price cap is binding is defined as follow:*

$$z_1(k) = \int_0^{t_1(k)} \frac{\partial q_1(t)}{\partial k}(p(q_1,t) - c) f(t)dt + \int_{t_1(k)}^{t_1^w(k)} r - \tilde{\phi(k)} f(t)dt + \int_{t_1^w(k)}^{+\infty} (p(k,t) - p^w) f(t)dt \tag{13}$$

Compared to the initial payment $z^w(k)$, only the third part of the optimal payment is directly related to the expected difference between the optimal wholesale price and the price cap. In this regime, the magnitude of the loss is impacted via $t_1(k)$, which means fewer periods during which the price cap is binding. The first part represents the loss associated with the shift of the threshold for on-peak/off-peak periods. It is negative as $\frac{\partial q_1(t)}{\partial k} \leq 0$. When $q_1(t)$ decreases due to the capacity price, the inframarginal rent is lower when the capacity starts binding at $t_1(k)$. The second term is positive, and it is directly related to the loss associated with the decrease of the demand during on-peak, which also decreases the inframarginal rent for any states of the world between $t_1(k)$ and $t_1^w(k)$.

4.2.2 With inefficient rationing

What is happening when we include the rationing cost? We still hold the same assumptions as previously with the rationing cost of the form $M(k)$. The new welfare function becomes:

$$W_1^{bo}(k) = W_1(k) - \int_{t_1^w(k)}^{\infty} J(\Delta_1 k) f(t) dt$$

With $\Delta_1 k$, the new function representing the difference between the quantity consumes at the price cap and the investment level. By construction, the main results for the rationing hold under the endogenous regime, especially in terms of the first-best solution, imply a higher investment level and lower welfare than the case without inefficient rationing. Note that the quantity at the price cap $q_0(k)$ does not depend on the level of investment under the exogenous regime. However, when the capacity price is allocated under the endogenous regime, the quantity $q_1(k)$ is indirectly affected by the investment level.

We focus our analysis on comparing the first-best solution under the exogenous regime and the first-best under the endogenous regime. The following lemma shows that with inefficient rationing, the effect of an endogenous regime is ambiguous on the social welfare, which depends on the size of the negative effect previously described of the capacity price and the gains in terms of avoided rationing cost.

Proposition 5. *The delta in welfare with respect to the delta with exogenous price is:*

$$\Delta W_1^{bo}(k) = \Delta W_1(k) - \int_{t_0^w(k)}^{t_1^w(k)} J(\Delta_0 k) - \int_{t_1^w(k)}^{+\infty} (J(\Delta_0 k) - J(\Delta_1 k)) f(t) dt \tag{14}$$

With $W^{bo}(k) - W_1^{bo}(k) = \Delta W_1^{bo}(k)$

The new surplus when the optimal level of investment is reached is higher than with an exogenous capacity price when the endogenization effect on the rationing cost (i.e., the two negative parts of $\Delta W_1^{bo}(k)$, as we always have $J(\Delta_1 k) \leq J(\Delta_0 k)$) is higher than on the initial welfare (i.e., $\Delta W_1(k)$). Recall that $\Delta W_1(k)$ represents the loss associated with the effect of the capacity price on consumer demand. The first negative part in $\Delta W_1^{bo}(k)$ stands for the lower occurrence of on-peak periods due to the lower demand, which reduces the rationing cost. The second negative part represents the loss avoided because a lower consumer demand implies a lower consumer surplus, hence a lower cost than the exogenous regime.

While there is an ambiguity regarding the new value of the welfare function, we find that the new optimal investment level is always lower than without inefficient rationing. That is we always have $\frac{\partial \Delta W_1^{bo}(k)}{\partial k} \geq 0$. Regarding the social welfare at the optimal investment level, it depends on the ambiguous effects described in Proposition 5.

4.3 Extension - an endogenous centralized demand function with ex-post requirements

The previous approach is based on ex-ante requirements, meaning that the quantity allocation onto the retailers (or directly to the consumers) is independent of the demand's current realization. The section analyzes a specific practical implementation of the reservation market demand curve where the capacity allocation depends on the retailers' realized quantity sold to the final consumers. The main difference with previous ex-ante requirements lies in the retailer profit function, where the capacity cost allocation act as an additional marginal cost. We study how this new cost adder for retailers modify the previous results in light of different degrees of competition in the retail market. Under competition *à la Cournot*, we find that having different numbers of retailers have a direct effect on the cost allocation sustained by final consumers. Therefore, the degree of competition determines the sign and the magnitude of the different outcomes described in the ex-ante regime.

The first implication of ex-post requirements concerns the last stage when the retail market clears. We rewrite the retailers' profit function by including an endogenous ratio in the retailer profit function. Contrary to the previous section, we do not need to assume any tariff hypothesis for the capacity cost allocation as it directly affects retailers' profit at the margin. We focus our analysis on symmetric equilibrium.

$$\pi_i^r(q_i, k) = q_i(p(q) - p^s) - p^c(k)k\frac{q_i}{q_i + q_{-i}}$$

We find the equilibrium using the first-order condition. The main results are stated in the following proposition:

Lemma 6. *When the retail equilibrium exists it is always unique and stable. The condition for the existence of an equilibrium is given by the following condition:*

$$-kp^c(k)\left(\frac{n-2}{n}\right)\frac{1}{q^2} \leq p_q(q) + \frac{q}{n}p_{qq}(q) - p^s$$

Using the first-order conditions and the symmetry between the retailers, the Cournot equilibrium in the retail markets allows to define the endogenous retailer demand function in the wholesale market:

$$\tilde{p_n}(q) = p(q) + \frac{q}{n}p_q(q) - p^c(k)k\frac{1}{q}\frac{n-1}{n}$$

The equilibrium in this market is similar to the case with the ex-ante requirement. Therefore, we can define the periodic threshold between on-peak/off-peak/binding price cap periods. We respectively denote them $t_n(k)$ and $t_n^w(k)$, with also $q_n(t)$ and $q_n^w(t)$ the quantity at which the wholesale price $\tilde{p_n}$ is equal to respectively the marginal cost and the price cap. Note that the wholesale market equilibrium does not necessarily exist as the demand function is not well defined on wholesale quantity and prices. To see this, recall the existing condition for the retail equilibrium. With a high value of q, some retail equilibrium does not exist. Therefore the retail threshold also applies to the wholesale market.

Given the inframarginal investment and the variable revenue from the reservation market defined as follow, we find the marginal opportunity cost that defines the supply function in the reservation market and hence the final capacity price:

$$p^c(k) = r - \left(\int_{t_n(k)}^{t_n^w(k)} (p(k,t) - c - p^c(k)\frac{n-1}{n}) \; f(t)dt + \int_{t_n^w(k)}^{+\infty} (p^w - c) \; f(t)dt \right)$$

Note the difference with the ex-ante requirement in the first part of the integrals, where the capacity cost adders are dependent on n. The following proposition summarizes the main effect of an ex-post allocation:

Proposition 6. *When allocating the reservation market cost is based on retailers' realized market share, it generates a lower depreciating effect on the demand. The ex-post regime provides an intermediate indirect effect between an ex-ante regime with exogenous capacity price and an ex-ante regime with endogenous capacity.*

To illustrate this proposition, the capacity cost adder when $n = 2$ is equal to half of the cost adder of equation 12, and it is increasing with n. When $n \to +\infty$, the capacity cost is entirely allocated to the consumer, mimicking the ex-ante exogenous equilibrium. This proposition states that increasing competition in the retail market increases the burden of consumers' capacity prices. Hence, the negative effect observes in the centralized regime with endogenous capacity prices is now shared between retailers and consumers.

Regarding the previous results found in the ex-ante regime, the proposition has ambiguous implications for the economic system. First, suppose there is no inefficient rationing. In that case, an increase in the number of retailers has two effects of opposite sign : (i) an increase of the social welfare, which is the common effect of higher competition in a canonical model *à la Cournot* (ii) a decrease in the social welfare due to the lowering of the consumption associated with a higher capacity cost allocated to the consumers. Both effects are defined in the following equation: the derivative of the welfare function with respect to the number of retailers. Under our framework and without the ex-post regime, the demand is only affected by the degree of competition during off-peak periods when $t \leq t(k)$,[12] and the derivative is always positive. However, the capacity cost allocation also has a depreciating effect on the demand level at the marginal cost (recall equation 13), hence the ambiguous sign of $\frac{\partial q(k)}{\partial n}$.

$$\frac{\partial W(k)}{\partial n} = \int_0^{t(k)} \frac{\partial q(k)}{\partial n}(p(q(k),t) - c)f(t)dt \tag{15}$$

A similar but more complex effect arises when considering the inefficient rationing. Paradoxically, a lower competition on the retail market makes the rationing occurrences less likely, which lowers the surplus loss. Note that as the surplus loss is only associated with on-peak periods, the lowering of competition in the retail market does not affect the size of the rationing cost, but only its occurrence. We provide in the following equation the marginal effect of an increase of the number of retailers on the social welfare when the rationing cost is based on a ratio as described in equation 7, as in the previous equation the sign of $\frac{\partial q^w(k)}{\partial n}$ is also ambiguous:

$$\frac{\partial W(k)}{\partial n} = \int_0^{t(k)} \frac{\partial q(k)}{\partial n}(p(q,t) - c) - \int_{t^w(k)}^{+\infty} (\frac{\partial q^w(k)}{\partial n} \frac{k}{q^w(k)^2} \int_0^k (p(q,t) - p^w)dq)f(t)dt \tag{16}$$

It is sufficient that the sign of the two derivatives be different so that the effect of imperfect competition on social welfare is clear. For instance if the increase of n increases the surplus during the off-peak periods ($\frac{\partial q(k)}{\partial n} > 0$) but decreases the rationing occurrence due to the capacity cost adder ($\frac{\partial q^w(k)}{\partial n} < 0$) then the social welfare increases with n. On the other hand, if n has the same effect on both quantity thresholds $q(k)$ and $q^w(k)$, then the role of imperfect competition on the outcome is ambiguous. For instance, if the increase of competition increases the welfare during off-peak periods ($\frac{\partial q(k)}{\partial n} > 0$) but also increases the demand at the price cap despite the capacity cost adder ($\frac{\partial q^w(k)}{\partial n} > 0$), then the effect is unknown and depend on the relative size of the welfare gain during off-peak periods and the loss occurring during rationing.

[12] We drop the subscript for simplicity.

5 Decentralized demand

We provide in this section an analysis of an alternative regime for the demand function in reservation markets. In this implementation, each retailer must purchase their capacities in the reservation market. The regulated entity only monitors the level of capacities and compares it to each retailer's consumption. We show that this market design can bring some other welfare due to its specific incentives, but one should note that this regime can also have pros and cons depending on additional hypotheses regarding the initial assumptions. For instance, the quality and quantity of information detained by private agents such as retailers in future states of the world can be seen as better or worse than the information detained by the regulated entity. While those specifications are outside the scope of this work, they should call for a deeper application of the model presented in this paper.

One of the critical features of this regime concerns the case when a retailer is in negative deviation, i.e., has sold more on the retail market than he has bought capacity in the reservation market. In this case, he suffers a penalty, which results in a payment from the retailer to the regulated entity[13] by a unitary amount of S, with $S \geq 0$ being an administratively fixed value[14]. When every retailer has bought enough capacity, we are under the no penalty case, and no other mechanism is implemented [15]. The price in the reservation market is still noted as $p^c(k)$, and the individual quantity contracted by a retailer i is k_i. Under this regime, we process as follows to describe the equilibrium: (i) We provide an analysis of the outcome when there is no uncertainty. We use a simple game theoretical framework to describe a game's equilibrium where agents must sequentially choose a fixed capacity first and then compete *à la Cournot* on a second game. (ii) Then, we extend our analysis to the initial framework developed in the model with investment and reservation decisions made before the demand is known.

[13] Which acts like the government is the model.

[14] We use in this paper a linear form of the penalty system, but some implementation can encompass nonlinearities depending on the effect desired for the penalty system

[15] Some remuneration mechanism can exist so to reward retailers who have provided additional capacity, but as we focus on symmetric equilibrium, they do not play a role in the outcome.

5.1 Market equilibrium without uncertainty

To provide the intuition for the general case with uncertainty, we start by describing the equilibrium in the case the demand is known when the retailer has to choose the level of capacity to be bought in the reservation market. We show that it is a dominant strategy to integrate the penalty value in the profit function as a marginal cost up to a point where it is optimal to stop buying capacity and sustain the penalty.

Given the assumptions and notation, the retailers' profit function during the last stage in the retail market is:

$$\pi_i^r(q_i, k_i) = q_i(p(q) - p^s) - p^c(k)k_i + \begin{cases} +0 \quad \text{if} \quad \forall i \quad q_i \leq k_i \\ \\ -S(k_i - q_i) \quad \text{if} \quad q_i > k_i \end{cases}$$

The equilibrium on the retail market is given by the first order condition:

$$p(q) + q_i p_q(q) - p^s = \begin{cases} 0 \quad \text{if} \quad \forall i \quad q_i \leq k_i \\ \\ S \quad \text{if} \quad q_i > k_i \end{cases}$$

As the equation shows, the decentralized market design's penalty system implies different discontinuous retailers' reaction functions. The first case will be called the penalty case, while the second one the noremuneration case. It depends on the capacities bought in the reservation market by the retailer and by his competitor and their strategies on the retail market. The penalty system always induces a lower reaction function, whatever is the sign of the retailer's deviation from its position in the reservation market. It is straightforward for the penalty system: a marginal increase in the retail market's quantity increases the marginal cost via the penalty. The capacities bought in the reservation market do not directly affect the reaction function's value, but it determines the form of the reaction function between the penalty/noremuneration cases. We summarize in the following proposition the central insight of this game equilibrium.

Lemma 7. *The set of dominant strategies in the retail market is:*

$$\begin{cases} [q^p, q^r] \qquad \textit{if} \qquad p^c(k) \leq S \\ \\ \{0,]q^p, q^r]\} \qquad \textit{if} \qquad p^c(k) > S \\ \\ \forall q \in]q^p q^r[\qquad q \textit{ is a solution of} \qquad p(q) + q_i p_q(q) - p^s(q) - p^c(q) = 0 \end{cases}$$

With q^r the equilibrium quantity offered on the retail market when the retailers are in the noremuneration case. This value is given by the solution of $p(q) + q_i p_q(q) - p^s(q) = 0$. q^p is the equilibrium quantity offered on the retail market when the retailers are in the penalty. This value is given by the solution of $p(q) + q_i p_q(q) - p^s(q) - S = 0$.

The optimal quantity on the retail market and the reservation market depends on the difference between the penalty value S and the capacity price $p^c(k)$. We assumed a linear penalty system, so if the penalty is lower than the capacity price $S \leq p^c(k)$, then it is a dominant strategy (strict if $S < p^c(k)$) to buy no capacity and sustain a penalty on all the quantity sold on the retail market. Indeed, with strict inequality, the profit function is a decreasing non-concave function with respect to k_i. On the other hand, when the penalty is higher than the price, the profit function is an increasing non-concave function with respect to k_i. The dominant strategy is to buy the same amount of capacity as the quantity q^p, which corresponds to the retail market's corresponding equilibrium in the penalty case.

In the decentralized case, the capacity price is passed on to the consumers, which increases the marginal cost equal to the capacity price. In turn, it reduces the demand function on the wholesale market. This result follows that the demand function in the reservation market strictly mimics an equilibrium quantity sold on the retail market when the marginal cost increases without a reservation market. When the cap S is not binding, those equilibria are given by the set $[q^p, q^r]$.

Under this no uncertainty assumption and relying on the set of dominant strategies, it is straightforward to extend the analysis to the optimal demand of capacity. Using the results of

7, the demand function on the reservation market is a linear decreasing function caped above at the penalty value while intersecting the null capacity price at exactly q^r which is the equilibrium without the reservation market. Any value between corresponds to the equilibrium given by the solution of $p(q) + q_i p_q(q) - p^s(q) - p^c(q) = 0$. To say it differently, given a capacity, retailers always buy the same amount of capacity that their equilibrium in the retail market unless the price is above the penalty value

On the supply side, following the marginalist approach, the supply function starts to be non-null and positive at the same value q^r. Therefore the equilibrium is always a null price with a level of investment strictly equal to the regime without any reservation market. Such counter-intuitive result stems from the fact that under no uncertainty and without any other refinement of the model, the reservation market is only a burden for retailers, which does not incite them to buy more capacities.

5.2 Social welfare under decentralized demand

In reality, retailers buy capacity before the demand is known. Hence, when building their demand function in the reservation market, they must consider a range of possible outcomes relative to the production and demand levels. They need to expect the occurrence of off-peak and on-peak periods and the rationing case when the price cap is binding. The last situation is critical as it determines the magnitude of the penalty payment bared by retailers. We start by providing the link between the previous analysis with no uncertainty and the general model. First, q^r is the value $q_0^w(t)$ as it is the *Cournot* outcome in the retail market[16]. Then, denote $q_d^w(t)$ the value equal to q_p, that is the threshold at which $p^s(q) - S$ is equal to the price cap p^w. In other terms, this is the *Cournot* equilibrium when the marginal cost for the retailers is equal to $p^w + S$. It is similar to assume a decrease in demand when retailers pass the penalty cost onto the consumers. Finally, denote $t_d^w(k)$ the state of the world when the price cap starts binding under the case the demand of final consumers is equal to $p^s(q) - S$.

Following the no uncertainty case, the set $[q_d^w(k), q_0^w(k)]$ defines the set of dominant strategies in the retail market for a wholesale price equal to p^w and any capacity level between 0

[16]In fact, every value between $q_0(t)$ and $q_0^w(t)$ are conceptually equal to q^r, but we focus on the threshold case between the periods where the price cap is binding and not binding

and the value q_0^w. To see this, let us distinguish three cases depending on the value of the installed capacity.

- (Case 1) When $k > q_0^w$, the price cap is never binding, and the outcome is strictly identical as a regime without a reservation market.
- (Case 2) For a value of k between $q_d^w(k)$ and $q_0^w(k)$, we observe a paradoxical outcome; rationing should have occurred as soon as k is below $q_d^w(k)$ without a penalty. It implies that retailers sustain the penalty, which is passed onto the consumers as a marginal cost, who lower their demand. However, rationing is not happening, which contradicts the demand's decrease due to the penalty. Therefore, as in the no uncertainty case, retailers follow the level of investment. To do so, they increase the price of their consumers by a unitary amount of $T(k)$ so that at any states of the world between $t_0^w(k)$ and $t_d^w(k)$ the demand is equal to the capacity k,[17] that is we have $p^s(q) - T(k) = p^w$.
- (Case 3) Finally, k is below $q_d^w(k)$, it is now optimal for the retailers to keep their strategy at q_0^w as it is the *Cournot* equilibrium given the penalty value S.[18].

The distinction between the two different cases ((2) and (3)) is crucial as in the former case (2) there is no inefficient rationing as the quantity sold by the retailers is equal k, while in the latter case (3) inefficient rationing necessary occurs because retailers ask for $q_d^w(k)$ which is above k.

Using this set of outcomes on the retail market, we can deduce the wholesale market outcomes. The following proposition describes how the new welfare function encompasses the previous implications regarding the retailer strategy when we assume that the initial capacity price is not passed onto the consumers via a unitary payment.

Proposition 7. *The decentralized regime ambiguously impacts the welfare function, with a negative distributional effect due to the penalty system and a positive effect due to reducing rationing costs. The rest of the welfare function is equal to the no reservation market regime.*

[17]We could also assume the reverse mechanism where retailers pay consumers $T(k)$ to reduce the demand in order to avoid the penalty

[18]Similarly, it is identical to the case where the benefice to pays consumers to lower their demand is above the cost generated by the penalty

$$W_1(k) = W^{bo}(k) \quad - \int_{t_d^w(k)}^{+\infty} S(q_d^w(t)+k) + \int_{t_0^w(k)}^{t_d^w(k)} J(\Delta_0 k) + \int_{t_d^w(k)}^{+\infty} J(\Delta_0 k) - J(\Delta_d k) f(t) dt \quad (17)$$

With $\Delta_d k$ the new difference between the installed capacity and the quantity bought by the retailers $q_d^w(k)$

The intuition behind the proposition is as follows. For the second negative part of the welfare function, the penalty S entirely affects the retailers' profit margin. It implies that both the retailers and the consumers suffered a surplus loss due to the demand reduction. Added to the penalty cost borne by the retailer of $S(q_d^w(k) - k)$, it gives the net loss for the welfare function. The rationing cost reduction is directly linked to the shift from $q_0^w(k)$ to $q_d^w(k)$. Indeed, as stated before, any capacity k in $]q_d^w(k)q_0^w(k)]$ implies equality between the quantity sold by the retailers and the capacity level k. Therefore, there is a net increase in welfare. Note that the last term of the welfare function is always positive as $q_d^w(k) \leq q_0^w(k)$. It is similar to the effect observed in the centralized case with endogenous capacity price, but it is limited to the rationing period in this regime. Therefore, the decentralized regime avoids the negative effect of having a lower demand when there is no rationing, and the wholesale price is optimal.

5.3 Investment decision and reservation market equilibrium with uncertainty

Assuming that the capacity price is not included in the variable price in the retail market[19], we states that the supply function is not impacted by the decentralized market design. Indeed, the previous analysis shows that the demand function on the wholesale market is impacted only when the price cap start binding under no reservation market regime (ie. when $k \leq q_0^w(k)$). In case (2) the adjusted demand follows the capacity level, while in the the case (3) the inefficient rationing exists but it still imply that demand equal capacity. Therefore, the occurrence and the magnitude of the inframarginal rent is not impacted by the decentralized market design. The supply function is given by the equation 10 and the undistorted inframarginal $\phi(k)$. Under this configuration,

[19] We provide in the numerical illustration some result with the impact of the capacity price included in the variable price

we define the aggregated retailer profit function when the demand is uncertain in the following equation:

$$\pi^r(k,t) = \int_0^{t_0(k)} -\frac{q_0(t)^2}{n} p_q(q_0(t),t)f(t)dt + \int_{t_0(k)}^{t_0^w(k)} -\frac{k^2}{n} p_q(k,t)f(t)dt \qquad \text{Case (1)}$$

$$+\int_{t_0^w(k)}^{t_d^w(k)} k(p(k,t) - p^w - T(k,t))f(t)dt \qquad \text{Case (2)}$$

$$+\int_{t_d^w(k)}^{+\infty} k(p(k,t) - p^w - S)f(t)dt - \int_{t_d^w(k)}^{+\infty} S(q_0^w - S)f(t)dt \qquad \text{Case (3)}$$

$$-p^c(k)k_i$$

The expected profit function comprises three main parts related to different values of k given a demand level (or a different level of demand given a value of k). The two first terms are the same with and without a reservation market as the price cap is not binding. The retail price rises while the wholesale price is fixed and equal to the price cap for the second term. As explained in the previous analysis, between the two states of the world $t_0^w(k)$ and $t_d^w(k)$, the demand decreases due to the retailers' actions to avoid paying the penalty. It is materialized by the transfer $T(k,t)$ [20]. When it is not profitable to reduce the demand given the penalty (when $t_d^w(k)$ is reached), then the new demand is given by $q_d^w(k)$, the retailer profit is in the fourth term, inefficient rationing is implemented, and the retailers pay the penalty in the fifth term. The last term is the capacity cost due to the retailer's obligation to buy their capacities. Given this expected profit, we can define the marginal value of a capacity for the retailer, which serves as the retailers' willingness to pay for an additional capacity.

Proposition 8. *Under a decentralized market design, the retailers aggregated demand function in the reservation market is equal to the marginal value of an additional capacity for their profit function.*

[20] Note that if we assume that retailers pay the consumers to reduce their consumption, only the sign changes

$$D^c(k) = \quad -\int_{t_0}^{t_0^w(k)} \left(\frac{2}{n}kp_q(k,t) + \frac{k^2}{n}p_{qq}(k,t)\right) f(t)dt \quad + \quad -\int_{t_0^w(k)}^{t_d^w(k)} \left(\frac{2}{n}kp_q(k,t) + \frac{k^2}{n}p_{qq}(k,t)\right) f(t)dt$$
$$+ \quad \int_{t_d^w(k)}^{+\infty} \left(p(k,t) - p^w + kp_q(k,t)\right) f(t)dt$$

The value of a capacity for a retailer depends on the effect a marginal variation brings to its profit function. When the level of capacity increases, we can distinguish three effects:

- (i) The decrease in the cost of the penalty during the case (3)
- (ii) An increase of the oligopolistic profit during on-peak periods when the quantity offered is equal to the capacity; and
- (iii) A change in the occurrence of off-peak/on-peak and price cap-binding periods.

The effects (i) and (ii) do not directly appear in the demand function as they are entirely offset. Indeed, for the penalty effect, while the increase of capacity lowers the marginal cost of penalty by S, the retailer gains at the same time a marginal profit equal to $p(k,t)+kp_q(k,t)-p^w-S$. Therefore, the marginal effect of the penalty is null, and the effect during the case (3) is limited to an increase of the marginal profit, as illustrated in the third term of the demand function. The effect (iii) of the occurrence of the different periods cancel each other out because when $q_0^w(k)$ is reached, the value of $T(k,t)$ is null. While, when $q_d^w(k)$ is reached the value of $T(k,t)$ is equal to the penalty. Finally, note that the first and second parts, which represent the net gain from an increase of capacity for retailers, are always positive as the marginal revenue is always decreasing following the initial assumptions regarding $p(.)$ with respect to q (i.e., $p_q(.) + qp_{qq}(.) < 0$).

The last part is ambiguous and depends on the value of k relatively to the monopoly outcome in this model. Indeed, note that $p(k,t) + kp_q(k,t) - p^w$ is the first-order condition of the retailer monopoly profit function with marginal cost equal to the price cap. Given its concavity, any value of k below the monopoly quantity implies a positive third part in the demand function, while a value of k above implies a negative third part. It is a sufficient condition that the *Cournot* outcome $q_0^w(k)$ bounds from below the monopoly outcome for the third part be always positive. In other terms, this condition holds whenever the penalty value is sufficiently high or with a low number of retailers. This last part of the demand shows that retailers are willing to pay for capacity if it allows them to reach (in expectation) the monopoly quantity when the price cap is binding. In other terms, they bid the expected marginal revenue that makes them indifferent between being at the monopoly outcome or no buying more capacity, given this third part is positive.

Finally, the general equilibrium in the system is found by solving $D^c(k) = p^c(k)$. As the supply function is entirely independent of the outcome of a decentralized market, the general outcome analysis is identical to the analysis of the reservation market demand function. For instance, it is sufficient to state that an increase of the demand function due to an increase of the penalty also increases the investment level. On the other hand, it can also indirectly affect the optimal level of investment given by maximizing the welfare function. Therefore, the comparative statistic for the decentralized regime's welfare effect boils down to the same approach described in the centralized case.

6 Closed form solution and application to the electricity market

6.1 Data

We provide in this section some insight on the previous general framework using a close-form solution and a numerical illustration. The main specification is the final consumers demand function, which is assumed linear, and where the uncertainty comes from the intercept of the linear function. We define the inverse demand function as follow :

$$p(q,t) = a(t) - bq$$

Where $a(t)$ is the uncertain intercept such as $a(t) = a_0 + a_1 e^{-t}$. We assume that t follows an exponential distribution which is characteristic of electricity consumption : $f(t) = -e^{-t}$. Other exogenous variables are summarized in the table and also follow the French data for the electricity system. The production cost is the one of a peak technology: the investment assumed to make the price on the electricity system when demand is high. We use the last report on the projected cost of generating electricity from the International Energy Agency XXXX. Regarding the coefficient value for the demand function, we use the data from (Léautier, 2014), and for the maximum realized demand, we use the data from the French system operator.

Coefficient intercept 1 ($)	a_0	18 827
Coefficient intercept 2 ($)	a_1	12 360
Maximal demand (GW)	Q_{Inf}	102
Marginal cost ($ /MWh)	c	79.55
Fixed cost ($ /MWh)	r	17.62
Demand slope	b	0.18

For the inefficient rationing, we use the specification defined in equation 7.

6.2 First-best and market equilibrium investment

Following the model specification, the welfare function is concave, which ensures a first-best solution exists. We provide in figure 1 an illustration of the relation between the first-best solution and the various inefficiencies that prevent the market equilibrium from reaching it. The black curve represents the initial social welfare for a different level of capacity as defined in equation 1 under perfect competition in the retail market. The upper black point gives the first-best investment level. Under imperfect competition (with $n = 40$)[21] we observe two different effects: (i) first the social welfare moves down to the red curve, which implies that given the level of imperfection on the retail market, the optimal is lower than under perfect competition (second black point below), (ii) a dissociation between the optimal level of investment and the market equilibrium. Market power in the retail market lowers the inframarginal rent, leading to the first red point from the right. Then the introduction of a price cap with a value of 200 $ widens the gap between the first-best and the market equilibrium with the second red point. [22]. Note that in that case, the welfare function curve is not directly impacted by the price cap as it only redistributes the welfare between producers and retailers.

[21]In 2021, there is 39 different retailers for the French retail market. This relatively high number should not hide that some of them have relatively high market shares due to historical reasons.

[22]The relatively low value implies that the risk of underinvestment could not be directly linked to explicit price caps in current power systems but because of implicit constraints such as risk aversion or uneconomic technical interventions of the system operator. In any case, we assume that an explicit price cap can capture all those inefficiencies. Such issue has already been addressed, for instance in Léautier (2016)

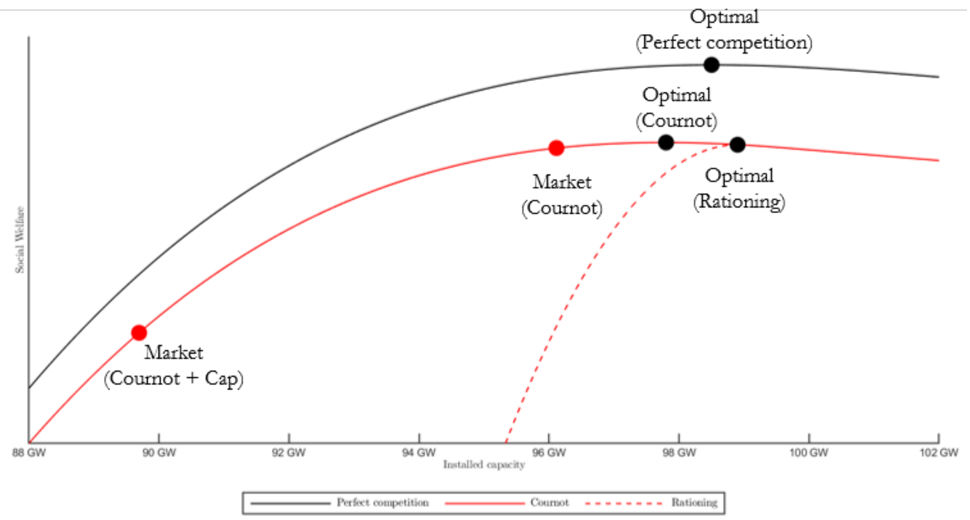

Figure 1: Social welfare function with respect to the investment level ($n = 40$,$p^w = 200$)

On the other hand, when we assume that the price cap can also create inefficient rationing, the welfare function is significantly lower by shifting to the right (red-dashed curve). In this case, the first-best solution implies a higher investment level than the case without inefficient rationing (black point to the right). Note that the social welfare value under inefficient rationing is equal to the social welfare with inefficient rationing when it reaches a certain level of investment. Above this value, the price cap is never binding in expectation, implying a null system cost.

We provide in figure 2 a comparison between the level of investment with respect to a different value for the price cap on the wholesale market and with perfect competition in the retail market. The black line represents the market investment equilibrium given the inframarginal rent of equation 4. As expected, the equilibrium converges toward the first-best as soon as the price cap increases. Around a value of 700 \$, the price cap stops binding in expectation which implies the equality between the first-best and the market equilibrium. When we introduce inefficient rationing, the rationing cost is negatively correlated with the price cap. Indeed, a lower cap means a higher chance in expectation to ration, which implies a higher surplus loss. Therefore, the first-best needs to increase to maximize social welfare. Similarly to the market investment, the system cost becomes null as soon as the price cap reaches a value of 700 \$ as it does not bind anymore.

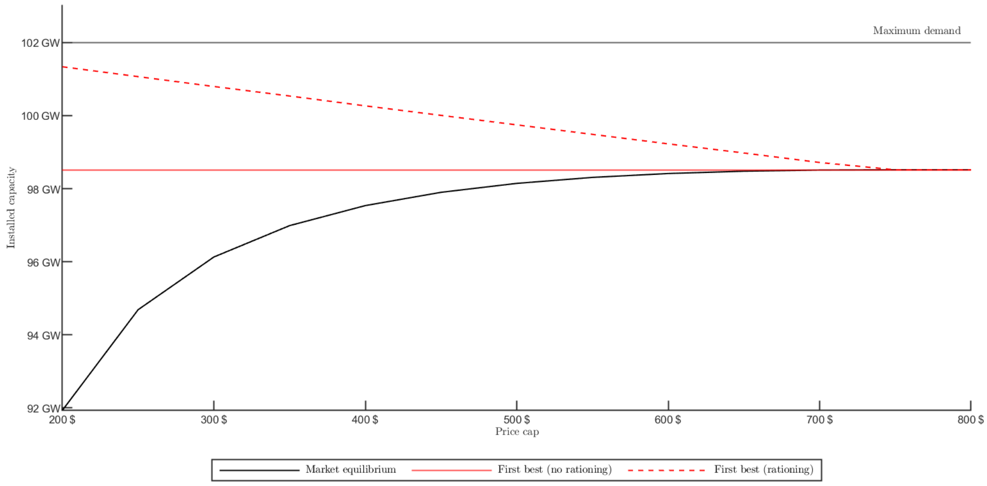

Figure 2: First-best investment and market equilibrium with respect to the price cap under perfect competition

6.3 Price equilibrium with a reservation market

One of the key components of the framework is the supply function build on the reservation market. Using a marginal approach, we assume that if the producer were to bids their investment availability, the supply function should represent the marginal opportunity cost of providing a capacity at the margin as in equation 10. We show in figure 3 the supply function (thick black line) on a reservation market with perfect competition on the retail market and with a price cap at 500 $. The supply function is null below a critical investment value equal to the market investment equilibrium (represented in figure 2 at the corresponding price cap value). Note that the supply function converges toward the fixed cost as at a certain level of investment, the additional inframarginal rent collected on the wholesale market becomes null. On the same figure, we have also represented the two optimal payments necessary to restore the first-best solution, both when the price cap does not generate inefficient rationing and when it does (with perfect competition in the retail market). As proven in the Proposition 2, the intersection between the supply curve and the optimal payments coincides with the first-best solution in terms of investment (represented with the two vertical lines). Consequently, the capacity price at the equilibrium under the exogenous regime is located at the two red dots depending on the inefficiency.

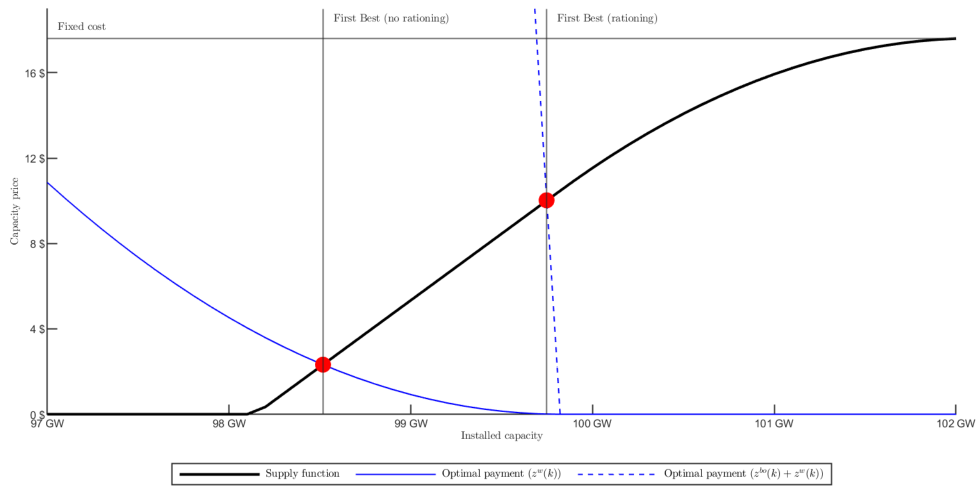

Figure 3: Supply function on a reservation market with the optimal payments ($p^w = 500$)

Following this framework, we derive some comparative statistics of the capacity price with respect to the price cap level on the wholesale market with perfect competition in the retail market and the number of retailers with a price cap equal to 500$. Figure 4 shows the different value for the capacity price given the price cap and the degree of competition on the retail market when we both consider only the direct price cap effect and then add the inefficient rationing. The price cap figure notes the respective convexity and concavity of the capacity price. This specific curvature stems from the value of the first-best solution with respect to the price, as shown in figure 2. While when only the price cap is considered, the first-best solution is constant with respect to the price cap. On the other hand, when inefficient rationing is considered, then the first-best decreases with respect to the price cap. As expected, the market structure on the retail market directly impacts the value of the capacity price. While it is significant when only the price cap is considered, note that it requires an important number of retailers to make the capacity price converge towards the perfect competition value (represented by the horizontal lines).

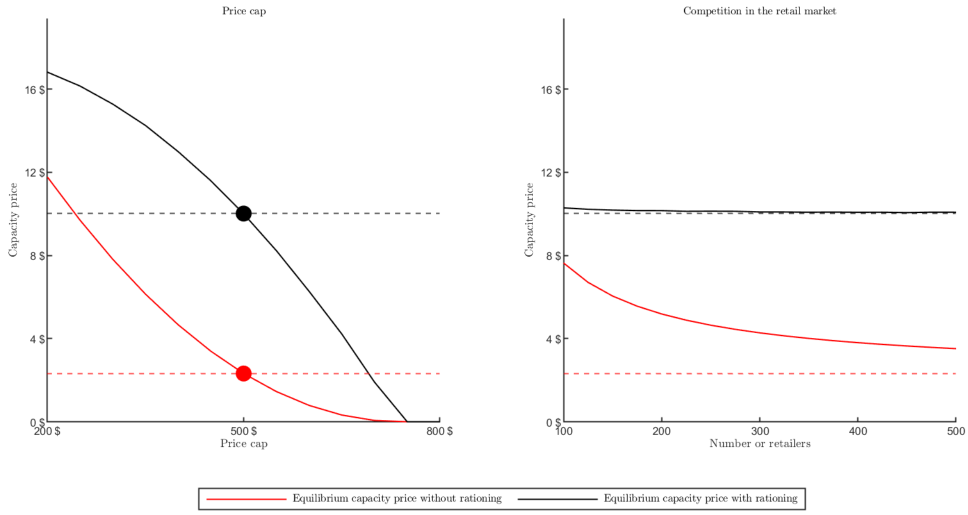

Figure 4: Capacity prices with respect to the price cap and to the number of retailers

6.4 Welfare effect of an endogenous ex-ante reservation market

We now study the effect of allocating the capacity price directly on the consumers via an increase in the unitary electricity price. We have demonstrated in Proposition 3 and Lemma 4 that the effect is negative when we only consider the price cap as a source of inefficiency. On the other hand, when rationing is considered, then an endogenous capacity price can bring the benefits in terms of social welfare at the first-best investment level, as shown in 5. We use the methodology in the proposition 4 to derive the equilibria for both cases. Following the previous figure 1 representation, we can analyze the effect of the endogenous market design with the value of the social welfare function. We illustrate in figure 5 such approach. The black line represents the initial social welfare function with a price cap but without inefficient rationing. When we implement the endogenous reservation market, the social welfare shifts below (red line). The new optimal investment under the endogenous (red point) level is slightly below the initial one with an exogenous market design (black point). However, one can see that the new welfare decreases. On the other hand, when the inefficient in considered, then we observe the reverse effect. We show the new social welfare given the surplus loss due to the rationing cost. Again, the black dashed line represents the exogenous case, while the red one then endogenous case. The new first best solution under the endogenous

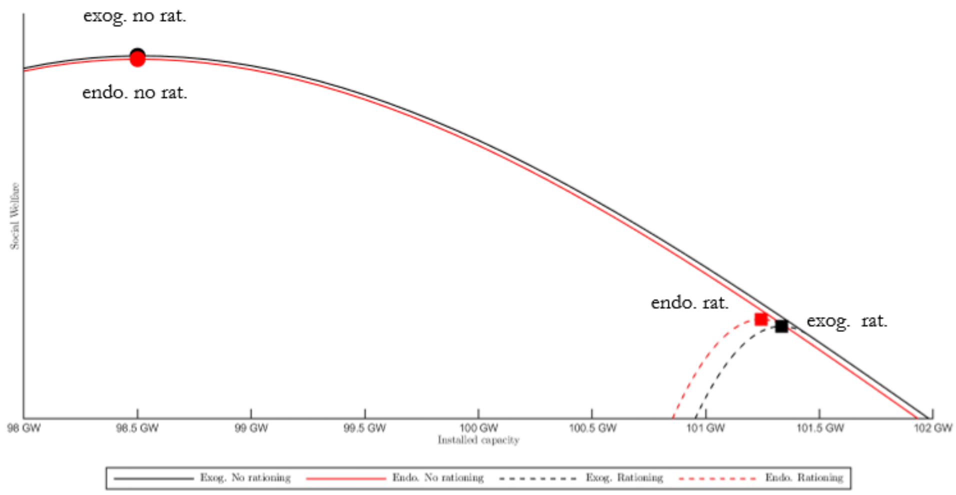

Figure 5: Social welfare function under exogenous and endogenous ex-ante market design

case (red square) is still bellow the exogenous investment level (black square), but this time the new welfare is higher.

Figure 6 illustrates the delta in social welfare at the first best investment level when comparing the exogenous ex-ante regime with the endogenous regime with respect to price cap values. The black line stands for the case when only the price cap creates an inefficiency, while the red line includes inefficient rationing. While they both converges to a similar value when the price cap increases, the numerical illustration shows that the endogenous regime brings a negative value in terms of welfare for the first case compared to the second case.

6.5 Retail market structure and ex-post requirements

The proposition 6 states that allocating the capacity price onto the retailers and their realized market share provide an intermediary indirect effect on the demand side. Namely, with a less concentrated retail market, retailers tends to pass more the capacity cost onto the consumers, hence mimicking the ex-ante regime with endogenous price. While we state that when the number of retailers tends to converges towards the same outcome as the ex-ante regime, the difference with an ex-ante regime with exogenous capacity price at a same degree of retail competition might be different with respect to the market structure. This relation is illustrated in figure 7 where

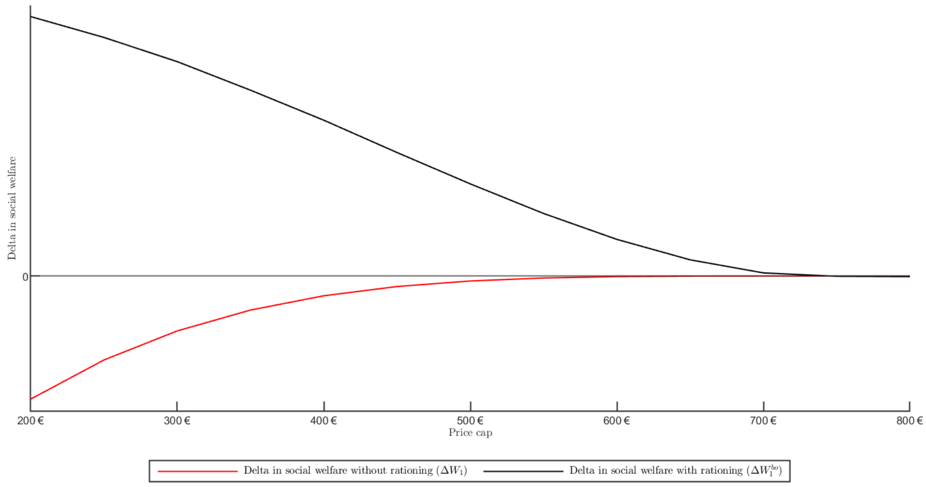

Figure 6: Delta in social welfare function at the optimal investment level between an exogenous and endogenous ex-ante market design

we draw the evolution of the first best investment level for different numbers of retailers. The red line stands for the exogenous case while the black line takes into account the realized market share in the outcome. While the difference in terms of investment is relatively small for a very concentrated retail market, it tends to widen as the number of retailer increases. Therefore, the point of convergence is different as shown in the figure.

One key element raised in this paper is the link between different market structure and the evolution of the social welfare for different retail market structure. In the analysis of the welfare function with respect to the market structure we found that the latter can have an ambiguous effect. Indeed, recall that increasing the number of retailer increase the welfare during off-peak periods but also increase the capacity price allocated towards the consumers, hence decreasing the welfare. A similar effect can be found when we take into account inefficient rationing. In figure 8, we show the couple price cap - number of retailers for which the effect is null. Namely, below the line on the first sub-figure, an increase of the number of retailers always increase the welfare, while above the line it always decreases the welfare. This threshold is decreasing with respect to the price cap. Note the inverse relationship when we take into account inefficient rationing (with still positive value below the line). This is due to the fact that in the first case only one effect is

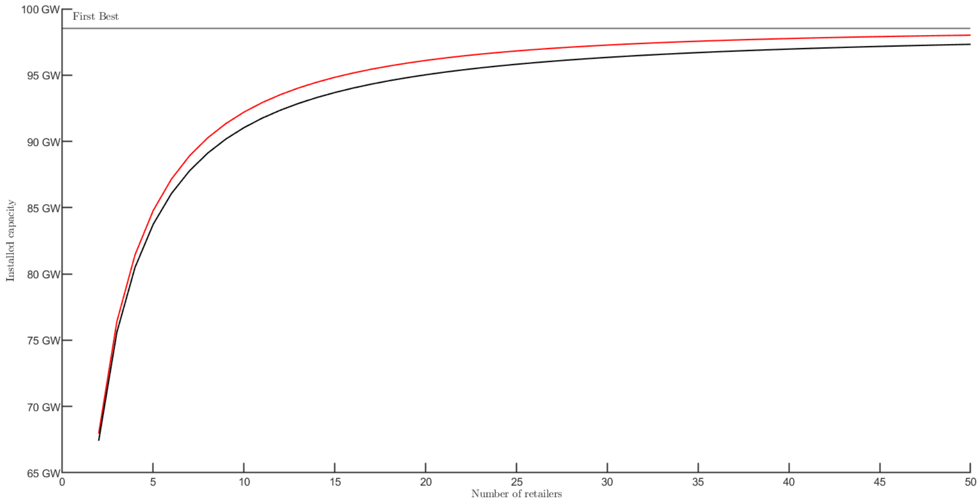

Figure 7: First-best investment level under retail Cournot competition between an ex-ante and ex-post market design

studied, while for the second to opposite effect are considered. As shown in the previous figures, the cost of inefficient rationing is significant in the social welfare, which explained the outweigh of the first term by the second on in the equation 16

6.6 Comparative statistics for decentralized demand

[INSERT HERE]

7 Conclusion and discussion

This paper built a tractable framework to analyze multiples markets' interdependence for an essential prone to underinvestment. We showed how the investment decisions are affected by those markets, their structure (such as the degree of competition), and, most importantly, their design. Our case study is the reservation markets that were implemented to encourage producers to invest by providing an additional remuneration. Most of the literature on reservation markets has focused on the supply side, where producers offer their availability on future transaction periods on the

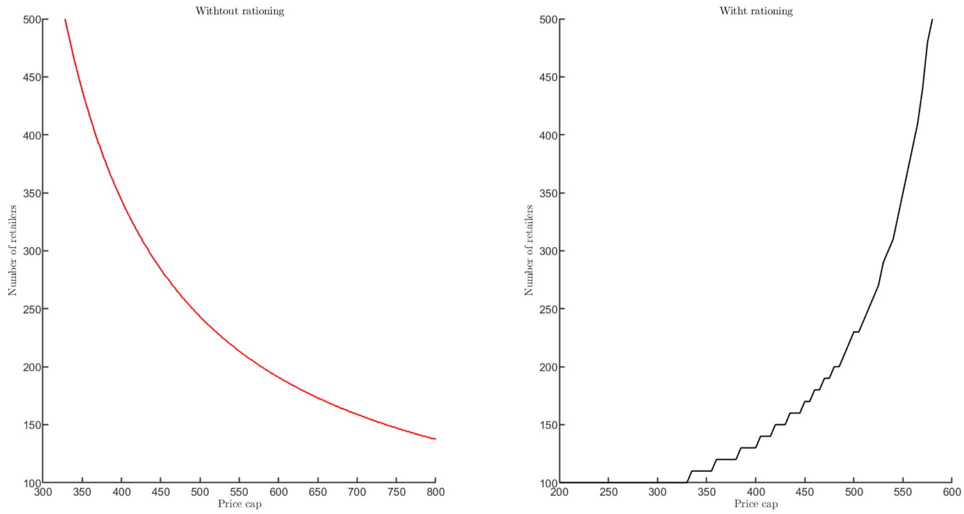

Figure 8: Threshold price cap and the market structure couples for the sign of the welfare function derivative with respect to the number of retailers

energy market. Therefore, the demand side has been overlooked, even though some system efficiency effects are well known. Current implementations show many options regarding the demand side's design on reservation markets, as consumers do not have proper incentives to buy capacities. Using our framework, we compare two market designs and their implications. The first regime is based on a single regulated entity that builds an administered demand function. The second regime is represented by retailers' obligation to cover their retail market sales by capacities bough in the reservation market. We underline the different parameters that can significantly affect the outcomes of a reservation market on investment decisions. The regulated entity's quantity can significantly affect prices and quantities on the three markets and redistribution welfare between agents for the centralized case. One of the advantages of this framework relies on the possible extensions that we can implement, besides providing a simple but complete vision. In the rest of the section, we discuss two issues that could be addressed in future research using this framework.

First, we initially assumed that consumers were fully reactive to retail prices. Such assumptions do not describe the reality yet as illustrated in the electricity system, as most small final consumers such as households are still under fixed-price contracts. The study of final consumers' heterogeneity and its implications for investment decisions in the power system is an emerging trend. Léautier (2014) and Léautier (2016) provide a relevant model close to the one presented in this paper. They show the effects of having those two types of consumers with different shares on investment decisions and a reservation market. However, the author does not compare demand design options for reservation markets and does not consider retailers. Therefore, implementing this new extension in our model could bring a new light on the issue associated with power systems' investment decisions. It could also have a significant impact on the decentralized market design option. Indeed, if we consider that some consumers cannot react to price, but retailers are still forced to cover their consumption, the demand function's formation in the reservation market is significantly impacted.

Finally, we assume that future final consumer demand is commonly shared between the different agents. The regulated entity and retailers could access a different quantity and quality of information. For instance, we can assume that the regulated entity has only a global vision of the future demand, and hence she is prone to make a more significant error forecast than retailers. On the other hand, retailers have private access to more precise information on their client portfolio while sharing common information on the world's future global states. Therefore, introducing these private/common elements in our model could shed new light on the effect of reservation markets and their market design options. Finally, in some current implementations, the regulated entity based its global forecast on retailers' information. Consequently, the comparison between the decentralized and centralized cases could be analyzed using game theory and signaling.

References

[Boiteux 1949] Boiteux, M..: *La tarification des demandes en pointe: Application de la théorie de la vente au coût marginal.* place Henri-Bergson, 1949

[Brown 2018] Brown, David P.: Capacity payment mechanisms and investment incentives in restructured electricity markets. In: *Energy Economics* 74 (2018), S. 131–142

[Bushnell u. a. 2017] Bushnell, James ; Flagg, Michaela ; Mansur, Erin: Capacity Markets at a Crossroads. In: *Energy Institute at Haas* (2017). – URL http://ei.haas.berkeley.edu. – .. – ISBN 926410867X

[Crew und Kleindorfer 1976] CREW, Michael A. ; KLEINDORFER, Paul R.: Peak load pricing with a diverse technology. In: *The Bell Journal of Economics* (1976), S. 207–231

[Fabra 2018] FABRA, Natalia: A primer on capacity mechanisms. In: *Energy Economics* 75 (2018), S. 323–335

[Hobbs u. a. 2007] HOBBS, Benjamin F. ; HU, Ming C. ; IÑÓN, Javier G. ; STOFT, Steven E. ; BHAVARAJU, Murty P.: A dynamic analysis of a demand curve-based capacity market proposal: The PJM reliability pricing model. In: *IEEE Transactions on Power Systems* 22 (2007), Nr. 1, S. 3–14. – ISSN 08858950

[Holmberg und Ritz 2020] HOLMBERG, Pär ; RITZ, Robert A.: Optimal Capacity Mechanisms for Competitive Electricity Markets. In: *The Energy Journal* 41 (2020), Nr. Special Issue

[Joskow und Tirole 2007] JOSKOW, Paul ; TIROLE, Jean: Reliability and competitive electricity markets. In: *The Rand Journal of Economics* 38 (2007), Nr. 1, S. 60–84

[Léautier 2014] LÉAUTIER, Thomas-Olivier: Is Mandating. In: *The Energy Journal* 35 (2014), Nr. 4

[Léautier 2016] LÉAUTIER, Thomas-Olivier: The visible hand: ensuring optimal investment in electric power generation. In: *The Energy Journal* 37 (2016), Nr. 2

[Leautier 2018] LEAUTIER, Thomas-Olivier: On the long-term impact price caps: Investment, uncertainty, imperfect competition, and rationing. In: *International Journal of Industrial Organization* 61 (2018), S. 53–95

[Llobet und Padilla 2018] LLOBET, Gerard ; PADILLA, Jorge: Conventional Power Plants in Liberalized Electricity Markets with Renewable Entry. In: *The Energy Journal* 39 (2018), Nr. 3

[Newbery 2016] NEWBERY, David: Missing money and missing markets: Reliability, capacity auctions and interconnectors. In: *Energy Policy* 94 (2016), S. 401–410

[Scouflaire 2019] SCOUFLAIRE, Charlotte: *Capacity remuneration mechanisms: analytical assessment of contemporary experiences and lessons for the future design of electricity markets*, PSL - Paris Dauphine University, Dissertation, 2019

[Weitzman 1974] WEITZMAN, Martin L.: Prices vs. quantities. In: *The review of economic studies* 41 (1974), Nr. 4, S. 477–491

[Zöttl 2011] Zöttl, Gregor: On optimal scarcity prices. In: *International Journal of Industrial Organization* 29 (2011), Nr. 5, S. 589–605