Optimal Mix of Policy Instruments and Green Technology Transitions *

Abstract

Although technological innovation are keys solution to fighting climate change, there are several challenges that need to be addressed for a green technology transition including market failures and the dynamic of the market structure. In the absence of public policies, markets alone are not sufficient to provide right incentives for the investment and diffusion of green technologies. In this context, what is the optimal combination of policy instruments that provides the right incentives to invest in green technologies? In this paper, we address this question by studying the interrelations between market structure, r&d policy and environmental policy to find the optimal mix of policy instruments. First, we conduct a welfare comparison and find that the socially desirable market structure is not a single solution and depends on the levels of damage and innovation. Second, given the socially desirable market structure, we analyse the optimal policy instruments that need to be implemented. We show that the taxation and subsidisation scheme is different depending on the market structure and the environmental damages. Finally, we study the fiscal implications of the optimal policy instruments. We show that both the market structure and the level of environmental damages influences the budget surplus or deficit. The regulator can use the fiscal implications in addition to social welfare, to decide on which market structure to implement, and at which stage of the green technology transition process. As policy implications, we would suggest that the regulator should implement a mix of policy instruments that should change over time depending on how the incumbents and entrants behave on the technology market. Therefore, the regulator should not commit to a static combination of policy instruments.

Keywords: Policy, Tax, Subsidy, Green Technology, Imperfect Competition, Technology Transition

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1 Introduction

Technological change plays an important role in climate change policy and is one of the keys solution to ensuring green growth (OECD, 2012, 2013). Green and efficient technologies reduce the intensity of carbon emissions in the production process and the use of less natural resource. The transition to green technologies is expected in different sectors that mainly contribute to climate change. In the mobility sector, e-vehicles are replacing diesel and gasoline cars, while the electricity sector is switching from fossil-fuels to renewable energy. The same transition has been observed in the cooling sector with the phase-out of CFC-based products in the 1980s and 90s.

In the absence of public policies, markets alone are not sufficient to provide right incentives for the investment and diffusion of green technologies because of the presence of market failures. First, there are fundamental market failures associated to the innovation process due to the public good nature of knowledge and information. Appropriate policy instruments such as patents, licensing and R&D subsidies have been used for the innovation market failures (Poyago-Theotoky, 2007). Second, there is an additional market failure associated with development of green technologies: the externality related to greenhouse gas emissions. Addressing these market failures altogether would necessitate the implementation of public environmental and technology policies with appropriate combination of policy instruments (Schneider and Goulder, 1997, Fischer and Newell, 2008 and Acemoglu et al., 2012).

On top of market failures, green technology transition often implies a change of the market structure as incumbents producing old technologies will face competition from new entrants that invest in the new technologies. As a response to this threat of competition, incumbents may decide to strategically invest in the new technology to avoid competition, to follow the new entrant or to become a leader in this new technology market (see Bondarev et al., 2020). This is the example of the mobility sector, more precisely the EV market. The environmental regulation has influenced incumbents behaviours in the EV market dynamic within three periods (Wesseling et al., 2015). First in the period of the 1990s, there was a significant number of innovation in EV by incumbents but sales did not follow the same trend. Later on, from 2000-2006, the innovation in EV was not active such that incumbents have developed only a low number of EV patents together with fewer sales. Finally, from 2007, both innovation and EV sales by incumbents have experienced a high increase.

While the EV innovation was inactive for the incumbents, Tesla Motors Inc. (Tesla), a new entrant into the car market, was created in 2003 and specializes in EV. Since then, Tesla has innovated a lot in EV market to become a first mover and has confirmed its position as a leader (Pontes, 2020, Holland, 2020). Tesla has then driven innovation in the EV market as a new entrant and has been followed by incumbents like General Motors, Toyota, Renault, Nissan, etc. However, the market structure is not static: local brands are becoming very competitive and contribute to drop EV prices. This is the example of The Renault Zoe in Europe (Taylor, 2020). To address the threat of increasing competition, Tesla and other incumbents like General Motor have decided to explore a better battery option (Mullaney, 2020) that would contribute to drastically reduce the EV cost (Dobuski and Schneider, 2020).

In this context, what is the optimal combination of policy instruments that provides the right incentives to invest in green technologies? Does this optimal combination depends on the market structure and the level of environmental damages? What are the fiscal implications of this optimal combination for the regulator? Under which conditions on the level of damages is it socially acceptable to switch to the green technologies? Our paper intends to study some of these challenges by mainly focusing on the interrelations between market structure, r&d policy and environmental policy.

To do that, we build our model on the framework developed by Bondarev et al. (2020) and include environmental and technology policies. We consider a small open economy in which the r&d investment is exogenous. We then limit our attention to the effects of environmental and technology policies on the local technology market. We assume that there are two types of technology on an imperfectly competitive market: old technology that is polluting and new technology that is clean. We consider two instruments: emissions tax on the old technology for the environmental policy and subsidy to the clean technology for the innovation policy. Two different firms can offer the technologies: the old technology is developed only by the incumbent, while the new technology can be developed by both the incumbent and the new entrant. Depending on the market conditions, the market structure can be a Cournot competition with both new entrant and incumbent (with old and/or new technologies) or a monopoly with only one of the firms serving the technology market. The decisions of both firms and consumers are affected by the level of the instruments used for the two policies.

Using this model, we first analyse the welfare optimisation problem and conduct a welfare comparison. We discuss under which conditions on the level of environmental damages, a transition to green technology is socially acceptable; or whether the innovation is sufficient or not to motive the green technology transition. We show that for a small accumulated level of innovation, the regulator would prefer only the old technology for small damages. The contrary happens if the market has accumulated enough innovation. In this case, the transition to only the new technology is welfare improving even for small damages. Furthermore, when the damages become large, the accumulated innovation of the new technology is no longer sufficient to compensate the high associated social cost of producing the old technology. Thus, it becomes welfare improving to rely only on the new technology. As a result, the socially desirable market structure is not a single solution and depends on the level of damage and the level of innovation. Moreover, the transition to the desirable market structure may include intermediary stages which enlarges the set of market structure that needs to be considered in the green technology transition process.

Second, we analyse the optimal policy instruments that needs to be implemented to achieve a given socially desirable market structure. We study how market structure can affect optimal policy instruments during the green technologies transition. We show that the tax/subsidy scheme is different depending on whether incumbent supplies only the old technology or the new technology; or the new entrant only supplies the new technology. We then derive the combination of instruments that the social planner would prefer to implement depending on the level of environmental damages and the market structure. We find that in a simple case where the old technology is available and the incumbent supplies the new technology, low damages require a subsidy while a tax is needed for high damages. While in the case only the entrant supplies the new technology, a tax is still needed for high damages but a subsidy is required for low damages only if the level of innovation of the entrant is sufficiently low. Furthermore, when the two firms share the market of the new technology and the old technology is phased out, there are some possibilities of negative r&d subsidies to the incumbent.

Finally, we study the fiscal implications of the social planner decisions associated with the optimal policies. While the social planner would decide whether environmental tax or/and subsidy is suitable to implement the first best, this decision also has fiscal implications. Whether the combination of tax and subsidy gives a budget surplus or deficit depends on how the effects of each instruments can compensate each other. We show that the budget surplus or deficit depends on the market structure and the level of environmental damages. The obvious case is when only the old technology is available. In this case, the budget is negative for small damages and positive for high damages. However, when both old and new technologies are available, the budget is still negative for small damages, while it can be negative or positive for high damages. Different patterns happen when the old technology is phased-out. For example, when both firms supply the new technology and the old technology is phased out, there is a budget surplus for sufficiently low level of damage, while both budget surplus and deficit is possible for sufficiently high level of damage. Therefore, the social planner can use the fiscal implications in addition to social welfare, to decide on which market structure to implement, and at which stage of the green technology transition process.

We derive the following policy recommendations from the above results. We would suggest the implementation of both emission tax and r&d subsidy as policy instruments to foster the development of green technology. The r&d subsidy should be different for the incumbents and the entrants such that the regulator has flexibility to discriminate between the different firms. However, to reflect the dynamics of the market structure, the regulator should not commit to a static combination of policy instruments. This mix of policy instruments should change depending on how the incumbents and entrants behave on the technology market.

The remainder of this paper is structured as follows. We provide a brief literature review in Section 2. The model is presented in Section 3. Section 4 focuses on the welfare optimisation including the optimal productions (Section 4.1) and the welfare comparison (Section 4.2). We analyse the equilibrium quantities of old and new technologies in Section 5. In Section 6, we study the optimal instruments to implement the first best levels of production of the old and new technologies. Section 7 studies the fiscal implications. Finally, Section 8 concludes the paper.

2 Literature review

The literature on optimal policies for a green technology transition encompasses two strand of literature: innovation and environmental economics. On the one hand, r&d market failures due to the public good nature of innovation is well known in the innovation literature and several studies have analysed various policy instruments to address these market failures, namely patents by granting intellectual property rights, licensing, r&d subsidies and the encouragement of cooperative r&d (see Lahiri and Ono, 1999, Toshimitsu, 2003 Kitahara and Matsumura, 2006 and Poyago-Theotoky, 2007). For instance, Lahiri and Ono (1999) has discussed whether the regulator should tax or subsidise r&d efforts. They find that the firm with initial cost advantage (i.e.; the efficient firm) requires subsidies while the inefficient firm should be taxed. Kitahara and Matsumura (2006) has extended the previous paper to include uncertainty related to r&d investments. While it is complicate to find optimal tax rules that equate efficient and equilibrium r&d levels in this setting, they derive an optimal policy based on simple subsidies which depend on the realized cost differences.

In the same vein, Toshimitsu (2003) explores how policies of r&d subsidisation affect social welfare depending on the market structure. They find that it is welfare improving to provide r&d subsidies to the firms producing the high-quality of product whether the competition is Bertrand or Cournot. This is not the case when the regulator provides r&d subsidies to the firm producing a low-quality product, which improves social welfare only in the case of Bertrand competition. Other factors may also influence the behaviour of incumbent and entrant during the innovation transition. In the broadband telecoms industry, Bourreau et al. (2012) finds that the access price has a non-monotonic effect on incentives to invest in the new generation network as a result of replacement effect, low opportunity cost of investment effect and retail-level migration effect. All these studies have focused on a specific market structure and do not include the externality market failure that is associated with green technologies.

The second strand of literature studies environmental policies with or without an implementation of r&d policy. Most studies of environmental policy mainly focus on market-based instruments to regulate emissions due to their static efficiency and the application of Pigouvian rule that ensures that the optimal price of pollution fully reflects the marginal social damage. However, the use of a Pigouvian tax for environmental regulations has been criticised, especially in imperfectly competitive markets (see Buchanan, 1969, Lee, 1975 and Barnett, 1980). A comprehensive discussion on optimal tax rules under imperfect competition can be found in Requate (2006). Furthermore, there is a debate on the implications on imprecise emissions tax on output. Fullerton et al. (2001) explores the effects of environmental taxes when the pollution target is imprecise. Several reasons may explain an imperfectly-targeted tax on output. For instance, policy-makers tend to primary focus on equity considerations by making sure that polluting industries pay for their polluting activities while neglecting that this may also affect their motivation to reduce emissions. Furthermore, the measurement of these emissions can be difficult and can be improved over time with more efficient technologies. Even if the pollution target is imprecise, Fullerton et al. (2001) finds that tax on output does not reduce the optimal rate of emissions tax.

Most of the studies that combines both technology and environmental policies have used a specific market structure, which may limit the applicability of their results. In a monopolistic setting, Krass et al. (2013) studies the role of three environmental policy instruments (taxes, fixed cost subsidies and consumer rebates) in the transition to innovative and “green" emissions-reducing technologies. They find that environmental taxes initially favour the transition to a greener technology, while a reverse transition may happen for a further tax increases. In addition, they find that the optimal policy can be a combination of the three policy instruments as they play complementary roles. Carraro et al. (2013) also finds that one environmental policy instruments is not enough to achieve optimal outcomes, similar to Schneider and Goulder (1997), Fischer and Newell (2008) and Acemoglu et al. (2012).

Carraro et al. (2013) contains papers that focus on different aspects of the issue within an oligopoly and duopoly market structure. For instance, Katsoulacos and Xepapadeas (1996a) analyses whether emission taxes are optimal in oligopolistic markets with a fixed or endogenous number of firms. They find that under a fixed number of firms, the environmental tax is sub-optimal (i.e.; less than marginal external damages) as in the case of monopoly (see Barnett, 1980), while it could be optimal to internalise excess of the external marginal damages under endogenous market structure. In this case, the second-best emission tax could exceed marginal external damages. Katsoulacos and Xepapadeas (1996b) analyses how taxes and subsidies could induce environmental innovation in a duopoly under R&D spillovers. They show that the optimal environmental tax is lower than the marginal external damage and it may be optimal to tax environmental innovation when spillovers are sufficiently small. In the case the environmental innovation already exists, Carraro and Soubeyran (1996) explores the role of environmental policy in providing the right incentives to their adoption. Their results show that while both environmental policy instruments generate less pollution, it is possible that the innovation subsidy is socially better than environmental taxes. While all these studies have analysed optimal policies for the green transition, they do not focus on the interactions between the incumbent that has the incentive to protect the old technology and the new entrant that has incentive to invest in the new technology. This may result in different market configuration with different optimal policies.

As an alternative to emissions tax, Denicolo (1999) studies pollution permits and compares the performance of two policies instruments in the green technology transition. They find that the two instruments are fully equivalent if the regulator can easily adjust the levels of the instruments after the innovation stage. In the case the regulator can pre-commit, taxes favour environmental innovation more than pollution permits. Furthermore, taxes provide more social welfare than permits if the social damage is not too high. Whether the regulator should tax r&d efforts may also depend on the level of emissions tax and the level of environmental damages. Petrakis and Poyago-Theotoky (2002) finds that it can be optimal to tax R&D efforts when environmental damages are large and emissions tax is low. In this case, the regulator faces both R&D market failures and pollution problem and she would implement a R&D tax to avoid over-production. Furthermore, they find that in terms of social welfare, a policy that provides R&D subsidies is more likely to be better than a policy that supports R&D cooperation. However, r&d cooperation maybe socially more desirable than non-cooperation r&d. Poyago-Theotoky (2007) shows that when firms cooperate, their environmental innovation efforts are more important than when they do not cooperate, except in the case of relatively large environmental damages and efficient innovation where the opposite is true. They also find the same ranking in terms of social welfare.

Overall, all these contributions on green technology transition confine attention to finding adequate policy instruments in a specific market structure and avoid the interactions between the incumbent and the new entrant on the technology market. To the best of our knowledge, there is no specific study that explores in details the interrelations between market structure, r&d policy and environmental policy including the fiscal implications and welfare comparison.

3 The model

In order to explore the optimal instruments for a transition from a polluting technology to a clean technology, we build our model on the framework developed by Bondarev et al. (2020). We assume that there are two types of technology on an imperfectly competitive market. One of the technologies is the old technology that is polluting like petrol-/diesel-based cars or coal/fossil fuels power and the other technology is a new technology like electrical vehicles or renewable energies.

The pollution from dirty technologies has become a predominant issue as global change or climate change has received relatively more attention than the exhaustion of natural resources. Carbon pricing policies have been implemented to reduce the motivation to develop dirty technologies and to provide incentives for the deployment of alternative solution like clean technologies. Environmental taxes on dirty technologies and subsidies for the deployment of the clean technologies are two different economic instruments that are general used to incentivise both consumers for the demand side and firms for the supply side.

On the supply side, we consider two different firms that can offer the technologies on the market depending on the market conditions. At the current stage, the old technology is developed only by the incumbent, while the new technology can be developed by both the incumbent and the new entrant. Depending on the market conditions, the market structure can be a Cournot competition with both new entrant and incumbent (with old and/or new technologies) or a monopoly with only one of the firms serving the technology market. The incumbent and the new entrant are different in two ways. First, only the incumbent can produce the old technology, while both firms can produce the clean technology. Second, we assume that the incumbent has a cost advantage for the production of the new technology relatively to the new entrant. More precisely, the incumbent benefits from a slightly lower production cost for the clean technology.

On the demand side, there is a continuum of heterogeneous consumers who decide on the technology to buy depending on both the quality and price. The decisions of the consumers are also affected by the level of the environmental tax and the subsidy for the clean technology.

3.1 Pollution Damage and Policy Instruments

We assume that the old technology is a polluting technology (for instance technology based on fossil fuel). Therefore, we introduce a damage function $Dam(q_l)$ that depends on the production of the old technology q_l. $Dam(q_l)$ is defined as:

$$Dam(q_l) = \frac{d}{2} q_l^a,$$

where d and a are positive and constant parameters. For simplicity, we assume a linear damage (i.e., $a = 1$) such that the marginal damage is given by $Dam' = d/2$.

The policy framework focuses on two types of instruments: tax and subsidies. We consider that the pollution is a by-product of the production technology so that taxing the amount of a polluting technology production is equivalent to an environmental tax. Due to the pollutant content of the old technology, we assume that a tax τ is paid by the incumbent on each unit of old technology q_l. Note that we do not make any specific assumptions on τ. A positive value would mean that the incumbent pays an environmental tax following the polluter-pays principle. Depending on the market structure, τ can be negative, which means that the incumbent is subsided for producing the old technology. Furthermore, we consider a subsidy as a second instrument that can motivate firms to produce a green technology. Depending on the market structure, the subsidies on the new technology production provided by the regulator to the incumbent or to the new entrant can be different. We assume that s_i and s_n are the subsidy on each amount of new technology production for the incumbent (q_{si}) and the new entrant q_{sn}, respectively.

3.2 Demand

Following Bondarev et al. (2020), we assume a continuum of consumers (i) with different utilities from buying the clean technology ($u_s(i)$) and the dirty technology($u_l(i)$) at each period t.[1] The utility for buying the old technology is given by:

$$u_l(i) = 1 - i - p_{l,t}, \tag{1}$$

where the quality of the old technology is constant and set to 1, and its price in period t is given by $p_{l,t}$.

The same type of consumer has the following utility from buying the new technology:

$$u_s(i) = k_t - \alpha i - p_{s,t}, \tag{2}$$

where k_t is the quality of the new technology, its price in period t is given by $p_{s,t}$ and the new technology can expand overall demand at the rate α with $0 < \alpha < 1$. The consumer decides to buy the type of technologies that provides a strictly higher and positive utility.

3.3 Supply

3.3.1 Incumbent

The incumbent offers a quantity $q_{l,t}$ of the old technology and $q_{si,t}$ of the new technology at each period t. We assume that the production of both technologies generates the same constant marginal cost c_i. The total profit of the incumbent with the economic instruments is given by:

$$\pi_{l,t} = (p_{l,t} - c_i)q_{l,t} + (p_{s,t} - c_i)q_{si,t} - \tau q_{l,t} + s_i q_{si,t} \tag{3}$$

[1]The dynamics of our model rely on how the accumulation of technology from previous periods affects the current level of innovation.

3.3.2 New entrant

The new entrant n has only access to the new technology market and offers a quantity $q_{sn,t}$ at a constant marginal cost c. With the implementation of the environmental taxes and the subsidy for the clean innovation, the profit of the entrant at each period t is given by:

$$\pi_{n,t} = (p_{s,t} - c)q_{sn,t} + s_n q_{sn,t} \tag{4}$$

3.4 Set of market structure

The set of the market structure possibilities is composed of seven types of markets ranging from Case 0 to Case 6. In the Case 0, there is no new technology available in the market, therefore, the incumbent supplies only the old technology. From Case 1 to Case 3, the old technology is still supplied by the incumbent. However, both firms (Case 1) or either incumbent (Case 3) or new entrant (Case 2) supplies a new version of the new technologies. Case 4 to Case 6 refer to the market structure in which the old technology is completely phased out. In this context, a new version of the new technology is supplied by both firms (Case 4) or by either the incumbent (Case 5) or new entrant (Case 6).

4 Welfare Optimisation

Using the model, we first focus on the welfare maximization problem in order to analyse the optimal production of the old and new technologies. Then, we conduct a welfare comparison to explore the socially desirable market structure.

4.1 Optimal production of the old and new technologies

We first derive the following general demand functions for the old and the new technologies from equations (1) and (2):

$$p_{s,t} = k_{t-1} + \delta_{i,t} - \alpha[q_{l,t} + (n-1)q_{r,t} + q_{si,t} + q_{sn,t}] \tag{5}$$

and

$$p_{l,t} = 1 - q_{l,t} - \alpha[(n-1)q_{r,t} + q_{si,t} + q_{sn,t})] \tag{6}$$

We define the welfare W as the sum of the consumer surplus and the producer surplus minus the damage from using the old technology. We first analyse the case 1 as the general problem in which new entrants and incumbent supply new technology while the old technology is still produced. From this case, we compute the FOCs of the welfare for the other cases with corresponding conditions.

The social planner solves the following welfare maximization problem:

$$\begin{aligned} \max_{q_l, q_{si}, q_{sn}} W_1(q_l, q_{si}, q_{sn}) &= \int_{x_{s,l}}^{x_{s,0}} (k_0 + \delta_i - \alpha x)\,\mathrm{d}x + \int_0^{x_{s,l}} (1-x)\,\mathrm{d}x - (c - \delta_n + \delta_i)q_{sn} - Dam(q_l) \\ st \quad x_{s,0} &= \frac{k_0 - p_s - \alpha x + \delta_i}{\alpha} \\ x_{s,l} &= \frac{-1 + k_0 + p_l - p_s + \delta_i}{\alpha - 1} \\ &Eqs.(5)\ and\ (6) \end{aligned} \tag{7}$$

The above program can be rewritten as:

$$\max_{q_l, q_{si}, q_{sn}} W_1 = \frac{(2 - d - q_{l,t})q_{l,t} + [2(k_{t-1} + \delta_{i,t}) - \alpha(2q_{l,t} + q_{si,t} + q_{sn,t})](q_{si,t} + q_{sn,t}) - 2(c + \delta_{i,t} - \delta_{n,t})q_{sn,t}}{2} \tag{8}$$

Solving the above program with the relevant conditions depending on the cases gives the following proposition.

Proposition 1. *When the environmental damage from using the old technology is considered,*

1- *The optimal quantities of the old technologies and the new technologies produced; and the optimal welfare are the following:*

   (a) *If only the old technology is available, then,* $q^*_{l,t,0} = 1 - d/2$; $q^*_{sn,t,0} = q^*_{si,t,0} = 0$ *and* $W^*_{0,t} = \frac{1}{8}(d-2)^2$

   (b) *If both new technology and old technology is produced, then for* $j = 1:3$,

   - $q^*_{l,t,j} = \frac{1}{1-\alpha}[-\delta_{i,t} + (1 - d/2) - k_{t-1}]$, *where* $\delta_{i,t} = \delta_{n,t} - c$, *if only the entrant produces the new technology.*

- $q^*_{sn,t,j} = \frac{1}{\alpha(1-\alpha)}[\delta_{n,t} - c - \alpha(1-d/2) + k_{t-1}]$, *if only the entrant produces the new technology, and* $q^*_{sn,t,j} = 0$*, otherwise,*
- $q^*_{si,t,j} = 0$ *if only the entrant produces the new technology, and* $q^*_{si,t,j} = \frac{1}{\alpha(1-\alpha)}[\delta_{i,t} - \alpha(1-d/2) + k_{t-1}]$*, otherwise.*
- $W^*_{j,t} = \frac{4\delta_{i,t}[-2c+\alpha(d-2)+2k_{t-1}]+8\delta_{n,t}(c+\delta_{i,t})-4\delta^2_{n,t}+\alpha(d-2)(d+4k_{t-1}-2)-4c^2+4k^2_{t-1}}{8(1-\alpha)\alpha}$.

(c) *If the old technology is phased out, then for* $j = 4:6$*,*

- $q^*_{l,t,j} = 0$*;*
- $q^*_{sn,t,j} = \frac{1}{\alpha}[\delta_{n,t} - c + k_{t-1}]$*, if only the entrant produces the new technology, and* $q^*_{sn,t,j} = 0$*, otherwise;*
- $q^*_{si,t,j} = 0$ *if only the entrant produces the new technology, and* $q^*_{si,t,j} = \frac{1}{\alpha}[\delta_{i,t} + k_{t-1}]$*, otherwise;*
- $W^*_{j,t} = \frac{(k_0+\delta_i)^2}{2\alpha}$*, where* $\delta_{i,t} = \delta_{n,t} - c$*, if only the entrant produces the new technology.*

2- *The optimal welfare is decreasing in the level of damage and increasing in the level of innovation.*

Proof. The optimal quantities of the old technology and the new technology (i.e., the first part of Prop.1) is obtained by solving the program in Eq.8 with relevant conditions $q_{l,t} = 0$, $q_{sn,t} = 0$, or $q_{si,t} = 0$ depending on the market structure. These optimal quantities are used to compute the optimal social welfare. More details are provided in Appendix A in which Table 3 summarizes the results. As we assume that the cost of producing the new technology by the new entrant is higher than that of the incumbent, whenever the two firms supply the new technology, the social planner will optimally prefer the incumbent to produce the new technology. This translates into the following optimal condition $\delta_{i,t} = \delta_{n,t} - c$. The second part of the Prop.1 comes from a negative derivative of the optimal welfare with respect to the level of damage and a positive derivative with respect to the level of innovation. Thus, the higher is the damage, the lower is the optimal welfare, whereas a higher innovation induces a higher optimal welfare. Note that this is only true under the conditions that define the existence of each cases. □

Three levels of interpretation can be provided for Prop.1. First, the level of damage and the level of innovation have positive and negative effects on the optimal production of the new technology and that of the old technology, respectively. Thus, the phased out of the old technology is motived by its high environmental damage and/or a high innovation. Second, the different market structure simplifies to three groups leading to a same welfare for market structure within the same group. Whenever a group of market structure is socially desirable, the social planner would then be indifferent regarding which one to choose within that group. Finally, from the second part of Prop.1, the environmental damage and the innovation compensates each other in term of welfare. Therefore, whether one group of market structure is more socially desirable than the other depends on if the level of damage or the level of innovation is high enough. Therefore, a welfare comparison is needed.

4.2 Welfare comparison

The different cases following the market structure give different social welfare, which depends on the level of damage, the level of innovation of both the incumbent and the new entrant and other cost and demand parameters. Moreover, under the condition $\delta_n = \delta_i + c$, the welfares from Case 1 to Case 3 are equivalent and the welfares from Case 4 to Case 6 also equivalent. Therefore, the welfare comparison will be analysed across three groups of cases: (i) Only old technology (Case 0), (ii) Old and new technologies (Cases 1-3) and (iii) Only new technology (Cases 4-6).

The best situation in this setting in term of welfare is that the society should refrain from using the old technology that generates pollution. Therefore, we use the group (iii) as a reference market structure. We are interested in under which conditions on the damage and level of innovation, the social planner should favour a transition from groups (i)-(ii) to that reference market structure. We also analyse the transition from group (ii) to group (ii) as an intermediary to the full transition to group (iii). We claim the following proposition.

Proposition 2. *In term of welfare comparison,*

1- *The social planner would prefer to switch from producing only the old technology to producing only the new technology whenever the following condition holds:* $\alpha d(d-4) < 4[(\delta_{i,t} + k_{t-1})^2 - \alpha]$. *This translates into the following range on damages and level of innovation:*

   (a) *If* $(\delta_{i,t} + k_{t-1})^2 < \alpha$, *then* $1 - \frac{1}{\sqrt{\alpha}}(\delta_{i,t} + k_{t-1}) < \frac{d}{2} < 1 + \frac{1}{\sqrt{\alpha}}(\delta_{i,t} + k_{t-1})$

   (b) *If* $(\delta_{i,t} + k_{t-1})^2 > \alpha$, *then* $\delta_{i,t} > (\frac{d}{2} - 1)\sqrt{\alpha} - k_{t-1}$.

2- *The transition from the production of both old and new technologies to the production of only the new technology is characterised by the following:*

(a) *The social planner would prefer to produce both technologies whenever the innovation is not sufficient.*

(b) *When the innovation is sufficient (i.e., $\delta_{i,t} > 1 - d/2 - k_{t-1}$), the social planner would switch to the sole use of the new technology.*

3- *For an intermediary situation, the social planner may decide first to switch from producing only the old technology to producing both technologies. This situation is characterised by the following:*

(a) *When the innovation is not sufficient with $\delta_{i,t} < -\alpha(d/2 - 1) - k_{t-1}$, the social planner will accept small environmental damages and rely only on the old technology.*

(b) *For sufficient innovation, i.e., $1 - (\delta_{i,t} + k_{t-1})/\alpha < d/2 < 1 - \delta_{i,t} - k_{t-1} < 1$, it is welfare improving to switch to the production of both technologies.*

Proof. For Part 1- of proposition 2, we compare the optimal social welfare that we obtained in the Section 4.1 for the case 0 (i.e., $W^*_{0,t}$) and cases 4-6 (i.e., $W^*_{j,t}$, with $j = 4, 5, 6$). This gives the first inequality. Depending on how large is the accumulated level of innovation (i.e., $\delta_{i,t} + k_{t-1}$) to cover the market share α, the range of the marginal damage $d/2$ is different. This gives the range of the damages and the level of innovation. In Part 2- of the proposition, the decision of the social planner to switch from producing both new and old technologies to producing only the new technology depends on the welfare difference. We then compare the optimal social welfare $W^*_{j,t}$ with $j = 1, 2, 3$ in the group of cases (ii) to the optimal social welfare $W^*_{j,t}$ with $j = 4, 5, 6$ in the group of cases (iii). We find that the minimum of the welfare from group (ii) is equivalent to the welfare of the group (iii). The social planner would then prefer to produce both technologies whenever the damage is low or the innovation is not sufficient. The reverse happens when the innovation is sufficient or the damage is sufficiently high (i.e., $d/2 > 1 - \delta_{i,t} - k_{t-1}$). The proof of part 3- of the proposition is similar to the previous one. We compare the optimal welfares in Case 0 and group (ii). We find that the social planner would be indifferent when $d/2 = 1 - (\delta_{i,t} + k_{t-1})/\alpha$. $\square$

Part 1- of proposition 2 shows that, for a small accumulated level of innovation, the social planner would prefer to produce only the old technology for small damages. In contrary, when the market has accumulated enough innovation, the transition to only the new technology is welfare improving even for small damages. Furthermore, this arbitrage needs to match with the condition that defines the cases of only new technology production. Here, we focus on the case 5 as the most relevant case for the social planner due to the cost advantage of the incumbent. This condition dictates that $d/2 > 1-(k_{t-1}+\delta_i)$. Given that $1-\frac{1}{\sqrt{\alpha}}(\delta_i+k_{t-1}) < 1 - (k_{t-1} + \delta_i) < 1 + \frac{1}{\sqrt{\alpha}}(\delta_i + k_{t-1})$, the social planner would prefer the situation with only the production of the new technology (namely with only the incumbent) which can be implemented under the following condition: $1-(k_{t-1}+\delta_{i,t})<\frac{d}{2}<1+\frac{1}{\sqrt{\alpha}}(\delta_{i,t}+k_{t-1})$. The above condition highlights different configurations regarding whether case 0 is already rolled out by the market existence condition or should be rolled out by the social planner. More precisely, for $d/2\in[1-(k_{t-1}+\delta_{i,t}),1]$, case 0 can still be implemented but the social planner would need to force the implementation of cases with only the new technology that give a higher welfare.

Part 2- of Proposition 2 shows that with small damages, the social planner would prefer to keep some amount of the old technologies in the market that would benefit the incumbent. The level of innovation would be sufficient to compensate the small damages. When the damages become large, the social cost of producing the old technology is so high such that it becomes welfare improving to rely only on the new technology. Note that the high damage translates into the impossibility of the economy to compensate the marginal damage (*i.e.*, $d/2$) with the remaining capacity for the innovation accumulation of the new technology $(1-\delta_{i,t}-k_{t-1})$.

As an intermediary situation, the social planner may decide a gradual transition to the production of only the new technology. In this case, the economy will first switch from producing only the old technology to producing both technologies before a full switch to only producing the new technology. This temporary transition will basically depend on the level of the damage. Part 3- of Proposition 2 shows that small damages for $d/2<1-(\delta_{i,t}+k_{t-1})/\alpha$, motivate the social planner to bear the social cost of pollution from the old technology. On the contrary, when the damages become so high, i.e., $1-(\delta_{i,t}+k_{t-1})/\alpha<d/2<1-\delta_{i,t}-k_{t-1}<1$, it is welfare improving to switch to the production of both technologies. Note that in this case, the damage is not sufficiently high to prevent the production of the old technology.

Section 4 (Propositions 1 and 2) shows, that the socially desirable market structure is not a single solution and depends on the level of damage and the level of innovation. Moreover, the

Table 1: *Equilibrium Price and Quantity*

Cases	Quantities			Prices	
	q_l	q_{si}	q_{sn}	p_l	p_s
Case 0	$\frac{1-\tau}{2}$	0	0	$\frac{1+\tau}{2}$	0
Case 1	$\frac{1}{2(1-\alpha)}[(1-\tau) -(\delta_{i,t}+s_i)-k_{t-1}]$	$\frac{1}{6\alpha(1-\alpha)}[(4-\alpha)(\delta_{i,t}+s_i) -2(1-\alpha)(\delta_{n,t}+s_n) -3\alpha(1-\tau)+2(1-\alpha)c +(\alpha+2)k_{t-1}]$	$\frac{1}{3\alpha}[2(s_n+\delta_{n,t}) -(s_i+\delta_{i,t}) -2c+k_{t-1}]$	$\frac{1}{6}(2c+\delta_{i,t} -2\delta_{n,t}-k_{t-1}+3 +s_i-2s_n+3\tau)$	$\frac{1}{3}(c+2\delta_{i,t} -\delta_{n,t}+k_{t-1} -s_i-s_n)$
Case 2	$\frac{1}{4-\alpha}[-(s_n+\delta_{n,t}) +2(1-\tau)+c-k_{t-1}]$	0	$\frac{1}{\alpha(4-\alpha)}[2(s_n+\delta_{n,t}) -\alpha(1-\tau) -2(c-k_{t-1})]$	$\frac{1}{\alpha-4}[-c+\delta_{n,t} +k_{t-1}+s_n +(\alpha-2)\tau-2]$	$\frac{1}{\alpha-4}[\alpha+(\alpha-2)c-2\delta_{n,t} -2k_{t-1}-\alpha\tau-(\alpha-2)s_n]$
Case 3	$\frac{1}{2(1-\alpha)}[-(\delta_{i,t}+s_i) +(1-\tau)-k_{t-1}]$	$\frac{1}{2\alpha(1-\alpha)}[(\delta_{i,t}+s_i) -\alpha(1-\tau)+k_{t-1}]$	0	$\frac{1}{2}(\delta_{i,t}+k_{t-1}-s_i)$	$\frac{\tau+1}{2}$
Case 4	0	$\frac{1}{3\alpha}[2(s_i+\delta_{i,t})-(s_n+\delta_{n,t}) +c+k_{t-1}]$	$\frac{1}{3\alpha}[2(s_n+\delta_{n,t}) -(s_i+\delta_{i,t}) -2c+k_{t-1}]$	0	$\frac{1}{3}(c+2\delta_{i,t}-\delta_{n,t} +k_{t-1}-s_i-s_n)$
Case 5	0	$\frac{\delta_{i,t}+s_{i,t}+k_{t-1}}{2\alpha}$	0	0	$\frac{1}{2}(\delta_{i,t} +k_{t-1}-s_i)$
Case 6	0	0	$\frac{s_n+\delta_{n,t}-c+k_{t-1}}{2\alpha}$	0	$\frac{1}{2}(c+\delta_{n,t} +k_{t-1}-s_n)$

transition to the desirable market structure may include intermediary stages which enlarges the set of market structure that needs to be considered in the green technology transition process. In the following sections, we analyse the optimal policy instruments that needs to be implemented to achieve a given socially desirable market structure.

5 Equilibrium of the old and new technologies production

To analyse the optimal policy instruments, we start analysing the equilibrium of the production of the old and new technologies. By applying the corresponding conditions on the quantities, we get the specific demand functions for each market structure. We then solve the corresponding profit-maximization problem of a Cournot or monopoly game by providing equilibrium quantities and prices using Eqs (3), (4), (5) and (6). The results are summarised in Table 1.

In Case 0, a higher tax on the old technology will increase the price of the old technology which will decrease the amount of old technology production from the incumbent. This is also the case in Case 1 where both firms receive subsidy. A higher subsidy on the new technology will reduce the equilibrium price of the new technology, will motivate more demand for the new technology and thus, firms that receive the subsidy will produce more. However, due to the interactions between the two firms as both are supplying to the same market, a higher subsidy to one firm will reduce the quantity of the new technology supplied by the other firm. Also note that the tax on the old technology increases the amount of the new technology supplied by the incumbent while the subsidy on the new technology reduces the supply of the old technology. Thus, for the incumbent, both subsidy and environmental tax play the same role for the production of the old and new technologies but the magnitude is different for the production of the new technology. This is because the higher effect of the subsidy to the incumbent needs to compensate for the negative effect of the subsidy to the new entrant. Finally, remark that the tax on the old technology does not affect either the equilibrium price of the new technology or the amount of the new technology supplied by the new entrant. This is because the effects of the environmental tax on the quantities produced by the incumbent compensate each other.

In Case 2, a higher environmental tax will reduce the production of the old technology as well, which will favour the production of the new technology from the new entrant. The effect on the equilibrium price of the new technology and old technology depends on how those effects on quantities balance each other. Since $\alpha < 2$, the decrease in the quantity of the old technology outweighs the increase in that of the old technology in absolute terms. Thus, environmental tax has a positive effect on both prices of old and new technologies. Furthermore, the subsidy on the new technology has the same effects on the quantity of new and old technology in term of the sign but are different in term of the size. Here, the effect of the subsidy on the quantity of the new technology outweighs that of the old technology. Consequently, subsidy decreases the equilibrium prices of the new and old technologies.

In Case 3, the environmental tax and subsidies have opposite effects on corresponding quantities of the technology production while environmental tax increases price of the old technology and subsidy reduces price of the new technology as in the Case 1. The results of Case 4 are similar to those of the Case 1, excepts for the amount of new technology supplied by the incumbent. More specifically, the effect of the subsidy is reduced as the incumbent does no longer benefit from the supply reduction of the old technology. A similar pattern is observed from Case 5 compared to Case 3. Finally, in Case 6 and compared to Case 2, the new entrant benefits from a lower effect of the subsidy on the quantity produced and a higher effect on the price. This is because the indirect effect of the subsidy on the price of the new technology through the reduction of the old technology production does no longer exist in this case.

The above results can be summarized in the following proposition:

Proposition 3. *When both environmental tax on the old technology and different subsidies for the incumbent and the new entrant on the new technologies are implemented, at the equilibrium:*

1- Environmental tax reduces the equilibrium quantity of the old technology

2- Subsidy on the new technology increases the equilibrium quantity of the new technology produced by the firm that receives the subsidy.

3- Subsidy on the new technology reduces the amount of the old technology while

- *(a) Environmental tax increases only the equilibrium quantity of the new technology produced by the incumbent with no effect on that of the new entrant when both firms share the new technology market;*
- *(b) Environmental tax increases the equilibrium quantity of the new technology produced by both the incumbent and the new entrant when the new technology market is not shared between the two firms.*

6 Optimal instruments

In this section, we focus on the optimal instruments to implement the first best production of new and old technologies. Basically, we compute the value for the tax on the old technology (τ) or the subsidies provided to support the production of the new technology (s_i and s_n) such that the productions in the equilibrium situation (Section 5) are equal to the one in

Table 2: *Optimal instruments*

Cases	Optimal instruments τ^*	s_i^*	s_n^*	Existence Conditions
Case 0	$d-1$	0	0	$d/2 < 1-(\delta_{i,t}+k_{t-1})/\alpha$
Case 1	$d-1$	$\delta_{i,t}+k_{t-1}$	$\delta_{i,t}-\delta_{n,t}+c$	For $0<d<2(1-k_{t-1})\leq 2$ such that $\delta_{i,t}>0$, $\alpha(1-\frac{d}{2})-k_{t-1}<\delta_{i,t}<(1-\frac{d}{2})-k_{t-1}$
Case 2	$\frac{1}{1-\alpha}[-c+d(1-\alpha/2)$ $+(-1+k_{t-1}+\delta_{n,t})]$	0	$\frac{1}{1-\alpha}[-c+\alpha(-1+d/2)$ $+k_{t-1}+\delta_{n,t}]$	For $0<d<2(1-\frac{k_{t-1}}{\alpha})<2$, such that $\delta_{i,t}>0$, $\delta_{i.t}<\alpha(1-\frac{d}{2})-k_{t-1}$ and $c-k_{t-1}+\alpha(1-\frac{d}{2})<\delta_{n,t}<c-k_{t-1}+(1-\frac{d}{2})$
Case 3	$d-1$	$\delta_{i,t}+k_{t-1}$	$\delta_{i,t}-\delta_{n,t}+c$	For $0<d<2(1-k_{t-1})\leq 2$ such that $\delta_{i,t}>0$, $\alpha(1-\frac{d}{2})-k_{t-1}<\delta_{i,t}<(1-\frac{d}{2})-k_{t-1}$
Case 4	0	$2(\delta_{n,t}-c)+k_{t-1}-\delta_i$	0	$\delta_{i,t}-2(\delta_{n,t}-c)>k_{t-1}+(1-\frac{d}{2})$
Case 5	0	$k_{t-1}+\delta_{i,t}$	0	$\delta_{i,t}+k_{t-1}>1-\frac{d}{2}$
Case 6	0	0	$c+k_{t-1}+\delta_{n,t}$	If $d<2$, $\delta_{n,t}-c+k_{t-1}>1-\frac{d}{2}$ If $d>2$, $\delta_{n,t}-c+k_{t-1}>0$

the welfare maximization problem (Section 4). Furthermore, we compute the condition of the existence of each case which depends on the demand and production cost parameters, the levels of innovation and the level of damages from the old technology. These conditions are non-negativity constraints on the level of production and price (i.e.; $q_{l,t}\geq 0$, $q_{si,t}\geq 0$, $q_{sn,t}\geq 0$, $p_l\geq 0$ and $p_s\geq 0$). In addition, whenever this is relevant, we also consider the condition to avoid copycats for the new technology developed by the incumbent or the new entrant following the condition $\delta_{i,t}+c-p_s\geq 0$ and $\delta_{n,t}-p_s\geq 0$. The results are summarised in Table 2.

The results show that, first, the tax and subsidy scheme is sufficient to internalize the negative damage from using the old technology (i.e.; price is equal to marginal damage) and to set the price of the new technology to be equal to the lower marginal production cost of the new technology. Second, a high damage would require a high tax in order to implement the first best solution in the equilibrium. Third, given that the existence condition of each market structure is different as well as the optimal instruments for some cases, the type of instruments (tax or subsidy) is also different.

More precisely, in Case 0 and for low damage (or insufficient innovation), i.e., $d/2<1/2<1-(\delta_{i,t}+k_{t-1})/\alpha$, the social planner would need to provide a subsidy for the production of the old technology. On the contrary, a tax would be required when the damage is high (or the innovation is sufficient) and is in the range of $1/2 < d/2 < 1 - (\delta_{i,t} + k_{t-1})/\alpha$. Case 1 is not optimally feasible when the damage is sufficiently high (or the innovation is sufficiently accumulated). Furthermore, a high share of the market (i.e., α) gives incentive to the incumbent to innovate more as the range of the low levels of innovation reduces. In Case 1, the social planner should tax the old technology if $1 < d < 2(1 - k_{t-1})$, otherwise (i.e., $d < 1$) she should provide subsidy on the old technology to the incumbent. For the new technology, she should provide a subsidy to the incumbent and no subsidy to the new entrant.

Case 2 is a bit different from the previous cases as the incumbent does no longer compensate the loss of profit by replacing the production of the old technology with that of the new technology. Whenever the new entrant has motivation to produce the new technology, this reduces the optimal production of the old technology from the incumbent. Consequently, a higher innovation by the new entrant will reduce the level of production of the old technology and will then increase the optimal tax on the old technology. In addition, the level of damage positively affect the optimal subsidy as this increases the optimal production of the new technology. The social planner decision is as follows:a tax on the old technology should be implemented if $\delta_{n,t} > 1 + c - k_{t-1} - (1 - \frac{\alpha}{2})d$, otherwise a subsidy should be provided to the incumbent. For the new technology, the social planner should always provide subsidy to the new entrant.

The tax and subsidy scheme in Case 3 is the same as the one in Case 1 as at the optimum, it is preferable for the social planner to let only the incumbent produce the new technology due to its cost advantage. For Case 4, the existence condition implies that higher damage does not affect the set of innovation for the new entrant but enlarges the possibility of innovation for the incumbent. The incumbent would prefer to innovate more to avoid a prohibitive tax on the old technology. The decision of the social planner in Case 4 is the following. For low level of damage (i.e., $d < 2$), providing a positive subsidy to the incumbent is not feasible, while a negative subsidy to the incumbent is feasible when $\delta_{i,t} - 2\delta_{n,t} > -2c + k_{t-1}$. For high level of damage (i.e., $d > 2$), the social planner should provide a positive subsidy to the incumbent when $-2c + k_{t-1} + (1 - \frac{d}{2}) < \delta_{i,t} - 2\delta_{n,t} < -2c + k_{t-1}$, or a negative subsidy to the incumbent when $\delta_{i,t} - 2\delta_{n,t} > -2c + k_{t-1} + (1 - \frac{d}{2})$.

Case 5 is similar to Case 1 in term of the new technology as here, the old technology is phased out. The implementation of the optimal production of the new technology would require a positive subsidy to the incumbent. Therefore, the only condition that is relevant is the condition under which the case 5 exists. For Case 6, given the existence condition, it is never feasible to implement a negative subsidy to the new entrant. The social planner will always provide a positive subsidy to the new entrant.

The above analysis can be summarised in the following propositions.

Proposition 4. *The optimal instruments that are required to implement the first best solution satisfy the followings:*

1- *As long as the incumbent supplies only the old technology or the new technology,*
   (a) *the optimal level of tax on the old technology is* $\tau^* = d - 1$
   (b) *and the optimal level of subsidy is* $s_i^* = k_{t-1} + \delta_{i,t}$

2- *When the new entrant only supplies the new technology,*
   (a) *the optimal level of tax on the old technology is given by* $\tau^* = \frac{-c+d(1-\alpha/2)+(-1+k_{t-1}+\delta_{n,t})}{1-\alpha}$
   (b) *and the optimal level of subsidy is given by* $s_n^* = \frac{-c+\alpha(-1+d/2)+k_{t-1}+\delta_{n,t}}{1-\alpha}$

Proposition 5. *Given the conditions that characterise the existence of each of the market structure, the full set of decisions for the social planner is described as follows:*

1- *When the old technology is available and the incumbent supplies the new technology, low damages require a subsidy while a tax is needed for high damages.*

2- *When the old technology is available and only the entrant supplies the new technology, a tax is needed for high damages. For low damages, a subsidy is required only if the level of innovation of the entrant is sufficiently low, i.e.,* $\delta_{n,t} < c - d(1 - \alpha/2) + (1 - k_{t-1})$. *Otherwise, a tax would be required.*

3- When the new technology is available and except the case in which the two firms share the market of the new technology and the old technology is phased out, subsidy should be provided to the relevant firm, i.e., either the incumbent or the new entrant.

4- When the two firms share the market of the new technology and the old technology is phased out,

(a) for low damages, a subsidy to the incumbent is not feasible, while a negative subsidy to the incumbent is feasible when $\delta_{i,t} - 2\delta_{n,t} > -2c + k_{t-1}$.

(b) for high level of damage, a positive subsidy to the incumbent is required when $-2c + k_{t-1} + (1 - \frac{d}{2}) < \delta_{i,t} - 2\delta_{n,t} < -2c + k_{t-1}$, or a negative subsidy to the incumbent when $\delta_{i,t} - 2\delta_{n,t} > -2c + k_{t-1} + (1 - \frac{d}{2})$.

For illustration purposes, we provide graphs in Fig 1-6 for different levels of damage to show the different decisions regarding the optimal instruments. The first three figures concern the cases when the old technology is still produced while the remaining three cases focus on the phase-out of the old technology.

For the cases with available old technology, we distinguish between cases 1 and 3 (one firm produces the new technology); and case 2 (the two firms share the new technology market). Figure 1 et 2 show that the possibility of the cases to exist reduces when the damage increases as the economy cannot accommodate with a high cost associated with the pollution from the old technology. For a very high damage, the cases are no longer possible. Moreover, when the damage is low, only subsidy is possible to implement the first best outcome. For a moderate damage when the case still exists, a tax would be required. A tax is still appropriate as instrument for a very high damage, however the case would not exist any more. In this case, there is no need to implement such instrument as the firms would choose another strategy.

As in the previous case, Figure 2 shows that as long as the damage becomes higher, the case 2 is less attractive and may not even be possible for a very high damage. Here the difference is that depending on the level of innovation, the optimal policy can be a tax or a subsidy on the old technology when the damage is low. More precisely, when the entrant has made a lot of effort in innovating, the social planner would prefer to tax the old technology. On the contrary, when the level of innovation is not enough, it is preferable to provide subsidy.

Figures 5 and 6 illustrate the first cases when the old technology is phased out. The two cases display the same patterns: subsidy is required rather for the incumbent or the new entrant to implement the first best outcome. This is not affected by the level of damage as the old technology is produced by one firm. The pattern described in Case 4 when the new technology market is shared between the two firms is different. Figure 4 shows that it is never optimal to provide subsidy on the new technology when the damage is sufficiently low or moderate. This is because the social planner may prefer to give incentive for the production of the old technology. In this case, the social planner would implement a tax on the new technology. However, the social planner may provide subsidy when the damage is sufficiently high. This is less likely to happen depending on the levels of innovation of the two firms. In fact, a tax on the old technology would already shift the motivation of the incumbent to produce the new technology. To prevent the incumbent to over-produce the new technology, a subsidy on the new technology would be needed.

7 Fiscal implications

While the social planner would decide whether environmental tax or/and subsidy is suitable to implement the first best, this decision also has fiscal implications. In this session, we calculate the fiscal implications in term of budget Ω_i as the total tax on the old technology T_i net of the total subsidy on the new technology S_i. We then discuss under which conditions the budget is positive or negative. When only tax or subsidy is implemented, this analysis is obvious. However, the analysis is interesting when the two instruments are implemented together such that the effects can compensate each other. We claim the following proposition.

Proposition 6. *In term of budget,*

1- *When only the old technology is available, the budget is negative for small damages and positive for high damages.*

2- *When both old and new technologies are available in the market, the budget is negative for small damages, while it can be negative or positive for high damages.*

3- When the old technology is phased out, the budget is negative, except when both firms supply the new technology.

4- When both firms supply the new technology and the old technology is phased out,

(a) for sufficiently low level of damage, the budget is positive

(b) for a sufficiently high level of damage, the budget can be negative or positive.

Proof. For part 1- of Proposition 6, only the tax is implemented as only the old technology is available. Therefore, the budget gives: $\Omega_0 = T_0 = -(1-d)(2-d)/2$. We can easily check that for $d < 1$, $\Omega_0 < 0$, that is the regulator provides subsidy to produce the old technology. When $1 < d < 2$, $\Omega_0 > 0$ and the regulator collects a tax from the production of the old technology.

In part 2- of the proposition, (i) both technologies are produced by the incumbent (forced by the social planner in Case 1) or the incumbent supplies the old technology and the new entrant supplies the new technology. For the first case (i), the total tax and total subsidy are given by $T_j = \tau_j^* q_{l,t,j}^* = (d-1)\left(2(1-\delta_{i,t}-k_{t-1})-d\right)/[2(1-\alpha)]$ and $S_j = s_{i,j}^* q_{si,t,j}^* = (\delta_{i,t}+k_{t-1})\left(2(\delta_{n,t}-c+k_{t-1}-\alpha)+\alpha d\right)/[2\alpha(1-\alpha)]$, with $j = 1,3$. The budget gives $\Omega_j = T_j - S_j = (\delta_{i,t}+k_{t-1})(-4\alpha-2c+3\alpha d+2\delta_{n,t}+2k_{t-1}) + \alpha(d-2)(d-1)/[2(\alpha-1)\alpha]$, with $j = 1,3$. The total tax is positive if $1 < d < 2(1-k_{t-1})$ and negative when $d < 1$. Moreover, the total subsidy is always positive. We can then deduce that the budget is negative for $d < 1$. The budget can be positive or negative depending on the extend to which the total tax compensates the total subsidy. We can then find a range of the damage within the interval of $1 < d < 2(1-k_{t-1})$ for positive or negative budget.

Similarly, in the second case (ii), the budget is given by

$\Omega_2 = \frac{\alpha(d(2-\alpha)-2(1-k_{t-1}-\delta_{n,t}+c))(2(1-k_{t-1}-\delta_{n,t}+c)-d)-(\alpha d-2(\alpha-k_{t-1}-\delta_{n,t}+c))^2}{4\alpha(1-\alpha)^2}$. As in the previous case, we can find a range of the damage within the interval $d > \frac{2}{2-\alpha} - k_{t-1} - \delta_{n,t} + c$ such that the budget is positive or negative. However, when $d < \frac{2}{2-\alpha} - k_{t-1} - \delta_{n,t} + c$, the budget is negative.

For Parts 3- and 4- of the proposition, the old technology is completely phased out. Consequently, there is no tax on the old technology collected by the regulator. The total budget in the three cases is given by the same following expressions: $\Omega_j = -s_{i,j}^* q_{si,t,j}^* = -(\delta_{i,t}+k_{t-1})^2/\alpha$, with $j = 4,5,6$. Although, the budget is the same in absolute value, for all the three cases, the sign can be different depending on the existence condition of the case that can trigger whether it is a negative or positive subsidy. Namely, when both firms supply the new technology, for sufficiently low level of damage (i.e., $d < 2$), only a negative subsidy to the incumbent is feasible (when $\delta_{i,t} - 2\delta_{n,t} > -2c + k_{t-1}$). In this case, the budget is positive as the regulator would implicitly tax the incumbent for producing a new technology. For a sufficiently high level of damage (i.e., $d > 2$), the subsidy can feasibly be negative or positive. Thus, the budget is negative when $-2c + k_{t-1} + (1-\frac{d}{2}) < \delta_{i,t} - 2\delta_{n,t} < -2c + k_{t-1}$ and positive when $\delta_{i,t} - 2\delta_{n,t} > -2c + k_{t-1} + (1-\frac{d}{2})$. In both the other cases, the existence condition dictates a positive subsidy to the incumbent or new entrant for producing the new technology. Therefore, the budget is always negative.

□

The Proposition 6 shows that the social planner has a second layer in term of fiscal implications to decide on which market structure to implement and at which stage of the green technology transition process. While a set of market structure may have the same social welfare, the regulator may prefer a situation that results in a budget surplus rather than a situation with budget deficit.

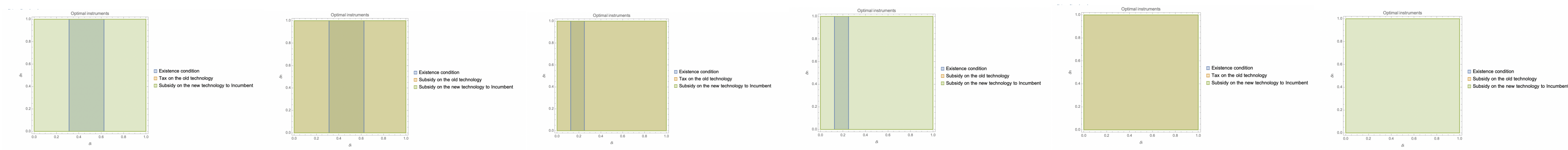
8 Conclusion

There are several challenges that need to be addressed for a green technology transition. First, a number of market failures such as the public good nature of innovation and environmental externalities are associated with this transition. In this sense, without public policies, markets alone are not sufficient to provide right incentives for the investment and diffusion of green technologies. Second, green technology transition often implies a change of the market structure as incumbents producing old technologies will face competition from new entrants that invest in the new technologies. As a response to this threat of competition, incumbents may decide to strategically invest in the new technology to avoid competition, to follow the new entrant or to become a leader in this new technology market. This is the example of EV market in the mobility sector and renewable energy in the electricity sector. In this context, finding the optimal mix of policy instruments that addresses together those issues is challenging.

This paper studies the interrelations between market structure, r&d policy and environmental policy to find the optimal mix of policy instruments. First, we compare optimal welfare for different market structure and find that the socially desirable market structure is not a single solution and depends on the levels of damage and innovation. For instance, we show that for small damages, small accumulated level of innovation would motivate the regulator to choose only the old technology. However, when the market has accumulated enough innovation, the transition to only the new technology is welfare improving even for small damages. Second, given the socially desirable market structure, we analyse the optimal policy instruments that need to be implemented. We show that the taxation and subsidisation policy differs depending on the market structure and the environmental damages. We also find that the level of environmental damages and the market structure affect how the social planner should combine the optimal policy instruments. Finally, we study the fiscal implications of the optimal policy instruments. We show that both the market structure and the level of environmental damages influences the budget surplus or deficit. The regulator can use the fiscal implications in addition to social welfare, to decide on which market structure to implement, and at which stage of the green technology transition process.

As policy implications, we suggest the implementation of both environmental tax on the old technology and r&d subsidy on the new technology-which should be different for the incumbent and the entrant-as policy instruments to support the green technology transition. However, to reflect the dynamics of the market structure, the regulator should not commit to a static combination of these two policy instruments. This mix of policy instruments should change depending how the incumbents and entrants behave.

This paper confines the attention to the effects of environmental and technology policies on the local technology market. This could be valid for small open economies in which innovation investment is exogenous. An interesting and complete approach could be to include innovation process and study how different strategies of incumbents and entrants would interact under r&d subsidies policies that reduce innovation cost. However, this would also influence the interactions that we have already studied in this paper. Thus, we left this for further research as the results show that optimal policy is already complex even when we take out r&d stage.



(b) $t > 0$ *and* $s_i > 0$ **(c)** $t < 0$ *and* $s_i > 0$

(c) *d=0.75*

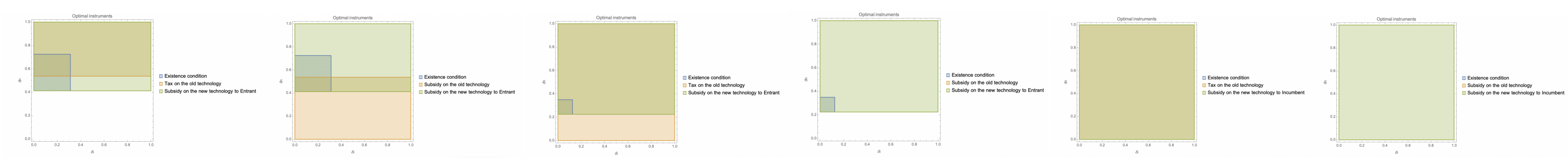
(e) $t > 0$ *and* $s_i > 0$ **(f)** $t < 0$ *and* $s_i > 0$

(f) *d=1.5*

(h) $t > 0$ *and* $s_i > 0$ **(i)** $t < 0$ *and* $s_i > 0$

(i) *d=10*

Figure 1: *Illustration of case 1 for different scenarios with respect to the values of the damage.*



(b) $t > 0$ *and* $s_n > 0$ **(c)** $t < 0$ *and* $s_n > 0$

(c) *d=0.75*

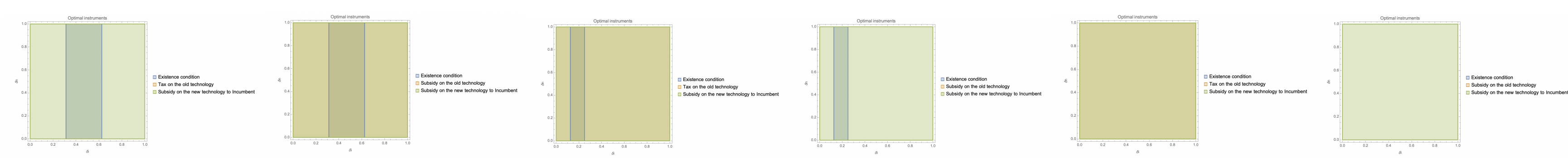
(e) $t > 0$ *and* $s_n > 0$ **(f)** $t < 0$ *and* $s_n > 0$

(f) *d=1.5*

(h) $t > 0$ *and* $s_n > 0$ **(i)** $t < 0$ *and* $s_n > 0$

(i) *d=10*

Figure 2: *Illustration of case 2 for different scenarios with respect to the values of the damage.*



(b) $t > 0$ *and* $s_i > 0$ **(c)** $t < 0$ *and* $s_i > 0$

(c) *d=0.75*

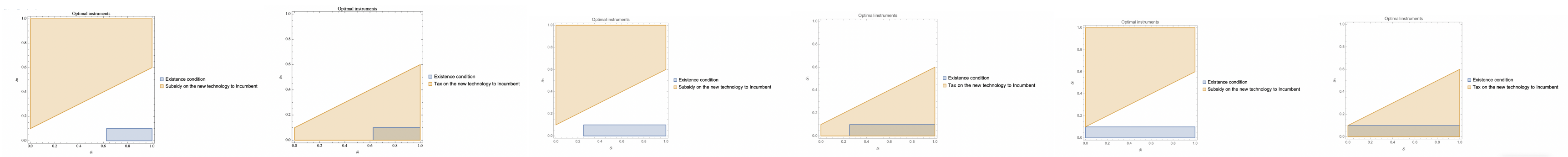
(e) $t > 0$ *and* $s_i > 0$ **(f)** $t < 0$ *and* $s_i > 0$

(f) *d=1.5*

(h) $t > 0$ *and* $s_i > 0$ **(i)** $t < 0$ *and* $s_i > 0$

(i) *d=10*

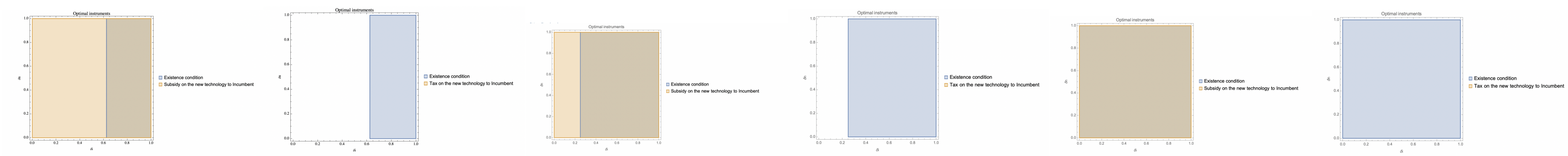
Figure 3: *Illustration of case 3 for different scenarios with respect to the values of the damage.*



(b) $s_i > 0$ (c) $s_i < 0$ (e) $s_i > 0$ (f) $s_i < 0$ (h) $s_i > 0$ (i) $s_i < 0$

(c) *d=0.75* (f) *d=1.5* (i) *d=10*

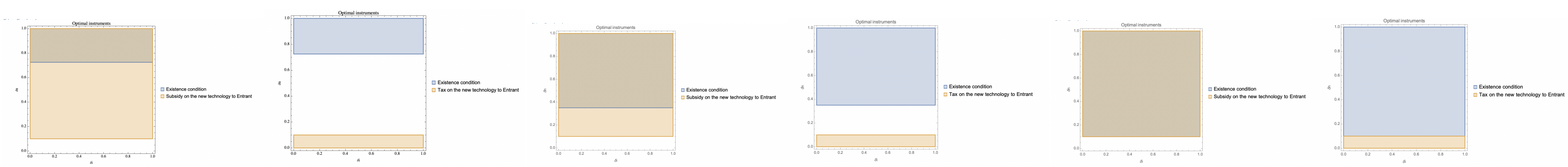
Figure 4: *Illustration of case 4 for different scenarios with respect to the values of the damage.*



(b) $s_i > 0$ (c) $s_i < 0$ (e) $s_i > 0$ (f) $s_i < 0$ (h) $s_i > 0$ (i) $s_i < 0$

(c) *d=0.75* (f) *d=1.5* (i) *d=10*

Figure 5: *Illustration of case 5 for different scenarios with respect to the values of the damage.*



(b) $s_n > 0$ (c) $s_n < 0$ (e) $s_n > 0$ (f) $s_n < 0$ (h) $s_n > 0$ (i) $s_n < 0$

(c) *d=0.75* (f) *d=1.5* (i) *d=10*

Figure 6: *Illustration of case 6 for different scenarios with respect to the values of the damage.*

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Appendix

A Social Optimum

A.1 Case 1: New entrants and incumbent supply new technology

Solving the program in Eq.8 and assuming that the total production of the new technology is given by $Q_{s,t} = q_{si,t} + q_{sn,t}$, gives:

$$q^*_{l,t,1} = \frac{1}{1-\alpha}[-\delta_{i,t} + (1 - d/2) - k_{t-1}], \tag{9}$$

$$Q^*_{s,t,1} = \frac{1}{\alpha(1-\alpha)}[\delta_{n,t} - \alpha(1 - d/2) - c + k_{t-1}] \tag{10}$$

and

$$\delta_{i,t} = \delta_{n,t} - c \tag{11}$$

As we assume that the cost of producing the new technology by the new entrant is higher than that of the incumbent, whenever the two firms supply the new technology, the social planner will optimally prefer the incumbent to produce the new technology. This translates into $q^*_{sn} = 0$ and $Q^*_s = q^*_{si}$. thus, the optimal welfare is given by:

$$W^*_{1,t} = \frac{4\delta_{i,t}[-2c + \alpha(d-2) + 2k_{t-1}] + 8\delta_{n,t}(c + \delta_{i,t}) - 4\delta^2_{n,t} + \alpha(d-2)(d + 4k_{t-1} - 2) - 4c^2 + 4k^2_{t-1}}{8(1-\alpha)\alpha} \tag{12}$$

From the derivative of the welfare with respect to d:

$$W^*_{1d} = \frac{d + 2(\delta_i + k_0 - 1)}{4(1-\alpha)},$$

we can easily show that the optimal level of the welfare is a decreasing function of the level of damage whenever the marginal damage is sufficiently low (i.e, $d/2 < 1 - \delta_{i,t} - k_{t-1}$). Note that this condition needs to match with the condition that defines the existence of Case 1 **(see Section....)**

A.2 Case 0: Only old technology

In the case the incumbent in a monopoly supplying only the old technology, the welfare maximisation problem is similar to the one in the Case 1 except that we solve for the FOC with respect to $q_{l,t}$ given the condition that $q_{sn,t} = q_{si,t} = 0$. The optimal level of the old technology production and the optimal welfare are given by:

$$q^*_{l,t,0} = 1 - d/2 \tag{13}$$

and

$$W^*_{0,t} = \frac{1}{8}(d-2)^2 \tag{14}$$

Taking the derivative of the optimal welfare with respect to d gives:

$$W_{0,d} = \frac{d-2}{4}$$

Thus, the higher is the damage, the lower is the optimal welfare if the marginal damage is low (i.e., $d < 2$). This condition is the also part of the conditions that define the existence of case 0, namely for positive $q^*_{l,t,0}$.

A.3 Case 2: New technology only sold by new entrants

From the welfare optimization in the Case 1, we solve for the FOCs with respect to $q_{l,t}$ and $q_{sn,t}$ given the condition that $q_{si,t} = 0$. This gives the following optimal amount of the old and new technologies:

$$q^*_{l,t,2} = \frac{1}{1-\alpha}[-\delta_{n,t} - d/2 + c + 1 - k_{t-1}] \tag{15}$$

and

$$q^*_{sn,t,2} = \frac{1}{\alpha(1-\alpha)}[\delta_{n,t} - \alpha(1-d/2) - c + k_{t-1}] \tag{16}$$

From the optimal levels of production, the optimal value for the welfare is given by:

$$W_{2,t} = \frac{\delta^2_{n,t} + [-2c + \alpha(d-2) + 2k_{t-1}]\delta_{n,t} + (c - k_{t-1} + 1)\alpha d + \alpha(d/2)^2 + \alpha + (c - k_{t-1})(2\alpha + c - k_{t-1})}{2\alpha(1-\alpha)} \tag{17}$$

Taking the derivative of the welfare with respect to d gives:

$$W_{2,d} = -\frac{-2c + d + 2(\delta_{n,t} + k_{t-1} - 1)}{4(\alpha - 1)}$$

As in the previous cases, we can easily show that the optimal level of the welfare is a decreasing function of the level of damage whenever the marginal damage is sufficiently low with $d/2 < 1 - k_{t-1} - \delta_{n,t} + c$. This condition should match with the one that defines the existence of Case 2 and similar to the one in Case 1 with $\delta_{i,t} = \delta_{n,t} - c$.

A.4 Case 3: New technology only sold by incumbent

The welfare maximisation of the Case 3 is similar to that of Case 1 by solving for the FOCs with respect to $q_{l,t}$ and $q_{si,t}$ given the condition that $q_{sn,t} = 0$, the optimal levels of production are given by:

$$q^*_{l,t,3} = \frac{1}{1-\alpha}[-\delta_{i,t} - d/2 + 1 - k_{t-1}] \tag{18}$$

and

$$q^*_{si,t,3} = \frac{1}{\alpha(1-\alpha)}[\delta_{i,t} - \alpha(1-d/2) + k_{t-1}] \tag{19}$$

The optimal value for the welfare is given by:

$$W^*_{3,t} = \frac{\delta^2_{i,t} + (-2\alpha + \alpha d + 2k_{t-1})\delta_{i,t} + \alpha(d/2)^2 - (1-k_{t-1})\alpha d + \alpha + k_{t-1}(k_{t-1} - 2\alpha)}{2\alpha(1-\alpha)} \tag{20}$$

Taking the derivative of the welfare with respect to d gives:

$$W_{3,d} = \frac{d + 2(\delta_{i,t} + k_{t-1} - 1)}{4(1-\alpha)}$$

This is the same as the one on the Case 1 with the same conclusion.

A.5 Case 4: New entrants and incumbent supply new technology and old technology is phased out

In the Case 4, the optimal levels of the new technology production are obtained by solving the FOCs of the welfare maximisation problem in the Case 1 with respect to $q_{sn,t}$ and $q_{si,t}$ given the condition that $q_{l,t} = 0$. The total optimal level of the new technology production is given by:

$$Q^*_{s,t,4} = \frac{1}{\alpha}[\delta_{n,t} - c + +k_{t-1}] \tag{21}$$

As in the case 1, the social planner will prefer the incumbent to supply the new technology such that $q^*_{sn,t,4} = 0$, $q^*_{si,t,4} = Q^*_{s,t,4}$ and Eq.11 holds. The optimal social welfare is given by:

$$W^*_{4,t} = \frac{-\delta^2_{n,t} + 2(c + \delta_{i,t})\delta_{n,t} - 2(c - k_{t-1})\delta_{i,t} - c^2 + k^2_{t-1}}{2\alpha} \tag{22}$$

Under Eq.11, the optimal level of production becomes $(\delta_{i,t} + k_{t-1})/\alpha$ and the optimal welfare becomes $(\delta_{i,t} + k_{t-1})^2/(2\alpha)$. We can easily show that innovation by the incumbent is welfare improving.

A.6 Case 5: Only incumbent supplies new technology and old technology is phased out

As in the previous cases, we solve for the FOC of $W_{1,t}$ with respect to $q_{si,t}$ given the condition that $q_{l,t} = q_{sn,t} = 0$. Under Eq.11, the optimal production of the new technology and the associated optimal welfare are the same as in the Case 4: $q^*_{si,t,5,} = q^*_{si,t,4,}$ and $W^*_{5,t} = W^*_{4,t}$.

A.7 Case 6: Only new entrant supplies new technology and old technology is phased out

The problem in the Case 6 is similar to the previous case in which we solve for the FOC of $W_{1,t}$ with respect to $q_{sn,t}$ given the condition that $q_{l,t} = q_{si,t} = 0$. The optimal production of the new technology by the new entrant and the optimal social welfare are similar to the ones in the Case 5, $W^*_{6,t} = W^*_{5,t}$ with $\delta_{i,t} = \delta_{n,t} - c$.

Table A summarizes the optimal quantities of the old technology and the new technology and the optimal social welfare.

Table 3: *Optimal quantities of the old technology and the new technology and the optimal social welfare.*

Cases	q_l
Case 0	$1 - d/2$
Case 1	$\frac{1}{1-\alpha}[-\delta_i + (1 - d/2) - k_0]$
Case 2	$\frac{1}{1-\alpha}[-\delta_n - d/2 + c + 1 - k_0]$
Case 3	$\frac{1}{1-\alpha}[-\delta_i - d/2 + 1 - k_0]$
Case 4	-
Case 5	-
Case 6	-

Cases	q_{si}
Case 0	-
Case 1	$\frac{1}{\alpha(1-\alpha)}[\delta_n - \alpha(1 - d/2) - c + k_0]$
Case 2	-
Case 3	$\frac{1}{\alpha(1-\alpha)}[\delta_i - \alpha(1 - d/2) + k_0]$
Case 4	$\frac{1}{\alpha}[\delta_n - c + +k_0]$
Case 5	$\frac{1}{\alpha}[\delta_i + k_0]$
Case 6	-

Cases	q_{sn}
Case 0	-
Case 1	0
Case 2	$\frac{1}{\alpha(1-\alpha)}[\delta_n - \alpha(1 - d/2) - c + k_0]$
Case 3	-
Case 4	0
Case 5	-
Case 6	$\frac{1}{\alpha}[\delta_n + k_0 - c]$

Cases	W
Case 0	$\frac{1}{8}(d-2)^2$
Case 1	$-\frac{4\delta_i[-2c+\alpha(d-2)+2k_0]+8\delta_n(c+\delta_i)-4\delta_n^2+\alpha(d-2)(d+4k_0-2)-4c^2+4k_0^2}{8(\alpha-1)\alpha}$
Case 2	$\frac{\delta_n^2+[-2c+\alpha(d-2)+2k_0]\delta_n+(c-k_0+1)\alpha d+\alpha(d/2)^2+\alpha+(c-k_0)(2\alpha+c-k_0)}{2\alpha(1-\alpha)}$
Case 3	$\frac{\delta_i^2+(-2\alpha+\alpha d+2k_0)\delta_i+\alpha(d/2)^2-(1-k_0)\alpha d+\alpha+k_0(k_0-2\alpha)}{2\alpha(1-\alpha)}$
Case 4	$\frac{-\delta_n^2+2(c+\delta_i)\delta_n-2(c-k_0)\delta_i-c^2+k_0^2}{2\alpha}$
Case 5	$\frac{(k_0+\delta_i)^2}{2\alpha}$
Case 6	$\frac{(k_0-c+\delta_n)^2}{2\alpha}$