Pitfalls of Insuring Production Risk: a Case Study on some Wind Power Auctions in France

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Prelimary and incomplete - Do not cite

Abstract

We consider auctions for procurement contracts that have both exogenous production risk and a payment rule depending on the winning bidder's self-reported expected production. We establish an incentive to over-estimate production when the payment rule is production-insuring (under truthful reporting), and that it is impossible to design a rule that fully insure strategic bidders. We then analyze equilibrium bidding behavior under several paradigms and illustrate our results on the pitfalls of production-insuring payment rules with some offshore wind power auctions in France. The estimated benefits under truthful reporting are much lower in magnitude than the potential losses due to misreporting, which exceed 3%. We consider variants of the French rule, in particular with punishments aimed to discourage misreporting, and find limited room for improving unit-price contracts.

Keywords: Production Risk; Insurance Provision Contracts; Auctions for Contracts; Market Design; Gaming; Renewable Energy; Wind Power.
JEL classification: D44; D47; D86; L94.

1 Introduction

The transition towards low-carbon economies has induced many countries to support renewable energy sources of electricity (RES-E) on a large scale, especially from wind and solar power. The corresponding subsidy contracts are often assigned through auctions.[1] Such production contracts involve risks which are known to induce precautionary bidding when producers are risk-averse (Eso and White, 2004). Among the various sources of risk, one is the determination by weather conditions of the quantity of electricity that will be produced from wind and solar sources. Letting

[1] In 2019, an estimated 115 GW (resp. 60 GW) of solar PV (resp. wind power) capacity was installed worldwide. RES-E subsidies were awarded through auctions in 48 countries according to REN21's 2020 global status report (www.ren21.net/wp-content/uploads/2019/05/gsr_2020_key_findings_en.pdf).

the public authority bear a larger share of this risk could reduce risk premiums and help to develop RES-E at a lower cost.

The benefits of insurance schemes that hedge contractors against exogenous risk, thereby deflating the risk premiums required by producers, have been analysed in the contingent auction literature in which payments are allowed to depend on ex post verifiable variables, such as volumes extracted in oil lease or timber auctions (Bhattacharya et al (2018) and Athey and Levin (2001)), revenues from tolls in auctions for highway franchising (Engel, Fischer and Galetovic, 2001), quantities of inputs in procurement for infrastructure project (Bolotnyy and Vasserman (2019) and Luo and Takahashi (2019)), coal prices in procurement for fossil power plant (Ryan, 2020), and lastly, electricity produced in the RES-E auctions. Various hedging instruments have been considered and/or used in practice. Engel, Fischer and Galetovic (2001) plead for least-present-value-auctions where the franchise term adjust to demand realizations: according to their estimations, such contracts could reduce public spending by more than 20% compared to fixed term contracts where contractors bid on tolls.[2] One of the most popular instruments to share profits between parties are royalties and unit-price (UP) contracts that specify, respectively, a percentage of the revenues and of the observable costs that accrue to the buyer. According to their structural model, Bhattacharya et al (2018) estimate that the optimal royalty rate is around 26% which is more than 50% higher than the one currently used in oil lease auctions. In procurement for transport infrastructure projects, Bolotnyy and Vasserman (2019) estimate that switching to a Fixed Price (FP) contract – where the contractor bears all the cost overruns – would more than double public spending compared to a UP scaling auctions where producers are partially insured against risk at the time of contracting (provided that unit prices reflect marginal production cost).[3]

RES-E are often subsidized through Feed-in-Tariff (FiT) contracts where producers receive a fixed subsidy for each MWh produced. Those contracts are analogous to the UP contracts used in infrastructure procurement, up to the twist that solar PV and wind power production do not involve any marginal costs. Henceforth, FiT contracts do not hedge producers against but rather greatly expose them to production risk. If we abstract from discounting and limit ourselves to the ex post risk after the equipment has been installed, the sole source of risk faced by producers in

[2] In a related vein, for offshore wind farm projects, the Danish government has used auctions where the duration of the contract adjust so that the public subsidy ends once a fixed quantity of energy has been produced (IRENA, 2017). After the end of the contract, the producer is then free to sell the electricity produced, which contrasts with auctions for highway franchising where the contractor does not collect any toll after the end of the contract.

[3] For the same kinds of contracts run by a different Department of Transportation, Luo and Takahashi (2019) show consistently that UP contracts are usually chosen by project managers for "complex projects", that are also more risky, while FP contracts are typically used for "simple projects". Furthermore, in scaling auctions where potential contractors bid on multiple unit-prices, there is another source of insurance provision by choosing appropriately the item(s) on which to bid more aggressively.

terms of revenues is the total quantity produced over the duration of the contract. To remunerate wind farms, some countries – including notably Brazil, France and Germany – have departed from linear FiT and adopted contract designs (further referred to more generally as "payment rules") where producers' yearly revenue is made less sensitive to yearly production variations within an interval around a production of reference. For instance, the payment rule for early offshore wind auctions in France was almost flat within an interval of plus or minus 10% around the production of reference. The rationale was presumably to insure producers against meteorological variations but also possibly against poor production predictions.[4] Under such production-insuring payment rules, the subsidy received by the contractor does not depend solely on the contract price but also on a so-called production of reference. In Brazil, this parameter is certified by a third party based on wind measurements in the planned location for the project.[5] In France, the production of reference is based on the firm's own data and calculation.[6] In any case, there is no guarantee that the reported production of reference will be the true one: Firms that are aware of the possibility to game the auction rules could be able to depart from truthful reporting.[7]

Our theoretical analysis considers a set of risk-averse firms bidding for risky contracts that remunerate production as a linear function of the auction price and also as a function of both the realized production and the production of reference, as this in a quite general way much beyond the very specific rule used in France. Our model abstracts from any moral hazard issues: firms are assumed to have no control on the quantity produced after having entered the auction. In addition to considering their price bid, we also consider that firms are either constrained to report truthfully their production of reference or are entirely free – at no cost – to make any possible report. The former (resp. later) firms are called truthful (resp. strategic).

We first formalize a fundamental conflict between insurance provision and strategy-proofness. For this we first consider the class of payment rules that would increase (resp. not change) the expected utility of any risk-averse (resp. risk-neutral) contractor for any symmetric single-peaked production distribution, provided that the contractor has reported truthfully its expected production. We characterize those payment rules that are referred to next as *production-insuring*

[4]The standard deviation of the yearly production of a wind farm represents at least 10% of the mean production (Newbery, 2012). Until recently, the accuracy of wind power forecasting was an important issue and suffered from an important over-prediction bias. See Lee and Fields (2020) for a survey.

[5]See the report D4.1-BRA (2016) of the AURES project for details. In Germany, the production of reference is also determined according to some administrative rules but without using the characteristics of the location (Bichler et al, 2020). The rationale for the German payment rule is very different: it consists in promoting regional diversification by giving bonus (resp. malus) to projects located in less (resp. more) windy areas.

[6]For that purpose, firms provide to the public authority the wind data they used and the technical characteristics of the wind turbine they considered. However, no certification by a third party is required regarding these data and calculations, and the technical characteristics reported are not entirely binding: some firms have finally selected a different wind turbine than the one initially declared to the authority in the auction process.

[7]Obtaining a more favorable production of reference could be obtained either directly through corruption or indirectly by reporting inaccurate technical characteristics about the project.

payment rules and that include in particular the payment rule used in France for offshore wind farms. We establish that for any given production-insuring payment rule and any given symmetric single-peaked distribution, risk-neutral producers strictly benefit from overestimating their expected production. When firms are risk-averse, the analysis of optimal misreporting is less clear-cut since overestimating production may also increase the variability of the revenue which is then detrimental to the contractor. To get more insights, we derive some comparative statics on the optimal misreporting for a specific class of production-insuring payment rules that are very similar to the one used in France.

Second, we consider the class of payment rules that are homogeneous of degree one in the realized production and the production of reference, a property also satisfied by the French wind auction contracts. Then for any production distribution and any form of risk aversion, we cannot design such a payment rule where strategic firms would be fully insured against production risk.

Those two negative results however do not claim that there is no hope to reduce risk thanks to a production-insuring payment rule or a homogeneous of degree one payment rule. We will partially address this question through our simulation exercises calibrated to our wind power contract application where we analyze the performance of a variation of the French rule.

The second part of our theoretical analysis is devoted to the impact of strategic misreporting on equilibrium prices and then on the buyer expected cost. In the benchmark case where all producers are risk-neutral and truthful, the buyer's expected cost is equal to the production (fixed) cost. If producers are (strictly) risk-averse, a risk premium should be added. Under truthful reporting, this premium is lower under a production-insuring contract than under the linear FiT contract. However, this result may no longer hold under strategic reporting: instead of evening out the producer's revenue (as it would be the case under truthful reporting), a production-insuring payment rule could have exactly the opposite effect, as illustrated in Section 2, and those risks are born ultimately by the buyer through an increased risk premium. When all firms are strategic, the equilibrium price decreases compared to the case where all firms are truthful, but this effect is deceptive because equilibrium prices no longer reflect the expected price per quantity produced when the reported production of reference differs from the true one. Quantitatively speaking, the impact of such risk premiums on the buyer's expected cost is nevertheless very limited for our wind power application. However, the picture is very different if firms are heterogeneous, i.e. if some are truthful and others strategic. We consider specifically two models as benchmarks: first a model where a single firm is strategic while the remaining firms are all truthful, and second, a model where each firm is truthful or strategic with some common probability and independently of each other. Such heterogeneity among bidders induces non-competitive rents. In our model with a single strategic bidder and for a typical level of risk aversion ($CRRA = 1$), our estimates of such non-competitive rents exceed 3% and are around fifteen times larger than the estimated theoretical gains that could be expected from the French production-insuring rule (compared to the linear FiT) in the best case when all bidders are assumed to be truthful. We obtain thus that the largest pitfall of production-insuring payment rules do not result from misreporting per se but relies rather on the possible heterogeneity in the way bidders misreport their production of reference. In other words, departing from linear contracts to reduce risk premium seems a quite risky bet for the buyer.

Related literature This paper contributes to the small (and still growing) literature on individual manipulations where bidders have opportunities to "game" the auction rules, which arise in complex environments where bids are multi-dimensional.[8] Yokoo et al. (2004) consider multi-object combinatorial auctions where bidders can benefit from using multiple identities to bid in the auction.[9] In scaling auctions, the score of a bid is computed based on ex ante estimates of the various underlying quantities. If bidders receive ex ante some information about the realized quantities, then they will benefit from skewing their bids (Athey and Levin, 2001).[10] Agarwal et al (2009) discuss such incentives and mention other manipulations as well in sponsored search auctions for online advertising. Last, Ryan (2020) considers a manipulation associated with a hedging instrument in procurement for fossil power plants where ex post risk comes from the future price of coal: a firm's bid can be viewed as the combination of a score (such that the lowest score wins the auction) and a hedging instrument. Ryan (2020) show that some firms do not use the hedging instrument having in mind their ability to renegotiate their contract in case of spikes in the price of coal. Overall, bid manipulations in procurement open the door to welfare inefficiencies by selecting – instead of the firms with the lowest cost – the best "manipulators", i.e. those who benefit the most from ex post renegotiation in Ryan(2020), those who benefit the most from skewing their bids in Luo and Takahashi (2019), or those who are able to fool the auctioneer by misreporting their production of reference in our analysis.

This paper also contributes to the theoretical literature on contingent auctions as surveyed by Skrzypacz (2013). Hansen's (1985) seminal contribution shows that auctions on royalties leave less informational rents to the winning bidder compared to cash auctions. Under a paradigm with risk-

[8]The standard auction formats (that prevail in auction textbooks, as Krishna (2002)) are immune to individual gaming strategies, but not to collective manipulations which are referred to as collusion and have received considerable interest (see for Correia-da-Silva (2017) for a survey).

[9]Such false-name bidding activity is sometimes referred to as shill bidding, a term that is also used for manipulation by the seller consisting in bidding in the auction (Lamy, 2013) in order to increase the selling price, although it is typically considered to be fraudulent activity.

[10]In Athey and Levin's (2001) bi-dimensional timber scaling auctions model, the optimal strategy of a risk-neutral bidder consists in bidding zero on the species whose percentage has been underestimated by the seller and paying the Forest Service only for the overestimated species. Such extreme unbalanced bids are not observed in practice, partly due to risk aversion (Athey and Levin, 2001). Bajari et al (2014) mention another explanation: the risk that a bid could be rejected when its skewness is too visible. Luo and Takahashi (2019) consider multidimensional UP contracts and argue that bidders form their bid portfolios to balance their risks.

neutrality and independent private signals, Laffont and Tirole (1986) and McAfee and McMillan (1987) develop models involving a trade-off between selecting the most efficient supplier and reducing moral hazard: in the optimal contract, the supplier and the buyer share the observable part of the profit. More recently, Abhishek et al. (2015) and Fioriti and Hernandez-Chanto (2020) consider risk-averse bidders and argue that steeper securities are beneficial not only because they reduce informational rents but also because they provide more insurance and thus reduce risk-premiums. Bhattacharya et al. (2018) consider a common value model auction for oil tracts contracts and analyze the trade-off between insurance-provision (pleading for larger royalties) and providing appropriate incentives to drill (or not) the tracts in an efficient manner (pleading for cash auctions). The trade-off we analyze is slightly different but still quite related: insurance-provision induces opportunities to game the auction rule and thus to increase the buyer's cost through non-competitive rents.

The remainder of the paper is organized as follows. Section 2 introduces the payment rule used by the French government and its caveats. Section 3 is devoted to the theoretical analysis of auctions for contracts with production risk: we analyse first how firms can optimally (mis)report their expected production, and second, the buyer's expected cost when the final contract results from competitive bidding under several paradigms regarding how bidders report expected production. Section 4 reports various estimates regarding the performance of the French rule compared to the linear FiT. Section 5 introduces a set of payment rules that are piecewise linear in the realized production (as the French rule). Those rules that are characterized by two parameters are designed in order to deter misreporting. We then present the results of some simulations in an attempt to determine whether and how such simple payment rules could outperform the linear FiT. Section 6 concludes.

2 Background: The French production-insuring payment rule

In 2011 and 2013, the French government auctioned up to 4 GW of capacity through six offshore wind farm projects.[11] For each retained project, the feed-in-tariff (FiT) contract specifies the yearly amount paid by the government to the winning firm as a function of its realized yearly production (in MWh). The French payment rule differs from standard FiT linear contracts where the payment is strictly proportional to total production: the yearly remuneration depends not only on the auction-determined price (per MWh) and the amount of electricity produced during

[11] The auction and contract rules are provided (in French) by the French Energy Regulatory Commission for both auction rounds from 2011 and 2013: http://www.cre.fr/Documents/Appels-d-offres/Appel-d-offres-portant-sur-des-installations-eoliennes-de-production-d-electricite-en-mer-en-France-metropolitaine and http://www.cre.fr/Documents/Appels-d-offres/Appel-d-offres-portant-sur-des-installations-eoliennes-de-production-d-electricite-en-mer-en-France-metropolitaine2.

the year, but also on how the latter compare to the *production of reference* reported by the producers in their bids. This production of reference corresponds to the capacity (which is a technical feature that is verifiable) times the expected *capacity factor* reported by the bidder, and corresponds thus to the expected production. The French payment rule was designed in a way that makes producers' yearly revenues vary little within a range of +/- 10% of the reported expected production. Therefore, this rule offers producers an opportunity to insure themselves against production risk. We presume that the original motivation for such a design was to lower producers' risk premium, that would be passed over to public authorities through higher price bids in the FiT auction.

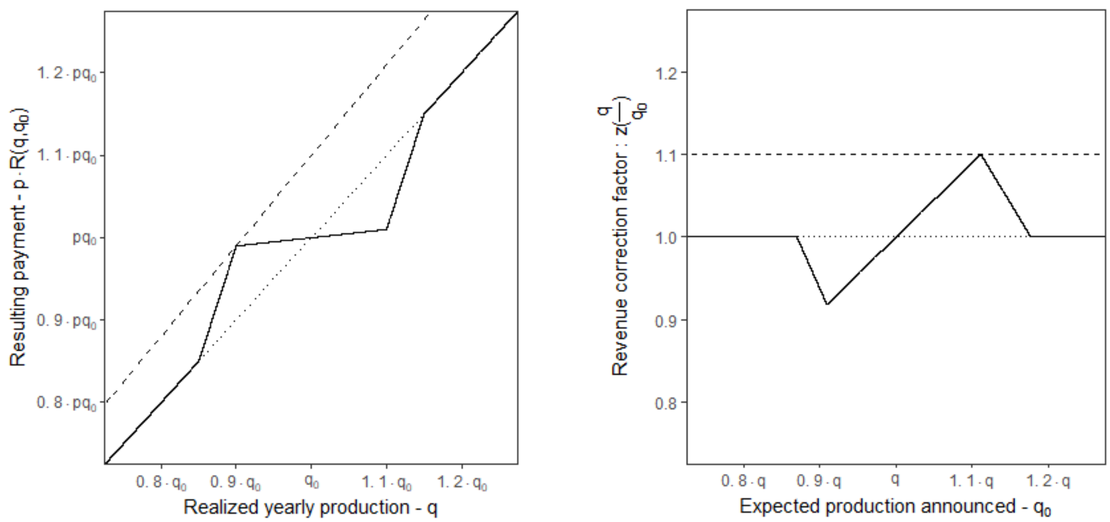
Formally, let p denote the price bid of the winning firm, q_0 the reported expected production and q_t the realized production in year t. Throughout our analysis, we consider payment rules where the revenue of the firm for each year t can be expressed as $p \cdot R(q_t, q_0)$. We also make the normalization $R(q_0, q_0) = q_0$ for any q_0. The French payment rule as a function of the realized yearly production is depicted in Figure 1a where the solid line shows the firm's (yearly) revenue depending on how its (yearly) production compares to q_0. Here $R(q_t, q_0)$ takes the form $q_t \cdot z(\frac{q_t}{q_0})$ where $z : \mathbb{R}_+ \to \mathbb{R}_+$ with $z(1) = 1$.

In order to hedge producers against the variation of q_t, we wish to set the correction factor z such that $z(\frac{q_t}{q_0}) \geq 1$ (resp. ≤ 1) if $q_t < q_0$ (resp. $q_t > q_0$), i.e. such that the payment is higher (resp. lower) than what it would have been under a standard linear payment rule for the same price (depicted by the dotted line Figure 1a) when the realized production is lower (resp. higher) than q_0. According to the French payment rule, we have $z(\frac{q_t}{q_0}) > 1$ (< 1) when production q_t is up to 15% below (resp. above) q_0 and $z(\frac{q_t}{q_0}) = 1$ when q_t lies outside the $[0.85q_0, 1.15q_0]$ range. As a consequence, for a given price p, the more often the yearly production lies in the range $[0.85 \cdot q_0, q_0]$, the higher will be the firm's revenue. For that purpose, if the firm knew ex ante that the realized production will be equal to q, or equivalently if there were no production risk, then reporting an expected production q_0 higher than the true expected production q would enhance revenues, as can be seen in Figure 1b which depicts the function $z(\frac{q_t}{q_0})$ as a function of q_0. This function is maximized when the realized production is equal to $0.9q_0$, i.e. when the firm overestimates its production by about 11%. Thanks to this strategic misreporting, the average subsidy per MWh produced would increase by 10% compared to the theoretical FiT p under truthful reporting. This shift corresponds to the difference between the slopes of the dashed and the dotted lines depicted in Figure 1a.

More generally, when production is risky, producers have no incentives to report truthfully their expected production $\mathbb{E}[q_t]$. E.g., a risk-neutral firm wish to report q_0 strategically in order to maximize $\mathbb{E}[z(\frac{q_t}{q_0})]$. If we also assume that the production is symmetrically distributed, then it is intuitive that bidders wish to overestimate q_0 to get more often a favorable correction factor

Figure 1: Payment rule in French offshore wind auction

(a) Payment to the firm (b) Correction factor z(.)



$z(\frac{q_t}{q_0})$ that is larger than 1 (and less often a correction factor below 1). By optimizing their report q_0, firms benefit from the *effective feed-in-tariff* $p \cdot \mathbb{E}[q_t z(\frac{q_t}{q_0})]/\mathbb{E}[q_t]$ which is larger than $p \cdot \mathbb{E}[q_t z(\frac{q_t}{\mathbb{E}[q_t]})]/\mathbb{E}[q_t]$. If q_t is symmetrically distributed, then the latter term is equal to p for the French rule since the function $q \to R(q, q_0)$ is symmetric around q_0 (for any reported q_0 and thus in particular for $\mathbb{E}[q_t]$). Naturally, the *effective feed-in-tariff* is bounded above by $p \cdot \max_x z(x \geq 0)$, the bound when future production is perfectly known ex ante.

To get a first-order approximation of the magnitude of the incentives to misreport expected production, we provide in Appendix 1 a methodology to model the yearly production distribution of a wind farm project, and this from an ex ante perspective. This distribution is built from historical data on local meteorological conditions, on the top of which we add a systematic error term reflecting imperfect knowledge on the wind resource at the ex ante stage. We then apply this methodology to five of the offshore wind farm projects auctioned by the French government.[12]

For three different payment rules and for a given fixed price bid (equal to the one awarded to the winning bidder in the corresponding project), Figure 2 depicts the pdf of the discounted revenue raised over 20 years for the offshore wind farm projects both in Le Tréport and in Saint-Nazaire. The payment rules we consider are: the linear FiT and the French payment rule both under truthful and strategic[13] reporting, i.e. when $q_0 = \mathbb{E}[q_t]$ and when $q_0 = q_0^* \in \operatorname{Arg\,max}_{q>0} \mathbb{E}[z(\frac{q_t}{q})]$,

[12]We leave out one of the sites for which the methodology cannot be applied (see Appendix 1).

[13]Here we consider to simplify that the optimal reported production of reference is the one that would maximize the expected revenue, or equivalently the expected payoff of a risk-neutral firm. More generally, the optimal

respectively. When firms report truthfully their expected production, then the revenue distribution is less spread out under the French rule than under the linear FiT. However, for any given price bid p, firms could benefit from a significant shift upward of their revenue distribution by strategically misreporting their expected production: for the five wind farms used in our simulations, we estimate that the optimal report of risk-neutral firms consists in overestimating their expected production by 11.9 to 12.5% which increases their expected revenue by 3.2 to 3.6% compared to truthful reporting. But by doing so, they also increase the standard deviation of their revenue distribution by 72 to 85% compared to truthful reporting, and which ends up being 10 to 13% larger than the standard deviation under the linear FiT. It is also noteworthy that the revenue distribution becomes quite asymmetric under strategic reporting.

If the contracts are awarded through competitive auctions, then the benefits from reporting an overestimated expected production would be competed away through the competition in the auction if all bidders are strategic. For a given price p under the linear FiT, and if bidders are risk-neutral, let p^s denote the price that yields the same expected revenue under the French rule under strategic reporting. We have then $p^s = p\mathbb{E}[q_t]/\mathbb{E}[R(q_t, q_0^*)]$. After this price rescaling, we find that the variance of $p^s \cdot R(q_t, q_0^*)$ is larger than the variance of $p \cdot q_t$ by 6.6 to 9.3% in the five wind farm projects included in our simulations. In other words, the alleged benefit from the French rule -insurance provision- can be largely offset by strategic reporting and is likely to fail completing its original objective of reducing firms' risk premiums. An in-depth analysis of risk premiums and of the expected equilibrium subsidy is developed in 4.

3 The model

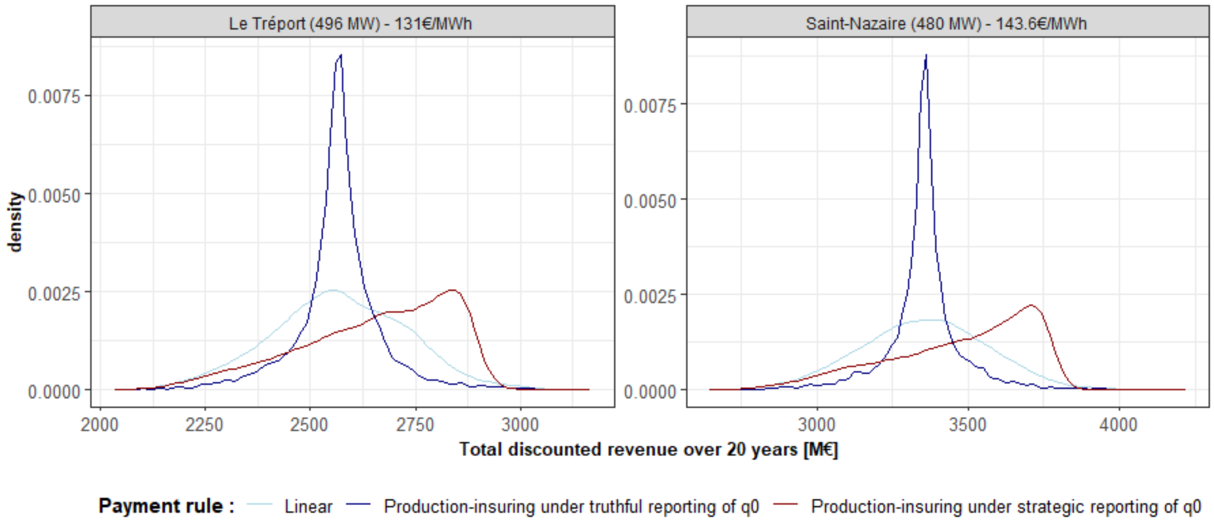
Let us enlarge our horizon beyond the specific intermittent RES-E application to develop a general theory of procurement/auctions for production contracts when the quantity produced ex post is determined by exogenous conditions and when the payment rule have an insurance provisions clause.[14] Namely, we consider the following setup:

Production risk: We assume that the quantity produced q is an exogenous random variable, and in particular that it does not depend on any efforts made ex post by the producer. The variable q is distributed on $\mathbb{R}_+$ according to the pdf f, which is assumed to be symmetric around its mean $\mathbb{E}_f[q] = \bar{q}$, with $\bar{q} > 0$. Formally, $f(\bar{q}+x) = f(\bar{q}-x)$ for any $x \in [0, \bar{q}]$ and the support of f is a subset of $[0, 2\bar{q}]$. Let F denote the corresponding (differentiable) cdf. We also assume that f

(mis)report would depend on bidders' risk aversion as developed later.

[14]In this section, we abstract from the (typical) inter-temporal nature of such contracts where producers are subsidized for their production over multiple years. Qualitatively, this dynamic aspect is rather innocuous from a theoretical perspective. Still, it matters quantitatively once producers are risk-averse, as averaging production over several years reduces the risk faced by producers.

Figure 2: Firm's revenue distributions depending on payment rule and firm's strategy



is single-peaked: formally, for a symmetric distribution f on $\mathbb{R}_+$, being single-peaked is equivalent to being non-decreasing on $[0, \bar{q}]$. Let $\mathcal{F}_{sp}$ denote the set of such symmetric and single-peaked distributions. For $f \in \mathcal{F}_{sp}$, the support of f corresponds then to the interval $[\bar{q}(1-\delta_f), \bar{q}(1+\delta_f)]$ where $\delta_f := \inf\{t \geq 0 | f(\bar{q}(1+t)) = 0\}$. Note that $\delta_f \in]0, 1]$.

The class of contracts: a producer receives from a buyer a payment taking the form $p \cdot R(q, q_0)$ where $p > 0$ is a per unit price, q the quantity produced ex post, and $q_0 \geq 0$ a parameter reported ex ante by the producer. The function $R(\cdot, \cdot)$ is called the payment rule.[15] Contracts are designed such that the buyer expect contractors to report $\bar{q}$ for q_0. Next, q and q_0 are referred to as the ex post production and the expected production reported by the producer, respectively. Throughout our analysis, we assume that the function $q \mapsto R(q, q_0)$ is positive, non-decreasing and continuous[16] with $R(0, q_0) = 0$, for any $q_0 \in \mathbb{R}_+$, and that the function $q \mapsto R(q, q)$ is strictly increasing with $\lim_{q \to +\infty} R(q, q) = +\infty$. Then without loss of generality, we can also make the normalization $R(q_0, q_0) = q_0$.[17] We also assume that $q_0 \mapsto R(q, q_0)$ is differentiable for any $q \geq 0$. Among these contracts, we call linear contracts those for which $R(q, q_0) = q$ for any q_0.[18] We

[15]In other kinds of applications, the variable q could corresponds to a measure for quality, or more generally any kind of uni-dimensional measure characterizing the producer's output.

[16]The assumption that $q \mapsto R(q, q_0)$ is continuous is not mandatory for most of our results and allows us to get rid of some technicalities: without it, some intermediate properties would hold almost everywhere instead of everywhere, which would be sufficient for most of our results.

[17]In general, what is referred to as the ex post production could thus correspond to a non-linear transformation of the quantity produced according to usual production measures.

[18]In our application, a simple FiT with a fixed per unit price is a linear contract.

say that the payment rule is homogeneous of degree one if $R(\lambda \cdot q, \lambda \cdot q_0) = \lambda \cdot R(q, q_0)$ for any $\lambda, q, q_0 \geq 0$.

Producers' payoff: We assume to simplify that the production costs resume to a fixed cost $C > 0$ that is sunk once the contract is signed. We also assume that producers value their profit according to an increasing concave utility function U. Under risk-neutrality, we consider that U is linear. If U is strictly concave then we say that producers are strictly risk-averse. If the producer wins the auction and has signed a contract characterized by the pair (p, q_0), then its expected payoff, denoted by $\Pi(p, q_0)$, is equal to $\mathbb{E}_f[U(p \cdot R(q, q_0))]$. If the producer loses the auction and thus does not sign any contract, its expected payoff is given by $U(C)$.[19]

Strategic behavior: We consider two kinds of producers: truthful (or non-strategic) producers who report $\bar{q}$ for q_0 and strategic producers who report a quantity q_0 belonging to the set $\operatorname{Arg\,max}_{q_0 \in \mathbb{R}_+} \Pi(p, q_0)$ given the price bid p.[20] In other words, for a given per unit price p, strategic producers face the menu of contracts $\{p \cdot R(q, q_0)\}_{q_0 \in \mathbb{R}_+}$ among which they pick the contract they prefer. Then, for a given distribution f, a given utility function U and a given contract price $p > 0$, we say that a payment rule is strategy-proof (resp. manipulable) if the producer does not benefit (resp. does strictly benefit) from misreporting its expected production, i.e., formally, if $\bar{q}$ belongs (resp. does not belong) to $\operatorname{Arg\,max}_{q_0 \geq 0} \mathbb{E}_f[U(p \cdot R(q, q_0)))]$. The linear contract is strategy-proof since the producer's payoff does not depend on q_0.

The buyer's payoff: We assume that the buyer is risk-neutral, and wish to minimize the expected transfer to the producer per quantity produced $p \cdot \mathbb{E}_f[R(q, q_0)]/\bar{q}$, which is next referred to as the buyer's expected cost (BEC) and taken as our criterium to evaluate the performance of different classes of contracts.

The auction rule: We consider the first-price auction where each producer submits a bid pair (p, q_0) and where the buyer selects the offer involving the lowest price bid p. Note that this rule selects the offer that minimizes the BEC when bidders reports truthfully their expected production $\bar{q}$ for q_0.

We are interested in payment rules that provide insurance against production variability compared to the linear contract. The latter appears as a natural benchmark since it is both commonly used and strategy proof.

Definition 1. *A payment rule $R(q, q_0)$ is production-insuring if for any $f \in \mathcal{F}_{sp}$, any increasing concave function U and any contract price $p > 0$,*

$$\mathbb{E}_f[U(p \cdot R(q, \bar{q}))] \geq \mathbb{E}_f[U(p \cdot q)] \tag{1}$$

[19] An alternative specification would consist in letting the winning producer's payoff be $\mathbb{E}_f[U(p \cdot R(q, q_0) - C)]$ and the losing producer's payoff be $U(0)$. Actually, such a specification would be equivalent to ours, thanks to a re-normalization of U that would not modify the concavity or the strict concavity properties.

[20] We assume implicitly that the class of contracts is such that this set is well defined for any price p.

and where the inequality is strict (resp. stands as an equality) if the producer is strictly risk-averse (resp. risk-neutral).

Eq. (1) reflects that, for a given price p, risk-averse producers should be better off with a production-insuring rule compared to the linear contract. We also wish the inequality to stand as an equality when producers are risk-neutral, or equivalently that the expected payment remains the same in both contracts to make them comparable. Formally, this means that $\mathbb{E}_f[R(q,q_0)] = \bar{q}$.

Comments: 1) In practice, the producer may be able to influence the quantity produced in some respect, in particular to reduce production ex post at no cost. Our assumption that $q \mapsto R(q,q_0)$ is non-decreasing guarantees that producers do not wish to intervene in this way.

2) In many environments, we would like to consider the production cost function $C + c(q)$, where $c : \mathbb{R}_+ \to \mathbb{R}_+$ is an increasing function with $c(0) = 0$. If the production cost depend linearly on the quantity ($c(q) = \bar{c}, \forall q$), then the analysis presented hereafter can apply by replacing the unit price p by $(p - \bar{c})$ in the payoff of the firm.[21]

3) A closely related alternative model would consist in considering that the quantity q_0 is fixed ex ante by the auction rules and that producers are able to downgrade (or upgrade) their production technology ex post. The pdf f_e and the associated mean production $\bar{q}_e$ would now depend on a costly effort e. Truthful producers would be those that set their effort e such that $\bar{q}_e = q_0$. By contrast, a strategic producer could increase its payoff by choosing the optimal effort, in particular by shirking such that $\bar{q}_e < q_0$ if the payment rule is production-insuring. Such a model would stand in line with the literature on the trade-off between moral hazard and insurance (Shavell, 1979).

3.1 Strategic misreporting in production-insuring payment rules

Before deriving the equilibrium in the auction, let us analyze bidders' incentives to misreport their expected production when the payment rule is production-insuring.

For any payment rule and any pair $q, q_0 > 0$, we can express the term $R(q,q_0)$ as $q \cdot z_{q_0}(\frac{q}{q_0})$ where the function $z_{q_0} : \mathbb{R}_+ \to \mathbb{R}_+$ can be viewed as a correction factor with $z_{q_0}(1) = 1$.[22] The following lemma establishes that a production-insuring payment rule would never deflate (resp. inflate) payments (compared to the linear contract) for production occurrences that are lower (resp. higher) than the reported expected production, or equivalently that the correction factor is larger (resp. smaller) than one when production is lower (resp. higher) than the reported expected

[21] In this case, there is no need for the buyer to know $\bar{c}$. For general cost functions $c(\cdot)$, our analysis would apply if the buyer knows the function c and is able to replace the payment rule $pR(q,q_0)$ (which is linear in p) by $pR(q,q_0) + c(q)$.

[22] Note that the payment rule is homogeneous of degree one if the function $z_{q_0}(\cdot)$ does not depend on q_0 and if $R(q,0) \equiv 0$.

production. Furthermore, the fact that these correction factors should compensate in expectation for any symmetric risk imposes a one to one relation between $z_{q_0}(1+\epsilon)$ and $z_{q_0}(1-\epsilon)$.

Lemma 1. *A payment rule is production-insuring if and only if we have for any* $q_0 > 0$*,* $z_{q_0}(1+\epsilon) \leq 1$*,* $z_{q_0}(1-\epsilon) \geq 1$ *and* $(1+\epsilon)\cdot z_{q_0}(1+\epsilon)+(1-\epsilon)\cdot z_{q_0}(1-\epsilon) = 2$ *for any* $\epsilon \in [0,1]$*, and* $\int_0^\epsilon z_{q_0}(1+t)dt < \epsilon$ *if* $\epsilon \in (0,1]$.

According to the French payment rule, the function $(1+\epsilon)z_{q_0}(1+\epsilon)$ can be written as $1+\epsilon+d(\epsilon)$ where the function $d : \mathbb{R} \to \mathbb{R}$ is a continuous odd function with $d(\epsilon) = 0$ if $|\epsilon| \geq 0.15$ and $d(\epsilon) < 0$ if $\epsilon \in (0, 0.15)$. We get thus from Lemma 1 that the French payment rule is production-insuring.

As a corollary of Lemma 1, we get that if there is no risk on production, overestimating (resp. underestimating) future production can only be beneficial (resp. detrimental) to the producer. Furthermore, the producer would also strictly gain from overestimating a bit production since the correction factor $z_{q_0}(x)$ is strictly larger than 1 for some values in the left neighborhood of 1. Next we generalize this insight for $f \in \mathcal{F}_{sp}$ and when producers are risk-neutral.

Proposition 2. *For any* $f \in \mathcal{F}_{sp}$ *and any contract price* $p > 0$*, any production-insuring payment rule is manipulable if the producer is risk-neutral. Furthermore, the producer weakly increases (resp. decreases) its expected payoff by overestimating (underestimating) its expected production.*

Proposition 2 states that, at least when producers are risk-neutral, any production-insuring payment rule is manipulable not only for a special distribution f, but for any $f \in \mathcal{F}_{sp}$. In other words, the production-insuring and strategy-proof properties are incompatible. No such result is derived when allowing for strict risk-aversion of the bidder, but Proposition 3 formalizes that in such case it is at least impossible to fully insure strategic producers against the production risk.

Proposition 3. *Consider a payment rule that is homogeneous of degree one and a contract price* $p > 0$*. For any* $f \in \mathcal{F}_{sp}$*, if producers (mis)report optimally their expected production, then they are not fully insured against production risk.*

Formally, if $q_0 \in \operatorname{Arg\,max}_{q \in \mathcal{R}_+} \Pi(p, q)$, then the variance of $p \cdot R(q, q_0)$ is strictly positive, which means that the producer's revenue is risky. The proof of Proposition 3 shows that starting from a quantity reported q_0 such that the producer is fully insured, then the producer would strictly benefit from reporting a slightly higher q_0.

The restriction to payment rules that are homogeneous of degree one is imposed here to avoid payment rules that are tailored specifically to the distribution f and that would thus fail to be robust.[23] An alternative interpretation of Proposition 3 is that it is impossible to insure

[23]Without the homogeneous of degree one restriction, there is an obvious strategy-proof payment rule for any $f \in \mathcal{F}_{sp}$ with the support $[(1-\epsilon)\bar{q}, (1+\epsilon)\bar{q}]$ (with $\epsilon \in (0,1)$): it is sufficient to specify the payment rule such that $R(q, q_0) = q$ if $q_0 \neq \bar{q}$ and $R(q, \bar{q}) = \bar{q}$ for any $q \in [(1-\epsilon)\bar{q}, (1+\epsilon)\bar{q}]$. To be able to implement such a solution, the contract designer should know $\bar{q}$.

strategic producers against production risk if the contract designer does not know the production distribution up to an homothetic transformation. Proposition 3 does not claim that it is impossible to reduce the risk faced by the producer compared to the linear contract, a question that we address through numerical simulations: it rather formalizes that there is no hope to fully annihilate the risk faced by the producers.

3.2 Auction prices

For a given payment rule $R(.,.)$ that is assumed to be either production-insuring or linear, we develop the equilibrium analysis of the auction game under several paradigms regarding whether producers are truthful or strategic: we characterize the bid pairs (p, q_0) submitted by the producers in equilibrium and then derive the corresponding risk premium, which is defined as the ratio of the expected cost for the buyer and the producing cost. In a first step, we consider that producers are either all truthful or all strategic. In a second step, we consider heterogeneous behavior in the auction: first, the case where a single producer is strategic (while all others are truthful), and second the case where each producer (independently of the others) is strategic with a given probability α. Throughout our analysis, we assume that $f \in \mathcal{F}_{sp}$. Nevertheless, our various equilibrium characterizations are valid more generally, in particular with asymmetric distributions.

Homogeneous producers

If producers are identical, then Bertrand competition leads to zero profit. Nevertheless, the buyer's expected cost depends on the payment rule and the presence of strategic bidding, as both result in different levels of insurance provision, and therefore different risk premiums.

First, we consider producers are non-strategic. Rationality imposes that winning the auction should raise a higher expected profit than losing the auction, while having competition leading to zero profit makes this constraint binding. Then the equilibrium price, denoted p^{NS}, is the unique solution of:

$$\mathbb{E}_f[U(p^{NS} \cdot R(q, \bar{q}))] = U(C) \tag{2}$$

When producers are risk-neutral, the production-insuring payment rule does not induce changes in the auction's outcome compared to the linear contract: in both cases, $p^{NS} = \frac{C}{\bar{q}}$ and the buyer's expected cost is equal to C. This equivalence, however, does not hold with risk-averse producers.

Proposition 4. *When all producers bid truthfully, the equilibrium price and the buyer's expected cost are smaller under a production-insuring payment rule than under the linear contract. They are strictly smaller if producers are strictly risk-averse, and equal if producers are risk-neutral.*

Since U is concave, we obtain from equation (2) and Jensen's inequality that $p \cdot \mathbb{E}_f[R(q, q_0))] \geq C$, or equivalently that the buyer's expected cost is necessarily larger than C. In the special case where the payment rule fully insures the producer (i.e. when payment remains identical for any possible value taken by q), then the auction's outcome is the same as in the risk-neutral case for any utility function U ($p^{NS} = \frac{C}{\bar{q}}$, and the cost for the buyer is C).

In such equilibrium, the difference between the buyer's expected cost and C corresponds to a risk premium. As formalized in Proposition 4, production-insuring payment rules reduce this risk premium. However, the fact that production-insuring payment rules outperform the linear contract relies crucially on the assumption that producers bid truthfully.

If bidders are strategic, then the equilibrium price, denoted by p^S, is characterized[24] as the unique solution of

$$\mathbb{E}_f[U(p^S \cdot R(q, q^S))] = U(C) \quad \text{with} \quad q^S \in \operatorname{Arg}\max_{q_0 \geq 0} \mathbb{E}_f[U(p^S \cdot R(q, q^S))] \tag{3}$$

As previously, when firms are risk-neutral the expected cost for the buyer $\mathbb{E}_f[p^S \cdot R(q, q^S)] = C$. However the equilibrium price differs : $p^S = \frac{C}{\mathbb{E}_f[R(q, q^S)]}$. The definition of q^S implies $\mathbb{E}_f[R(q, q^S)] \geq \bar{q}$, which in turn implies $p^S \geq p^{NS}$ (with strict inequalities if the payment rule is manipulable).

Proposition 5 generalizes this result to the case of risk-averse firms. However, a lower equilibrium price does not imply a lower cost for the buyer: it remains identical in the risk-neutral case while we most often expect it to be inflated in the risk-averse case, as strategic reporting may induce producers to take more risks and thus increase the risk premium. Proposition 5 points out a case where such a phenomenon is known to happen.

Proposition 5. *The equilibrium price (p^S) is smaller when all producers bid strategically than the equilibrium price when all producers bid truthfully (p^{NS}).*[25] *If the payment rule provides a full insurance against production risk under truthful reporting of q_0, is homogeneous of degree 1 and if producers are strictly risk-averse, then the buyer's expected cost is greater under strategic bidding than under truthful bidding.*

Example 1 in Appendix 2 exhibits a case where the buyer's expected cost under strategic producers is lower than under truthful producers. In this example, the insurance provided by the payment rule is vanishing under truthful reporting while the payment rule is flat further away from the production expected value and thus provides insurance when producers are misreporting their expected production.

The lesson from the propositions 4 and 5 is that from the buyer's perspective, production-insuring rules outperform the linear contract under truthful reporting, but that strategic behavior

[24]See the proof of Proposition 5 in Appendix 2 for details on this characterization.
[25]The inequality is strict if the payment rule is manipulable.

may reap out all those benefits. Which effect dominates and their order of magnitude is an empirical question that is investigated in our simulations. Last, under risk-neutrality all those effects have no impact on the buyer.

Nevertheless, we argue next that heterogeneity among bidders regarding strategic behavior threatens considerably the use of production-insuring rules and still have a sizable effect even if producers are risk-neutral.

Heterogeneous producers

Suppose first that a single producer bids strategically, knowing all other producers bid truthfully. In order to avoid equilibrium existence problems, that are well-known in continuous games (Simon and Zame, 1990), we assume that ties are always broken in favor of the strategic producer. Then the equilibrium takes the following form: truthful producers bid $(p^{NS}, \bar{q})$ exactly as in the equilibrium where all producers are truthful, while the strategic producer bids (p^{NS}, q_0^*) where $q_0^* \in \operatorname{Arg\,max}_{q_0 \geq 0} \Pi(p^{NS}, q_0)$. Next we let $\Delta := \Pi(p^{NS}, q_0^*) - \Pi(p^{NS}, \bar{q}) = \mathbb{E}_f[U(p^{NS} \cdot R(q, q_0^*))] - U(C) \geq 0$. Δ represents the payoff reaped out by the strategic producer from misreporting the expected production q_0. The buyer's expected cost is then equal to $p^{NS} \cdot \mathbb{E}_f[R(q, q_0^*)]$.[26]

If producers are risk-neutral, then the percentage increase of the buyer's expected cost is equal to $\frac{\mathbb{E}_f[R(q,q_0^*)]}{\bar{q}} - 1$.[27] This increase is bounded above by $(\sup_{q,q_0}\{z_{q_0}(q/q_0)\} - 1) \cdot C$, which could be reached only for a deterministic q.[28] In the French rule, we have $\max_{q_0} z_{q_0}(q/q_0) = \frac{1}{0.9}$ and the buyer's expected cost increase due to such strategic misreporting cannot exceed 12%. [29]

Let us now consider the case where there are $N \geq 2$ producers, each independently strategic with probability α, α being common knowledge. Truthful producers are forced to report $\bar{q}$, or equivalently do not have access to the technology or knowledge that enables to misreport the expected production. Then they should always bid p^{NS}, whether they are or are not aware other bidders may bid strategically. According to this framing, our analysis is analogous to the equilibrium analysis developed by Maskin and Riley (1985) of first price auctions with two (possibly risk-averse) symmetric bidders having binary valuations: being strategic in our setup corresponds to having a high valuation in Maskin and Riley (1985).[30] There are nevertheless three

[26] As long as the payment rule is manipulable, $\mathbb{E}_f[R(q, q_0^*)] > \mathbb{E}_f[R(q, \bar{q})]$, and then the buyer's expected cost would have been strictly lower if a non-strategic producer were selected.

[27] From Proposition 2, this term is strictly positive and represents thus an increase if the payment-rule is production-insuring.

[28] Provided the existence of $\sup_{q,q_0}\{z_{q_0}(q/q_0)$.

[29] As illustrated with Example 1 in Appendix 2, the buyer's expected cost is not guaranteed to increase with a strategic producer under general production-insuring payment rules: the producer may misreport q_0 (by underestimating production) in a way that reduces $E_f[R(q, q_0] < \bar{q}$ and this in order to improve its insurance against the production risk. We conjecture that such effects have very poor empirical relevance.

[30] Maskin and Riley (1985) do also consider correlated valuations which we do not. Doni and Menicucci (2012) extend the analysis to two asymmetric bidders when bidders are assumed to be risk-neutral.

differences: 1) We consider any number of bidders, 2) Under risk aversion, the payoff of a bidder is no longer linear in the bid price p insofar as the optimal report for the expected production could now depend on p,[31] 3) Our analysis is framed into a procurement setup. In our framework, being non-strategic (resp. strategic) corresponds to having a low (resp. high) valuation in Maskin and Riley (1985).

The equilibrium takes the following form (see details in the Proof of Proposition 6):

- Non-strategic producers bid the price-quantity pair $(p^{NS}, \bar{q})$,
- Strategic producers bid price-quantity pairs $(p, q_0^*(p))$ where p is distributed according to the distribution $G(p) = 1 - \frac{1-\alpha}{\alpha}\left(\sqrt[N-1]{\frac{\Pi(p^{NS}, q_0^*(p^{NS})) - U(C)}{\Pi(p, q_0^*(p)) - U(C)}} - 1\right)$ and where $q_0^*(p) \in \operatorname{Arg\,max}_{q \geq 0} \Pi(p, q)$.

In equilibrium, we have that the expected payoff of a non-strategic (resp. strategic) producer is null (resp. is equal to $(1-\alpha)^{N-1}\Delta$). Next proposition converts those rents into increased buyer's expected costs.

Proposition 6. *Suppose that producers are strategic with probability α, independently of each other.*

If producers are risk-averse (resp. risk-neutral), then the buyer's expected cost is larger than (resp. equal to) the sum of the buyer's expected cost with truthful producers and the term:

$$N\alpha(1-\alpha)^{N-1} \frac{[\Pi(p^{NS}) - U(C)]}{U'(p^{NS} \cdot \mathbb{E}_f[R(q, \bar{q})])} > 0.$$

When producers are risk-neutral, the buyer's expected cost is equal to C plus a term that vanishes in the two polar cases where α is equal to 0 or 1 (as covered by Propositions 4 and 5). When $N = 2$, the extra cost resulting from such "miscoordinated" heterogeneity is less than the half of the extra cost when one producer is strategic and the other is truthful. This bound is reached for the worst case when $\alpha = 0.5$. More generally, for any $N \geq 2$, the extra cost can be as large as 36% of the extra cost when one producer is strategic and the other is truthful.[32]

The lesson from Proposition 6 is that the rents captured by the producers are smaller with such miscoordinated heterogeneity, but still could have a sizable effect of the same order of magnitude. In our simulations we consider the case with a single strategic producer, while having in mind that the rents of the producers or equivalently the buyer's expected cost should be mitigated.

[31] Revenue equivalence between first-price and second-price auctions holds in Maskin and Riley (1985) when the binary valuations are drawn independently, and then analogously in our environment when the optimal report for the expected production does not depend on the price bid p. This later property is always satisfied under constant relative risk aversion (Appendix 3).

[32] The worst case is when $\alpha = 1/N$ and the bound comes from the fact that $(1 - 1/N)^{N-1} > \exp(-1)$ which results from a standard logarithm inequality.

4 Performance analysis of the French rule

As described in Section 2, the French government has used a production-insuring payment rule in the auctions for six offshore wind farm sites. These contracts were awarded separately through first-price sealed bid auctions: the firm asking for the lowest per unit subsidy p was declared the winning bidder and was subsidized according to this price.[33]

A first mild difference with our theoretical analysis is that we now explicitly consider multi-year contracts: the length is 20 years, during which the production-insuring payment rule $R(.,.)$ defined in Section 2 applies for each year separately, based on the expected yearly production q_0 reported freely by producers in their bid.[34] A second difference is that we consider both a (fixed) investment cost IC occurring before production (which corresponds to C is our model), and (fixed) operating costs OC occurring each year. The values we use for our analysis are reported in Appendix 1. For a given bid (p, q_0), the producer's expected payoff difference between winning and losing the auction can then be expressed as:

$$\Pi(p, q_0) = \mathbb{E}\left[U(\sum_{t=1}^{20} \frac{[p \cdot R(q_t, q_0) - OC]}{(1+r)^t})\right] - U(IC) \tag{4}$$

where the expectation is made w.r.t to the vector of yearly production $(q_1, \cdots, q_{20})$ and where r denotes producers' interest rate which is set equal to 5.7%.[35] Producers' risk aversion is captured through the CRRA utility function, i.e. $U(x) = \frac{x^{1-\gamma}}{1-\gamma}$, where the parameter $\gamma \geq 0$ is the relative risk aversion coefficient. When the payment rule is homogeneous of degree one, as is the French rule, these utility functions induce convenient properties detailed in Appendix 6. Through the specification of equation (4) we assume the initial wealth of the firm is equal to IC.[36]

[33] These auctions were actually *scoring* auctions: in addition to the per unit subsidy bid p, other criteria such as local environmental impact or carbon footprint were taken into account to determine the winning bid. We abstract from such "multidimensional bidding" aspects given that they do not interfere with the production-insuring payment rule. In contrast to France, most countries use auctions that are only price-based once projects are declared to be eligible to bid through a pre-qualification phase.

[34] From a practical perspective, the report of q_0 corresponds to the report of the expected capacity factor. Naturally, unrealistic capacity factors would lead to disqualification (even if there were no formal rule about this). Our analysis implicitly assume that the optimal report of q_0 is not unrealistic.

[35] Our choice is a choice based on an estimation of the cost of capital for onshore wind projects in France made by Angelopoulos et al. (2016) which accounts for taxation and for compensation for other kinds of risks. Note that our analysis abstract for many kinds of risks, including cost overruns or delays that are not entirely under the control of the producers (e.g. connection to the grid). Those risks could induce much larger risk premiums but they are orthogonal to the design of the payment rule.

[36] Considering different level of initial wealth induce small variations in the risk premium. For instance, considering the total net present cost (including the discounted cumulative operating costs) instead of the sole investment cost as initial wealth induces risk premiums about 25-40% lower for $\gamma \in [1; 3]$. However, it as very little impact on the outcome in the presence of asymmetric bidder. Additional results illustrating this are presented in Appendix 4.

We calibrate the distribution of the vector of yearly production $(q_1, \cdots, q_{20})$ based on historic production simulated by models developed by Staffell and Pfenninger (2016) and whose outputs are easily accessible through the site `www.renewables.ninja`. The calibration procedure, detailed in Appendix 1, consider a wide range of possible yearly production based on recombinations of quarterly production values randomly drawn in historic data, to which a random variable is added to account for likely ex ante misevaluation of each site's wind resource.

We then compute equilibrium bids under 3 paradigms analyzed in Section 3.2 : when firms all bid truthfully, when firms all bid strategically, and when one firm bids strategically while knowing other firms bid truthfully.[37]. We then evaluate the payment rule's performance regarding the resulting buyer's expected cost, while considering the linear contract as a benchmark. The buyer's expected cost depending on the winning firm's bid (p, q_0) is given by[38] $BEC(p, q_0) = p \cdot \frac{\mathbb{E}[R(q_t, q_0)]}{\mathbb{E}[q_t]}$. In equilibrium, it takes as a lower bound the actual production cost per expected quantity produced (i.e. $\frac{C}{\bar{q}}$), and takes higher values depending on either or both of a risk premium (when producers are risk-averse) and a positive noncompetitive rent captured by a strategic winning bidder (when producers are heterogeneous).

It is first noteworthy that for the 5 wind farm sites considered, the risk premium under a linear contract are rather small: for $\gamma = 1$ they are comprised between $0.3 - 0.4\%$, and they only reach $0.9 - 1.1\%$ for $\gamma = 3$.[39] When all producers are truthful, the BEC is as expected lower for the French rule than for the linear payment rule (Proposition 4). However, the gain from such a production-insuring rule happens to be very small in practice: a decrease by $0.2 - 0.3\%$ when $\gamma = 1$, and by $0.6 - 0.7\%$ when $\gamma = 3$. Furthermore, these (limited) gains are entirely lost when firms bid strategically and this for any reasonable level of risk aversion, as shown in Figure 3: only for unrealistic degree of risk aversion ($\gamma > 5$) do we find that the French payment rule slightly outperforms the linear contract under strategic reporting.[40]

As shown in section 3.2, heterogeneity among producers induces noncompetitive rents that inflate the buyer's expected cost. Our simulations support that such rents are of a larger order of magnitude than the risk premium reduction that the buyer could save if producers were truthful: with a single strategic producer, we find a BEC $3.3 - 3.6\%$ larger than under a linear contract when firms are risk-neutral, and $2.6 - 2.9\%$ larger when firm's risk aversion is up to $\gamma = 3$. For any risk aversion level in between, the increase in BEC when a single firm is strategic is more than four times larger than the cost reduction thanks to insurance provision when all firms bid

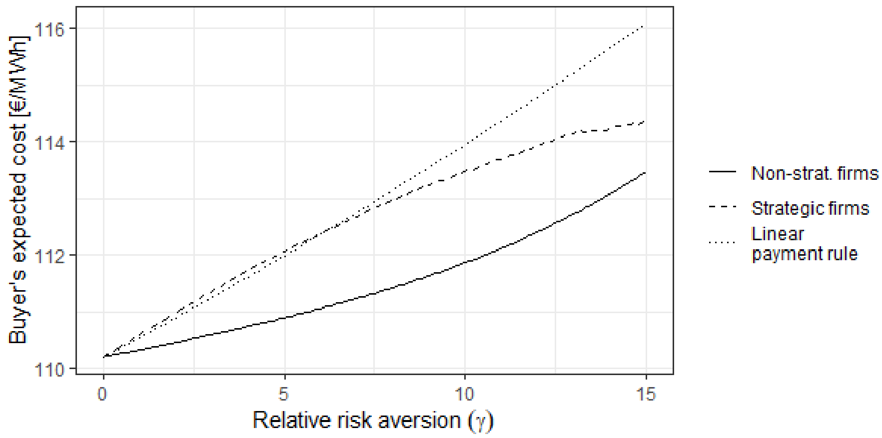
[37] The latter paradigms should be seen as a worst case of the heterogeneous paradigms presented in section 3.2.

[38] To get this simple expression, we use here the assumption that q_t is drawn independently across years. Note also that $BEC(p, \bar{q}) = p$ for any production-insuring rule R.

[39] All ranges presented here are the smallest ranges comprising results obtained for all five sites simulated. Detailed results for each site are presented in Appendix **??**.

[40] Using data on labor supply behavior, Chetty (2006) argues that the coefficient γ is bounded by 2.

Figure 3: Buyer's expected cost with homogeneous producers



truthfully. For a relative risk aversion $\gamma = 1$, this increase is more than 15 times larger.

5 Alternative payment rules that provide insurance

If all producers are strategic, our simulations show that for any reasonable level of risk aversion the production-insuring payment rule chosen by the French government does not outperform the linear contract. Furthermore, under such payment rules bidders' asymmetry regarding the adoption of strategic behavior induce a loss for the buyer much larger than the gain obtained in the best case scenario, when all firms are truthful. These results suggest that auctioneers should give up production-insuring contracts in the face of strategic misreporting of expected production. In an attempt to find a better performing class of contract, possibly by discouraging such misreporting of expected production, we propose the following class of homogeneous of degree 1 payment rules $R_{(w,\eta)}$ parameterized by the pair of coefficients $(w, \eta) \in [0, 1]^2$ and defined in the following way:

- $R_{(w,\eta)}(q, q_0) = q_0$ if $q \in [q_0(1-w), q_0(1+w)]$,
- $R_{(w,\eta)}(q, q_0) = (1-\eta)q + \eta q_0(1+w)$ if $q > q_0(1+w)$
- $R_{(w,\eta)}(q, q_0) = \max\{\frac{1}{1-\eta}q + (1 - \frac{1}{1-\eta})q_0(1-w), 0\}$ if $q < q_0(1+w)$.[41]

The parameters w and η capture respectively the width of a range around q_0 where producers are fully insured and the strength of the punishment when the realized production lies outside this range. If $\eta = 0$, then the payment rule matches the linear payment rule outside the insured range and the payment rule $R_{(w,\eta)}$ is production-insuring. On the contrary, when $\eta > 0$ then

[41]If $\eta = 1$, then we adopt the convention that $R_{(w,\eta)}(q, q_0) = 0$ if $q < q_0(1+w)$.

payment to the firm decreases more rapidly (resp. increases more slowly) when production falls below (resp. goes above) the insured range.[42] Therefore misreporting its expected production may come at the price of an increased risk of ex post production falling outside the insured range, which would be punished by a per unit payment lower than p.

This specification is closely related to the payment rule adopted by Brazil in 2013 in order to punish departures from the contracted production. If the realized spot market price stands below the (auction-determined) contract price p, the contractor should pay $1.06 \cdot p$ for each quantity that he/she fails to deliver at the end of the contract, which corresponds to $\eta \approx 5.7\%$. In the case of overproduction, the contractor sells the surplus on the spot market.[43] The rationale behind such punishments is to provide incentives to producers to stick to their initial production plan.

As before, we study this new class of payment rules through simulations. Throughout this section, we consider a single year contract[44] and a CRRA utility function with $\gamma = 0.9$. The production distributions considered are, first, a normal distribution where the standard deviation is equal to 20% of the mean (Figure 4) and, second, a uniform distribution on the interval $[0.5\bar{q}, 1.5\bar{q}]$[45] (Figure 5). Since U is a CRRA and the payment rules considered are homogeneous of degree one, the simulations' outcome are strictly proportional to the production cost C and the mean of the production distribution $\bar{q}$ (Lemma 7 in Appendix 3) which we both normalize to 1. In the Figures 4 and 5 we report our results for the parameters (w, η) varying over the square $[0, 0.5]^2$.

In Figures 4 and 5, the three panels (a), (b), and (c) depict the buyer's expected cost, respectively when all firms are truthful, when all firms are strategic, and last when only one firm is strategic. All these values lie above $C = 1$ as the buyer's expected cost includes a risk premium in both first cases and both a risk premium and a rent captured by the winning firm in the third case. Panel (d) depicts the reported expected production when firms are strategic q_0^*.[46]

With truthful bidders, the impact of both parameters on the BEC through the risk premium is straightforward: the larger is the insurance range and the less punished firms are, the lower is the risk premium, as shown in panels (a) in both Figures 4 and 5. It is not as straightforward in the presence of strategic bidders. Panels (d) in both Figures 4 and 5 shows that, overall,

[42]Note that $R_{(w,\eta)}$ is not production-insuring when $\eta > 0$, since $\mathbb{E}_f[R(q, \bar{q})] < \bar{q}$ if the support of f is not a subset of $[q_0(1 - w), q_0(1 + w)]$ which stands in contraction with (1).

[43]Note that the Brazilian rule does not depend solely on the auction price but also on the electricity market price. It is indeed slightly more intricate since both under and over-production can be compensated (partly) across years.

[44]Unlike previous simulations in section 4 and as in the theoretical section, costs are to be covered on a single realization of production and not over several years.

[45]Then the standard deviation is equal to $\sqrt{\frac{1}{12}} \approx 29$ % of the mean.

[46]Since the chosen utility function is a CRRA, the optimal quantity reported by a strategic firm is independent of the price p (as detailed in Appendix 3). Therefore the values presented in panel (d) correspond to the equilibrium bid of the winning firm both when all firms are strategic and when only one is strategic.

Figure 4: Auction outcome depending on payment rule for a normally distributed production

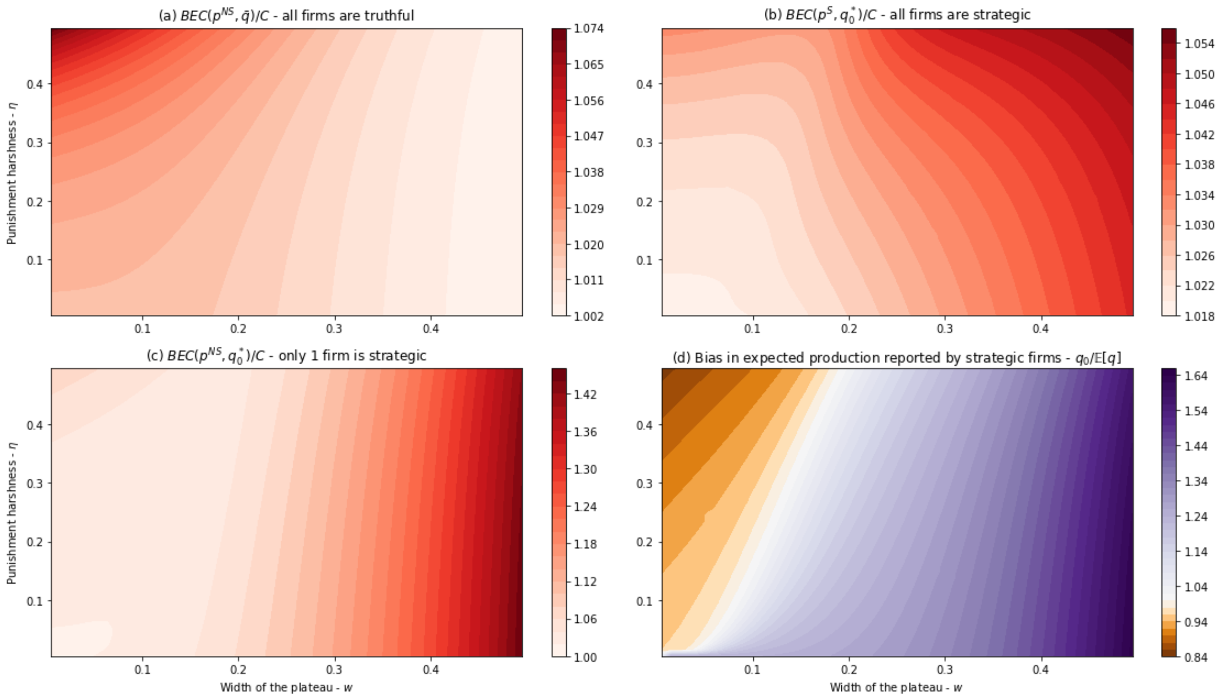
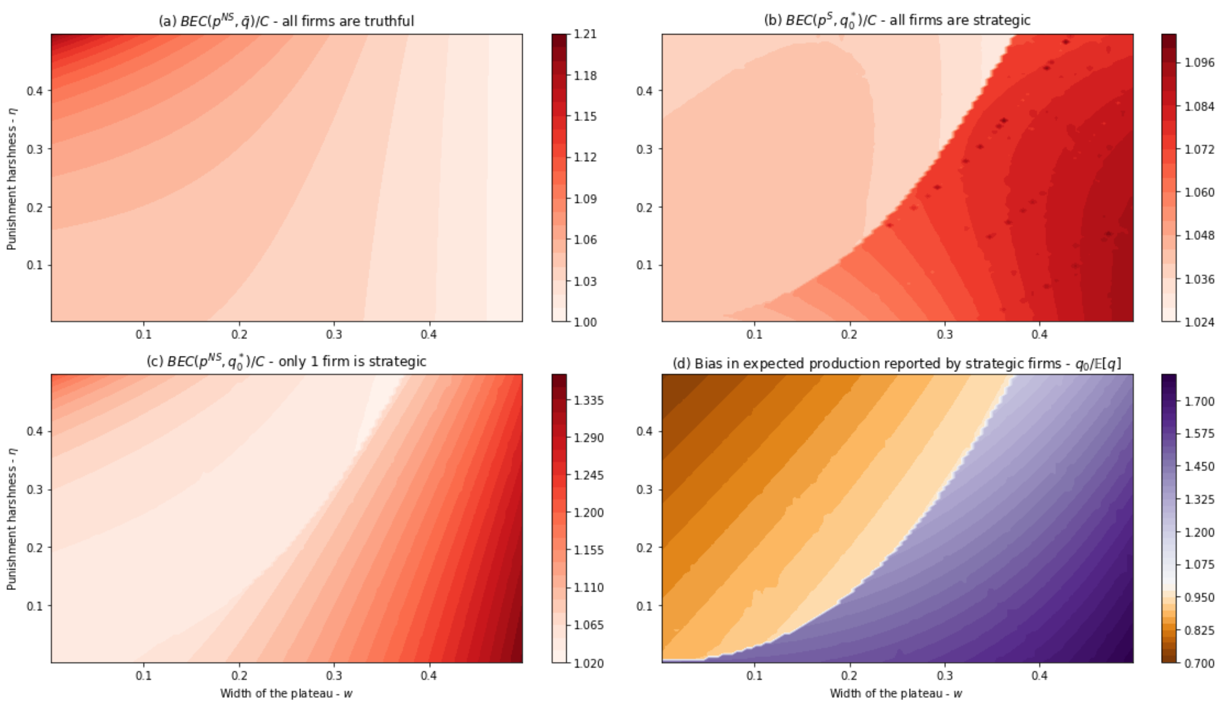


Figure 5: Auction outcome depending on payment rule for a uniformly distributed production



the larger is the insurance range w the more firms overstate their expected production, in an attempt to maximize their expected benefit from the insurance mechanism. On the other hand, harsher punishments η induce firms to understate their expected production in an attempt to avoid realized production falling below the lower bound of the insurance range (which would be punished by rapidly decreasing revenue). A striking difference between both figures is the discontinuity regarding the optimal q_0^* depending on the payment rules parameters, which appears only for the uniform distribution (i.e. in Figure 5). This discontinuity is due to the existence of two local maximums, each moving in different directions with w and η. The first local maximum (dominating for small w and large η) reflects a strategy which consists in insuring oneself against low production realizations (punished by rapidly decreasing revenue) by declaring q_0^* such that a limited part of realization falls below (and not too far below) the lower bound of the plateau. The second strategy (local maximum), which dominates for large w and small η, consists in overstating expected production so that the lower part of the plateau inflates the expected revenue while the upper part of the plateau does not deflate it as much. The global maximum switches from the first to the second when crossing the discontinuity, as w grows or η decreases (see Figure 6). Under the normal distribution case, we note a set of parameters (w, η) leading to truthful reporting (i.e. a set of strategy-proof payment rules) which corresponds roughly to the line where $\eta = 0.2 \cdot w$ in Figure 4(d). However, as a consequence of the discontinuity in q_0^*, under the uniform distribution case such a set does not exists (Figure 5(d)).

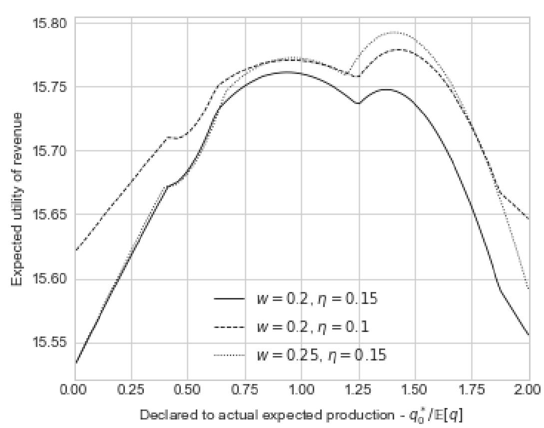


Figure 6: Bias in expected production maximizing expected utility of revenue under uniform production distribution

Under the normal distribution and when all firms are strategic, we see in Figure 4(b) that a higher w or a higher η are everywhere associated with a higher BEC, resulting from a higher risk faced by the producer. Then, the lowest BEC is obtained through the linear contract $(w, \eta = 0)$.

We note however that the pace at which the buyer's expected cost increases is the slowest when moving along the set of strategy-proof payment rules. Moreover, if a strategic firm is facing truthful firms in the auction the strategy-proof set of payment rules is associated with the lowest values of the buyer's expected cost (Figure 4(c)). Intuitively, the further is the payment rule from the strategy-proof set the larger is the rent the strategic firm will be able to capture (due to its ability to adjust its reported q_0). But among strategy-proof payment rules, where the rent of the strategic firm is null, the lowest BEC is obtained when the producer faces the lowest risk, which is, as seen before on panel (b), under the linear contract. Overall, in the presence of strategic bidders, no payment rule within the set considered seem preferable to the linear contract: all increase the risk premium and are equally or more exposed to the risk that a strategic firm may capture a large rent. Even though punishments mitigate firms' incentive to take more risk by overstating their expected production, the additional risk punishments represent by themselves exceeds this gain.

The interpretation is more complex in the case of the uniform distribution as such a set of strategy-proof payment rule does not exist, and the space of payment rules is divided in two regions where two different strategies dominate. Figure 5(b) suggests the strategy consisting in insuring oneself against low realizations of q (which dominates for low w and high η) is the least risky: any payment rule (w, η) inciting firms to chose this strategy appear to induce less risk than any payment rule inducing the choice of the second strategy (as they result in lower BEC). In addition, as long as the payment rule ensures this strategy remains preferable, increasing the width of the plateau or the harshness of punishments (within $[.5, .5]^2$) lowers the BEC.

Two paths appear to bring more rapidly decreasing risk premiums. The first one is when only punishments (η) grows while w remains small (or null): then firms understate their expected production so that production is most likely to fall above the plateau, where revenue is getting flatter as punishment get harsher. For very high η, firms benefit this way from nearly full insurance. However, payment rules imposing a narrow plateau and strong punishments are subject to other concerns: They result in very large risk premiums if firms are truthful (see panel (a)), and may allow strategic firms to capture large rents if they do not face other strategic firms in the auction (see panel (c)).

The second path is when both w and η grow along the left side of the discontinuity line: Firms then chose to benefit from the insurance offered by the plateau by declaring an expected production rather close to the real one. The best contract within the square $[0.5, 0.5]^2$ ($w = 0.375, \eta = 0.5$) even induce a BEC lower (1.0325) than the linear contract do (1.0462) when all firms are strategic. For the same contract, we note that when firms are all truthful the BEC is slightly lower (1.0341) than under the linear contract as well (see panel (a)), while a strategic firm facing only truthful bidders would barely be able to capture any rent from it: the BEC is only 1.0319 (see panel (c)).

However, such payment rules might be risky in practice: The regulator would most likely not have sufficient information to precisely determine the optimal payment rule, and a slight mistake may result in companies choosing the risky strategy, which would dramatically increase the BEC. For instance, if the bidder's relative risk aversion is $\gamma = 0.9$ instead of 1, then the contract that was optimal in the previous setup actually may result in a BEC much larger than the linear contract do: if one or all firms are strategic, the BEC is 1.0730 while it would be only 1.0413 under the linear contract.

In conclusion, such production-insuring payment rules may bring a better outcome than a standard linear payment rule in some cases, namely for widely spread distribution of q. However, adopting such payment rules would remain risky as imprecise information about the production distribution or firms' preferences may induce the regulator to choose an inadequate payment rule resulting in significantly inflated costs.

6 Conclusion

This paper analyses the pitfalls of a kind of procurement contract which attempts to insure the supplier against an exogenous risk, the expected value of which being reported by the supplier in its bid. Such contracts have in particular been used by the French government in some auctions for wind farm projects, to insure the producer against the production risk due to weather or estimation errors. However, to the best of our knowledge, such contracts have not been studied from an economic perspective. Expanding the peculiar payment rule used in France for wind farm projects to a general class of production-insuring payment rules, we find that such contracts are likely to fail their initial objective because they give producers incentive to misreport their expected production. Furthermore, when firms are heterogeneous regarding their "gaming" abilities, strategic firms would capture rents in the auctions for such production-insuring contracts, and this at the expense of the buyer. We estimate that gaming in the production-insuring payment rule used in France could have inflated by 3 to 4% the project cost compared to the linear FiT contract which is immune to gaming.

Beyond the specific case of RES electricity production, such contracts (and their pitfalls) could be extended to other applications with little variation needed in the model. Considering a positive marginal cost of production, we would reach equivalent conclusions[47] if production-insuring payment rules were to be used in the auctioning of leases for other natural resources whose extraction is exposed to an exogenous risk (availability of the resource), e.g. oil auctions or timber auctions. Moreover, such contracts could be used in procurement auction, e.g. for infrastructure projects, where the risk to be insured against would be the quantity of input instead

[47] In particular when the marginal cost is assumed constant, as emphasized at the beginning of section 3.

of the production. Payment rules analogous to the production-insuring payment rule discussed here could be use to partially insure the contractor against cost-overruns, but similar pitfalls would then result from the contractor benefiting from an underestimation of the needed quantity of inputs.

An apparently easy answer to these pitfalls would be to drop the feature that firms self-report their expected production: One could argue that the auctioneer may be better off by estimating the expected production herself (or through a third party). But such rules would still open the door to welfare inefficiencies: An advantage would still be granted to the firm which actively succeeds in making the production of reference the closest to its optimal report (e.g. through manipulation or corruption of the third party), or if the firm simply happens to be lucky in the determination of that production of reference. Then such a "lucky" firm might win the auction even though it is not the most efficient one.

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Appendix 1: Modelling production risk and our assumptions on producers' costs

Our simulations of producers' equilibrium bidding behaviour and then of the corresponding expected public spending are based on a production distribution built from historic simulated data and this for each of the six offshore wind farm sites that were actually auctioned under the production insuring payment rule we have presented in Section 2. The characteristics of those projects (name, location, size in MW) are listed in Table 1.

Hourly electricity productions of these farms are simulated for 19 years (from 2000 to 2018) using the model developed by Staffell and Pfenninger (2016) and this thanks to the website https://www.renewables.ninja/ to which the location and the characteristics of the turbines have been given as inputs. The production is simulated considering the full capacity of each farm.[48] In most cases, data needed to simulate production with the turbine type actually implemented by the winning bidder (most often the Adwen AD 8-180 turbine) was not available. For the six projects, we consider instead the Vestas V164 8000 turbine which seems the most closely related kind of turbine for such projects.

Historic hourly production obtained from the simulator is then aggregated at the quarterly level. Then we bootstrap our 19 years of aggregated quarterly data to generate the distribution

[48] Staffell and Pfenninger's (2016) model is for an isolated turbine. Therefore, the production of each farm (which consists of many turbines) is likely to be slightly overestimated due wake effects.

of yearly production: quarters are randomly drawn and summed to generate yearly production points. This resampling approach to generate more than our 19 original years of production is relevant if there is no significant autocorrelation between quarterly aggregate production.[49]

At the bidding stage, firms do not have a perfect knowledge on their average capacity factor which does not depend solely on their technological choice (e.g., the size and height of the turbine) but also on the local meteorological conditions which are estimated from measurement mats. In the past, such estimations suffer from important bias: Lee and Fields' (2020) survey report an over-prediction of the median of the capacity factor distribution around 4%. The methodologies have been improved with the aim to reduce bias, but they involve economically relevant errors: e.g. Jourdier and Drobinski (2017) show that the commonly used statistical model based on Weibull distributions lead to a mean average error around 4 or 5% of the electricity production. In order to account for such noise in the estimation of the capacity factor, the distribution of the vector of yearly-production $(q_1, \ldots, q_{20})$ is build in the following way: each yearly-production q_t is the product of a yearly-dependent production drawn independently across years according to the bootstrapped distribution defined above with $1 + \epsilon$ where ϵ is a non-year-dependant noise distributed according to a centered normal distribution with the variance σ^2. We assume that $\sigma^2 = 6.3\%$, which matches a mean average error of 5%. The noise ϵ for the capacity factor estimation is the main driver for the risk premiums relative to net present value of the subsidy contracts: contrary to meteorological risk, this additional risk is not averaged out over the 20 years of production.

Table 1: Characteristics on the wind farm projects (source : European Commission (2019) and French Energy Regulatory Commission (2011, 2013)

Site	Location (lat.,long.)	Capacity in MW	IC (CAPEX) M €	OC (OPEX/year) M €	FiT awarded €/MWh
Le Tréport	(50.1, 1.1)	496	2000	105	131
Ile d'Yeu	(46.9, -2.5)	496	1860	110	137
Fécamp	(49.9, 0.2)	497	1850	75	135.2
Courceulles	(49.5, -0.5)	448	1600	69	138.7
Saint-Brieuc	(48.8, -2.5)	496	2200	63	155
Saint-Nazaire	(47.2, -2.6)	496	1800	78	143.6

We consider throughout the paper that producers are fully homogeneous, meaning :

- Producers do not receive any private information on future production distribution which does not depend on the winning bidder's identity. The revenue distribution derived from any given contract is thus the same across all producers.

[49]The Saint-Brieuc site suffers from significant autocorrelation between quarterly aggregate production. Therefore we do not further consider results related to this site which differ importantly from the other sites.

- Producers have the same costs made of two components: a fixed cost C_0 (reflecting the initial investment) and a yearly operational cost C_t, $t = 1, \cdots, 20$ (reflecting operation and maintenance). Our assumptions for the cost for the various projects come from a reported of the European Commission.[50] are reported in Table 1.

Appendix 2: Proofs

Proof of Lemma 1

"Only if" part For a given $q_0 > 0$ and a given $\epsilon \in [0, 1)$, let $f^*_{q_0,\epsilon}$ denote the uniform distribution on the interval $[q_0(1-\epsilon), q_0(1+\epsilon)]$. We have that $f^*_{q_0,\epsilon} \in \mathcal{F}_{sp}$ and that $\bar{q} = q_0$.

Applying definition 1 to the contract price $p = 1$ and when U is linear, we have that :

$$\mathbb{E}_{f^*_{q_0,\epsilon}}[q \cdot z_{q_0}(\frac{q}{q_0})] = \int_{q_0(1-\epsilon)}^{q_0(1+\epsilon)} q \cdot z_{q_0}(\frac{q}{q_0}) \cdot \frac{dq}{2q_0\epsilon} = \frac{1}{2\epsilon}\int_{-\epsilon}^{\epsilon} q_0(1+t) \cdot z_{q_0}(1+t)dt = \mathbb{E}_{f^*_{q_0,\epsilon}}[q] = q_0.$$

We obtain then that $\int_0^\epsilon [(1+t) \cdot z_{q_0}(1+t) + (1-t) \cdot z_{q_0}(1-t)]dt = 2\epsilon$ for any $\epsilon \in [0, 1)$. The left-hand side of this later equation has a derivative almost everywhere which is equal to $(1+\epsilon) \cdot z_{q_0}(1+\epsilon) + (1-\epsilon) \cdot z_{q_0}(1-\epsilon)$, and which should thus be equal to the derivative of the right-hand side almost everywhere. Since the function $z_{q_0}(.)$ is continuous (because the function $q \to R(q, q_0)$ is assumed to be continuous), we obtain that

$$(1+\epsilon) \cdot z_{q_0}(1+\epsilon) + (1-\epsilon) \cdot z_{q_0}(1-\epsilon) = 2 \tag{5}$$

for any $\epsilon \in [0, 1)$.

In order to show that $z_{q_0}(1+\epsilon) \leq 1$ for any $\epsilon \in [0, 1]$, let us proceed by contradiction. Suppose on the contrary that $z_{q_0}(1+\epsilon) > 1$ for some $\epsilon \in [0, 1]$ and let then $\underline{\delta} := \inf\{\epsilon \in [0, 1] | z_{q_0}(1+\epsilon) > 1\}$. Since $z_{q_0}(.)$ is continuous, we have then $\underline{\delta} < 1$ and we can also define $\overline{\delta} \in (\underline{\delta}, 1]$ such that $z_{q_0}(1+\epsilon) > 1$ for any $\epsilon \in]\underline{\delta}, \overline{\delta}[$. Since $z_{q_0}(\cdot)$ is continuous, we also have $z_{q_0}(1+\underline{\delta}) = 1$.

Consider then $f^*_{q_0,\overline{\delta}}$ the uniform distribution on $[q_0(1-\overline{\delta}), q_0(1+\overline{\delta})]$. Consider a continuous function U such that $U(x) = x$ for $x \leq q_0(1+\underline{\delta})$ and $U'(x) \in]0, 1[$ being strictly decreasing for $x > q_0(1+\underline{\delta})$.[51] Note that U is then increasing and concave.

Given that the function $q \mapsto q \cdot z_{q_0}(\frac{q}{q_0})$ is non-decreasing and that $z_{q_0}(1+\underline{\delta}) = 1$ (which implies $z_{q_0}(1-\underline{\delta}) = 1$ given (5)), we have that $q \cdot z_{q_0}(\frac{q}{q_0}) \in [q_0(1-\underline{\delta}), q_0(1+\underline{\delta})]$ for any $q \in [q_0(1-\underline{\delta}), q_0(1+\underline{\delta})]$. Therefore using $U(x) = x$ for $x \in [q_0(1-\underline{\delta}), q_0(1+\underline{\delta})]$, the symmetry of $f^*_{q_0,\overline{\delta}}$ around q_0, and making the change of variable $\epsilon = \frac{q}{q_0} - 1$ in (5) we get:

[50] https://ec.europa.eu/competition/state_aid/cases1/201933/265141_2088479_221_2.pdf

[51] How to build a function U satisfying such properties (which will guarantee then its existence) is left to the reader.

$$\int_{q_0(1-\underline{\delta})}^{q_0(1+\underline{\delta})} U(q\cdot z_{q_0}(\frac{q}{q_0}))dF^*_{q_0,\overline{\delta}}(q) = \int_{-\underline{\delta}}^{\underline{\delta}} q_0(1+\epsilon)\cdot z_{q_0}(1+\epsilon)dF^*_{q_0,\overline{\delta}}(q_0(1+\epsilon))$$
$$= q_0\int_0^{\underline{\delta}}[(1+\epsilon)\cdot z_{q_0}(1+\epsilon)+(1-\epsilon)\cdot z_{q_0}(1-\epsilon)]dF^*_{q_0,\overline{\delta}}(q_0(1+\epsilon))$$
$$= 2q_0\cdot[f^*_{q_0,\overline{\delta}}(q_0(1+\underline{\delta}))-\frac{1}{2}] = \int_{q_0(1-\underline{\delta})}^{q_0(1+\underline{\delta})} U(q)dF^*_{q_0,\overline{\delta}}(q).$$

We obtain thus that the difference $\mathbb{E}_{f^*_{q_0,\overline{\delta}}}[U(q)] - \mathbb{E}_{f^*_{q_0,\overline{\delta}}}[U(q\cdot z_{q_0}(\frac{q}{q_0}))]$ resumes to

$$\int_{q_0(1-\overline{\delta})}^{q_0(1-\underline{\delta})}[U(q)-U(q\cdot z_{q_0}(\frac{q}{q_0}))]\frac{1}{2\overline{\delta}}dq + \int_{q_0(1+\underline{\delta})}^{q_0(1+\overline{\delta})}[U(q)-U(q\cdot z_{q_0}(\frac{q}{q_0}))]\frac{1}{2\overline{\delta}}dq$$

Thanks to the change of variable $\epsilon = 1-\frac{q}{q_0}$ and $\epsilon = \frac{q}{q_0}-1$ in the first and second integrals, respectively, we get :

$$\mathbb{E}_{f^*_{q_0,\overline{\delta}}}[U(q)] - \mathbb{E}_{f^*_{q_0,\overline{\delta}}}[U(q\cdot z_{q_0}(\frac{q}{q_0}))] = \frac{q_0}{2\overline{\delta}}\int_{\underline{\delta}}^{\overline{\delta}}[U(q_0(1-\epsilon)) - U(q_0(1-\epsilon)z_{q_0}(1-\epsilon))]d\epsilon$$
$$+ \frac{q_0}{2\overline{\delta}}\int_{\underline{\delta}}^{\overline{\delta}}[U(q_0(1+\epsilon)) - U(q_0(1+\epsilon)z_{q_0}(1+\epsilon))]d\epsilon.$$

Let us show below that in the first (resp. second) integral the function U is applied to values where it is linear (resp. strictly concave).

For $\epsilon\in[\underline{\delta},\overline{\delta}]$, we have $z_{q_0}(1+\epsilon)\geq 1$. From (5), we get for any $\epsilon\in[\underline{\delta},\overline{\delta}]$ that $z_{q_0}(1-\epsilon)\leq 1$, which further implies that $q_0(1-\epsilon)z_{q_0}(1-\epsilon)\leq q_0(1-\epsilon)\leq q_0(1+\underline{\delta})$. For $q\leq q_0(1+\underline{\delta})$, U is defined such that $U(q)=q$: in the first interval, the function U is thus applied only for values below $q_0(1+\underline{\delta})$. We have thus that $\forall\epsilon\in[\underline{\delta},\overline{\delta}]$, $U(q_0(1-\epsilon)-U(q_0(1-\epsilon)z_{q_0}(1-\epsilon)) = q_0(1-\epsilon)-q_0(1-\epsilon)z_{q_0}(1-\epsilon)$.

Since the function $\epsilon\mapsto q_0(1+\epsilon)z_{q_0}(1+\epsilon)$ is non-decreasing and $z_{q_0}(1+\underline{\delta})=1$, then for $\epsilon\in[\underline{\delta},\overline{\delta}]$, we have that $q_0(1+\epsilon)z_{q_0}(1+\epsilon)\geq q_0(1+\underline{\delta})z_{q_0}(1+\underline{\delta}) = q_0(1+\underline{\delta})$. Besides, we note that $q_0(1+t)\geq q_0(1+\underline{\delta})$. For $q\geq q_0(1+\underline{\delta})$, U is strictly concave ($U'(q)<1$): in the second interval, the function U is thus applied only for values above $q_0(1+\underline{\delta})$. We have thus that $\forall\epsilon\in(\underline{\delta},\overline{\delta}]$, $U(q_0(1+\epsilon)) - U(q_0(1+\epsilon)z_{q_0}(1+\epsilon)) \geq [q_0(1+\epsilon)-q_0(1+\epsilon)z_{q_0}(1+\epsilon)]\cdot U'(q_0(1+\epsilon)) > q_0(1+\epsilon)-q_0(1+\epsilon)z_{q_0}(1+\epsilon)$.

Finally, using $(1+\epsilon)z_{q_0}(1+\epsilon)+(1-\epsilon)z_{q_0}(1-\epsilon)=2$ and the inequality above, we get:

$$\mathbb{E}_{f^*_{q_0,\overline{\delta}}}[U(q)] - \mathbb{E}_{f^*_{q_0,\overline{\delta}}}[U(q\cdot z_{q_0}(\frac{q}{q_0}))] > \frac{q_0^2}{2\overline{\delta}}\int_{\underline{\delta}}^{\overline{\delta}}\underbrace{[2-(1-\epsilon)z_{q_0}(1-\epsilon)-(1+\epsilon)z_{q_0}(1+\epsilon)]}_{=0}d\epsilon.$$

We have thus shown that $\mathbb{E}_{f^*_{q_0,\bar{\delta}}}[U(q)] > \mathbb{E}_{f^*_{q_0,\bar{\delta}}}[U(q \cdot z_{q_0}(\frac{q}{q_0}))]$, which stands in contradiction with the production-insuring assumption.

On the whole we have shown that $z_{q_0}(1+\epsilon) \leq 1$ for any $\epsilon \in [0,1]$. From (5), we get then that $z_{q_0}(1-\epsilon) \leq 1$ for any $\epsilon \in [0,1]$.

The remaining part of Lemma 1 to be shown is that z_{q_0} can not be equal (uniformly) to one in the neighborhood of one or equivalently (given that we have shown that $z_{q_0}(1+\epsilon) \leq 1$ for $\epsilon \in [0,1]$ and that z_{q_0} is continuous) that for all $\epsilon \in (0,1]$ we verify $\int_0^\epsilon z_{q_0}(1+t)dt < \epsilon$. Suppose that $z_{q_0}(t) = 1$ for any $t \in [-\epsilon, \epsilon]$ (with $\epsilon > 0$) and let us establish a contradiction. Consider a strictly concave payoff function U, the contract price $p = 1$ and the uniform distribution $f^*_{q_0,\epsilon}$, the production-insuring property guarantees that $\mathbb{E}_{f^*_{q_0,\bar{\delta}}}[U(q)] < \mathbb{E}_{f^*_{q_0,\bar{\delta}}}[U(q \cdot z_{q_0}(\frac{q}{q_0}))]$ or equivalently

$$\int_0^\epsilon [U(q_0(1+t)z_{q_0}(1+t)) + U(q_0(1-t)z_{q_0}(1-t))]\, dt > \int_0^\epsilon [U(q_0(1+t)) + U(q_0(1-t))]\, dt.$$

This inequality can not hold once $z_{q_0}(t) = 1$ for any $t \in [-\epsilon, \epsilon]$ which concludes the proof.

"If" part

Consider first the case where U is linear. If Eq. (5) holds for any $q_0 > 0$ and $\epsilon \in [0,1]$, then for any contract price p and any symmetric distribution f with expected value $\bar{q}$ (such that the support of f is a subset of $[0, 2\bar{q}]$), using the change of variable $q = \bar{q}(1+\epsilon)$, we obtain below that Eq. (1) stands as an equality (note that it is the first and the last equality that uses that U is linear):

$$\begin{aligned}
\mathbb{E}_f[U(pqz_{\bar{q}}(\frac{q}{\bar{q}}))] = U\left(\mathbb{E}_f[pqz_{\bar{q}}(\frac{q}{\bar{q}})]\right) &= U\left(p\bar{q}\int_{-1}^{1}(1+\epsilon)z_{\bar{q}}(1+\epsilon)f(\bar{q}(1+\epsilon))d\epsilon\right) \\
&= U\left(p\bar{q}\int_0^1 [(1+\epsilon)z_{\bar{q}}(1+\epsilon) + (1-\epsilon)z_{\bar{q}}(1-\epsilon)]\, f(\bar{q}(1+\epsilon))d\epsilon\right) \\
&= U\left(p\bar{q}\int_0^1 2f(\bar{q}(1+\epsilon))d\epsilon\right) = U(p\bar{q}) = \mathbb{E}_f[U(pq)].
\end{aligned}$$

Let us now consider the case where U is strictly concave. Consider the function $\varphi : \lambda \to U(p\bar{q}\lambda) + U(p\bar{q}(2-\lambda))$. If U is strictly concave, then $U'(p\bar{q}\lambda) < U'(p\bar{q}(2-\lambda))$ as long as $\lambda > 1$. We have thus that $\varphi'(\lambda) = p\bar{q}\,[U'(p\bar{q}\lambda) - U'(p\bar{q}(2-\lambda))] < 0$ for $\lambda > 1$.

Moreover, since f is symmetric, we have both following equations for any function U:

$$\mathbb{E}_f[U(p\cdot q)] = \int_0^1 \overbrace{[U(p\cdot\bar{q}(1+\epsilon)) + U(p\cdot\bar{q}(1-\epsilon))]}^{=\varphi(1+\epsilon)} dF(\bar{q}(1+\epsilon))$$

$$\mathbb{E}_f[U(p\cdot q\cdot z_{\bar{q}}(\frac{q}{\bar{q}}))] = \int_0^1 \underbrace{[U(p\cdot\bar{q}(1+\epsilon)z_{\bar{q}}(1+\epsilon)) + U(p\cdot\bar{q}(1-\epsilon)z_{\bar{q}}(1-\epsilon))]}_{=\varphi((1+\epsilon)z_{q_0}(1+\epsilon))} dF(\bar{q}(1+\epsilon))$$

In addition to (5), we also assume that $z_{q_0}(1+\epsilon) \leq 1$ for any $\epsilon \in [0,1]$ and that for any $\epsilon' > 0$, there exists a subset S of $[0,\epsilon']$ with positive measure such that $z_{q_0}(1+t) < 1$ for any $t \in S$. We obtain then that $(1+\epsilon)z_{q_0}(1+\epsilon) \leq 1+\epsilon$ for any $\epsilon \in [0,1]$ and that for any $\epsilon' > 0$, there exists a subset S of $[0,\epsilon']$ with positive measure such that $(1+t)z_{q_0}(1+t) < 1+t$ for any $t \in S$. Moreover, since $q \mapsto q\cdot z_{q_0}(\frac{q}{q_0}$ is non decreasing, we have $1 \leq (1+\epsilon)z_{q_0}(1+\epsilon)$ for $\epsilon \in [0,1]$.

The function φ is strictly decreasing $[1,2]$. From the previous inequalities, we have thus that φ is strictly decreasing on the interval $[(1+\epsilon)z_{q_0}(1+\epsilon), 1+\epsilon]$ for any $\epsilon \in [0,1]$.

Finally we have for any $\epsilon \in [0,1]$,

$$U(p\cdot\bar{q}(1+\epsilon)z_{\bar{q}}(1+\epsilon)) + U(p\cdot\bar{q}(1-\epsilon)z_{\bar{q}}(1-\epsilon)) \geq U(p\cdot\bar{q}(1+\epsilon)) + U(p\cdot\bar{q}(1-\epsilon)).$$

Furthermore, for any $\epsilon' > 0$, there exists a subset S of $[0,\epsilon']$ with positive measure such that for any $t \in S$:

$$U(p\cdot\bar{q}(1+t)z_{\bar{q}}(1+t)) + U(p\cdot\bar{q}(1-t)z_{\bar{q}}(1-t)) > U(p\cdot\bar{q}(1+t)) + U(p\cdot\bar{q}(1-t)). \qquad (6)$$

Since $f \in \mathcal{F}_{sp}$, then there exists $\epsilon' > 0$ such that f is strictly positive on $[0,\epsilon']$. Therefore, by integration we get the strict inequality:

$$\int_0^1 [U(p\cdot\bar{q}(1+t)z_{\bar{q}}(1+t)) + U(p\cdot\bar{q}(1-t)z_{\bar{q}}(1-t))]dF(\bar{q}(1+t))$$
$$> \int_0^1 [U(p\cdot\bar{q}(1+t)) + U(p\cdot\bar{q}(1-t))]dF(\bar{q}(1+t))$$
$$\Leftrightarrow \mathbb{E}_f[U(p\cdot q\cdot z_{\bar{q}}(\frac{q}{\bar{q}}))] > \mathbb{E}_f[U(p\cdot q)]$$

Last, in the remaining case where U is concave, it is straightforward according to the arguments above (it is sufficient to integrate the inequality (6)) that the inequality $\mathbb{E}_f[U(p\cdot q\cdot z_{\bar{q}}(\frac{q}{\bar{q}}))] = \mathbb{E}_f[U(p\cdot R(q,\bar{q}))] \geq \mathbb{E}_f[U(p\cdot q)]$, i.e. Eq. (1), holds for any symmetric distribution f (even if it is not single-peaked). On the whole, we have established that any payment rule associated to the correction factors $\{z_{q_0}(.)\}_{q_0>0}$ is production-insuring.

Q.E.D.

Proof of Proposition 2

Let us first show that if $q_0 \geq \bar{q}$, then $\mathbb{E}_f[R(q, q_0)] \geq \mathbb{E}_f[R(\bar{q}, q_0)] = p \cdot \bar{q}$ or equivalently $\mathbb{E}_f[q \cdot z_{q_0}(\frac{q}{q_0})] \geq \bar{q}$. Take $q_0 \geq \bar{q}$ and let $\alpha := 1 - F(q_0)$.

Suppose first that $\alpha = 0$. Then $\mathbb{E}_f[R(q, q_0)] = \int_0^{q_0} q \cdot z_{q_0}(\frac{q}{q_0})dF(q) \geq \int_0^{q_0} qdF(q) = \bar{q}$, since from Lemma 1 we have $\forall q \leq q_0$, $z_{q_0}(\frac{q}{q_0}) \geq 1$.

Suppose now that $\alpha > 0$. Let $G_{q_0} : \mathbb{R}_+ \to \mathbb{R}_+$ denote the function defined by:

$$\text{for } q \geq q_0, \quad G_{q_0}(q) = \frac{1 + F(q) - 2F(q_0)}{2\alpha}$$
$$\text{for } q \leq q_0, \quad G_{q_0}(q) = 1 - G_{q_0}(2q_0 - q).$$

Since the CDF F is non-decreasing, then G_{q_0} is also non-decreasing. Since $f \in \mathcal{F}_{sp}$, we have $F(2\bar{q}) = 1$ and then $F(2q_0) = 1$ (since $q_0 \geq \bar{q}$). We have then that $G_{q_0}(2q_0) = 1$, and therefore $G_{q_0}(0) = 0$. Finally, we have that G_{q_0} is a symmetric CDF function with expected value q_0. Let g_{q_0} denote the corresponding pdf. We have then that $\mathbb{E}_{g_{q_0}}[q \cdot z_{q_0}(\frac{q}{q_0})] = \mathbb{E}_{g_{q_0}}[q] = q_0$.

Let us define the function $H_{q_0} : \mathbb{R}_+ \to \mathbb{R}_+$ by $H_{q_0}(q) := F(q) - 2\alpha \cdot G_{q_0}(q)$, and therefore $f(q) = H'_{q_0}(q) + 2\alpha \cdot g_{q_0}(q)$. Then we may write :

$$\mathbb{E}_f[qz_{q_0}(\frac{q}{q_0})] = \int_0^{2\bar{q}} qz_{q_0}(\frac{q}{q_0})dF(q) = \int_0^{2q_0} qz_{q_0}(\frac{q}{q_0})dF(q)$$
$$= \int_0^{2q_0} qz_{q_0}(\frac{q}{q_0})H'_{q_0}(q)dq + 2\alpha \int_0^{2q_0} qz_{q_0}(\frac{q}{q_0})dG_{q_0}(q) = \int_0^{2q_0} qz_{q_0}(\frac{q}{q_0})H'_{q_0}(q)dq + 2\alpha \cdot q_0$$

where the second equality uses the assumption $q_0 \geq \bar{q}$ and the last the fact that $g \in \mathcal{F}_{sp}$ with the expected value q_0.

For $q \geq q_0$, $2\alpha g_{q_0}(q) = f(q)$ and therefore $H'_{q_0}(q) = 0$. Moreover, $\forall q \leq q_0$, $z(\frac{q}{q_0}) \geq 1$. We obtain therefore :

$$\begin{aligned}
\mathbb{E}_f[q \cdot z_{q_0}(\frac{q}{q_0})] - 2\alpha \cdot q_0 &= \int_0^{q_0} q \cdot z_{q_0}(\frac{q}{q_0})H'_{q_0}(q)dq \\
&\geq \int_0^{q_0} q \cdot H'_{q_0}(q)dq = \int_0^{q_0} qdF(q) - 2\alpha \int_0^{q_0} qdG_{q_0}(q) \\
&= \bar{q} - \int_{q_0}^{2\bar{q}} qdF(q) - 2\alpha \int_0^{q_0} qdG_{q_0}(q) \\
&= \bar{q} - 2\alpha \underbrace{\left(\int_{q_0}^{2\bar{q}} qdG_{q_0}(q) + \int_0^{q_0} qdG_{q_0}(q) \right)}_{=\mathbb{E}_{g_{q_0}}[q] = q_0} = \bar{q} - 2\alpha \cdot q_0.
\end{aligned}$$

Finally, $\mathbb{E}_f[R(q,q_0)] \geq \bar{q} = \mathbb{E}_f[R(q,\bar{q})]$ (for any $q_0 \geq \bar{q}$). By symmetry, we can show that $\mathbb{E}_f[R(q,q_0)] \leq \bar{q}$ for any $q_0 \leq \bar{q}$.

To prove that the payment rule is manipulable, then for any given $f \in \mathcal{F}_{sp}$, let us build $q_0 > \bar{q}$ such that $\mathbb{E}_f[R(q,q_0)] > \bar{q}$.

Consider first the case where there exists a pair (q_0, ϵ) with $q_0 > \bar{q}$ and $\epsilon > 0$ such that $f(q_0 - t) > f(q_0 + t) > 0$ for any $t \in (0, \epsilon]$. The pdf f is either continuous and strictly decreasing around q_0, or discontinuous. The existence of $\epsilon > 0$ such that $f(q_0 + \epsilon) > 0$ ensures that $\alpha > 0$. Therefore using the same arguments as above, in order to show that $\mathbb{E}_f[R(q,q_0)] > \bar{q} = \mathbb{E}_f[R(q,\bar{q})]$, it is sufficient to show that $\int_0^{q_0} q z_{q_0}(\frac{q}{q_0}) H'_{q_0}(q)dq > \int_0^{q_0} qH'_{q_0}(q)dq$.

For $q \in [\bar{q}, q_0]$, we have $H'_{q_0}(q) = f(q) - 2\alpha g_{q_0}(q) = f(q) - f(2q_0 - q)$. Since f is non-increasing for $q > \bar{q}$, then $\bar{q} < q < q_0 < 2q_0 - q$ implies $f(q) > f(2q_0 - q)$ and therefore $H'(q) > 0$ for any $q \in [\bar{q}, q_0]$. Moreover we know from Lemma 1 that there is a subset of $[\bar{q}, q_0]$ with positive measure in which $z_{q_0}(\frac{q}{q_0}) > 1$. We obtain then $\int_{\bar{q}}^{q_0} q z_{q_0}(\frac{q}{q_0}) H'_{q_0}(q)dq > \int_{\bar{q}}^{q_0} qH'_{q_0}(q)dq$ which further implies $\int_0^{q_0} q z_{q_0}(\frac{q}{q_0}) H'_{q_0}(q)dq > \int_0^{q_0} qH'_{q_0}(q)dq$ (since $z_{q_0}(q) \geq 1$ and $H'_{q_0}(q) = 2F(q_0) - 1 \geq 0$ for $q \leq q_0$ given that $q_0 \geq \bar{q}$).

Consider the other case where f is locally constant for any $q > \bar{q}$ on its support. Therefore, there exists a threshold $q' > \bar{q}$ such that f is constant and strictly positive on $(\bar{q}, q')$ and then $f(q) = 0$ for $q > q'$. In other words, f is a uniform distribution on the interval $[2\bar{q} - q', q']$. Then $z_{q'}(\frac{q}{q'}) \geq 1$ for any realization of q on the support $[2\bar{q} - q', q']$. Furthermore, from Lemma 1, there is a subset of $[2\bar{q} - q', q']$ with positive measure on which $z_{q_0}(\frac{q}{q_0}) > 1$. Finally, we have shown that $\mathbb{E}_f[q \cdot z_{q'}(\frac{q}{q'})] > \mathbb{E}_f[q]$. **Q.E.D.**

Proof of Proposition 3

Suppose there is payment rule homogeneous of degree 1 such that for $p > 0$, $f \in \mathcal{F}_s p$ and $q_0^* \in \operatorname{Arg\,max}_{q_0} \Pi(p, q_0)$, the bidder is fully insured against production risk, meaning $\operatorname{Var}_f[p \cdot R(q, q_0^*)] = 0$. Note first that the payment rule being homogeneous of degree 1 implies that the function $z_{q_0}(\cdot)$ does not depend on q_0: $\forall \lambda, q, q_0 > 0, R(\lambda q, \lambda q_0) = \lambda R(q, q_0) \Rightarrow \lambda q z_{\lambda q_0}(\frac{q}{q_0}) = \lambda q z_{q_0}(\frac{q}{q_0})$.

Denote $[q_{min}, q_{max}]$ the support of f. Having $p > 0$, the bidder being fully insured against production risk implies that $\forall q \in (q_{min}, q_{max}), R(q, q_0^*) = q \cdot z(\frac{q}{q_0^*}) = k \in \mathbb{R}^{+*}$ a constant. Therefore z is defined on $(\frac{q_{min}}{q_0^*}, \frac{q_{max}}{q_0^*})$ by $z(x) = \frac{k}{q_0^*}\frac{1}{x}$.

Take $q'_0 \geq q_0^*$, then the bidders payoff is given by:

$$\Pi(p, q'_0) = \int_{q_{min}}^{q_{max}} U\left(pqz(\frac{q}{q'_0})\right) dF(q) = \int_{q_{min}}^{q_{min}\frac{q'_0}{q_0^*}} U\left(pqz(\frac{q}{q'_0})\right) dF(q) + \int_{q_{min}\frac{q'_0}{q_0^*}}^{q_{max}} U\left(pk\frac{q'_0}{q_0^*}\right) dF(q)$$

$$= \int_{q_0^*}^{q'_0} U\left(pq_{min}\frac{x}{q_0^*} z(\frac{q_{min}}{q'_0}\frac{x}{q_0^*})\right) dF(q_{min}\frac{x}{q_0^*}) + U\left(pk\frac{q'_0}{q_0^*}\right)\left[1 - F(q_{min}\frac{q'_0}{q_0^*})\right]$$

From the payment rule $R(\cdot, q_0)$ being continuous and increasing we know it is derivable almost everywhere. As $z(x) = R(x \cdot q_0, q_0)/x \cdot q_0$, it is also derivable almost everywhere, and therefore $z(\frac{q_{min}}{q_0'} \frac{x}{q_0^*})$ admits a derivative with respect to q_0' for some $q_0' > q_0^*$ close to q_0^*. Then, the derivative of the bidder's payoff can be written for such q_0' as:

$$\begin{aligned}\frac{\partial \Pi(p, q_0')}{\partial q_0'} =& U\left(pq_{min}\frac{q_0'}{q_0^*}z(\frac{q_{min}}{q_0^*})\right)\frac{q_{min}}{q_0^*}f(q_{min}\frac{q_0'}{q_0^*}) + \int_{q_0^*}^{q_0'} \frac{\partial U(pq_{min}\frac{x}{q_0^*}z(\frac{q_{min}}{q_0'}\frac{x}{q_0^*}))}{\partial q_0'}\frac{q_{min}}{q_0^*}dF(q_{min}\frac{x}{q_0^*}) \\ &+ \frac{pk}{q_0^*}U'\left(pk\frac{q_0'}{q_0^*}\right)[1 - F(q_{min}\frac{q_0'}{q_0^*})] - U\left(pk\frac{q_0'}{q_0^*}\right)\frac{q_{min}}{q_0^*}f(q_{min}\frac{q_0'}{q_0^*})\end{aligned}$$

Then the derivative to the right in q_0^* is given by:

$$\begin{aligned}\frac{\partial \Pi(p, q_0')}{\partial q_0'} \underset{q_0' \to q_0^*}{\longrightarrow} & U\left(pq_{min}z(\frac{q_{min}}{q_0^*})\right)\frac{q_{min}}{q_0^*}f(q_{min}) + \frac{pk}{q_0^*}U'(pk)[1 - F(q_{min})] - U(pk)\frac{q_{min}}{q_0^*}f(q_{min}) \\ =& \frac{pk}{q_0*}U'(pk) + \left[U(pq_{min}z(\frac{q_{min}}{q_0^*}) - U(pk)\right]\frac{q_{min}}{q_0^*}f(q_{min}) \\ =& \frac{pk}{q_0^*}U'(pk) > 0\end{aligned}$$

since by continuity of $R(\cdot, q_0^*)$ we have $q_{min}z(\frac{q_{min}}{q_0^*}) = k$. [52] Therefore the bidder can increase its expected payoff by increasing its report of q_0, which stands in contradiction with $q_0^* \in \operatorname{Arg\,max}_{q_0 \in \mathbb{R}^+} \Pi(p, q_0)$.

Q.E.D.

Proof of Proposition 4

Let us denote p_{lc} the equilibrium price under the linear contract. Based on the same argument as for equation (2), p_{lc} is characterized by $\mathbb{E}_f[U(p_{lc} \cdot q)] = U(C)$. Therefore combining with equation (2) we get $\mathbb{E}_f[U(p_{lc} \cdot q)] = \mathbb{E}_f[U(p^{NS} \cdot R(q, \bar{q}))] = U(C)$. From definition 1, we know that for any production-insuring payment rule $\mathbb{E}_f[U(p_{lc} \cdot R(q, \bar{q}))] \geq \mathbb{E}_f[U(p_{lc} \cdot q)] = \mathbb{E}_f[U(p^{NS} \cdot R(q, \bar{q}))]$, the inequality being strict if producers are strictly risk-averse and stands as an equality if producers are risk-neutral.

Moreover, we know on the one hand that U is strictly increasing, and on the other hand that $R(q, \bar{q})$ does not take negative values and takes a strictly positive value with a probability strictly superior to zero. Therefore the function $p \mapsto \mathbb{E}_f[U(p \cdot R(q, \bar{q}))]$ is strictly increasing. From there and the inequality $\mathbb{E}_f[U(p_{lc} \cdot R(q, \bar{q}))] \geq \mathbb{E}_f[U(p^{NS} \cdot R(q, \bar{q}))]$ we get that $p^{NS} \leq p_{lc}$, and in consequence that the expected buyer's cost $\mathbb{E}_f[p^{NS} \cdot R(q, \bar{q})] \geq \mathbb{E}_f[p_{lc} \cdot R(q, \bar{q})]$. The previous

[52] Note that the assumption of $R(\cdot, q_0)$ being continuous could here be replaced by the continuity of f as it would imply $f(q_{min}) = 0$. However the present proof would still require to have $z(\cdot)$ derivable to the left of $\frac{q_{min}}{q_0^*}$ to be able to express $\frac{\partial \Pi(p, q_0')}{\partial q_0'}$.

inequalities are strict if producers are strictly risk-averse, and stands as equalities of producers are risk-neutral.

Q.E.D.

Proof of Proposition 5

Let us first show that the characterization of the equilibrium (p^S, q^S) is valid. Based on the same argument as in the proof of Proposition 4, we obtain that the function $p \mapsto \mathbb{E}_f[U(p \cdot R(q, q_0))]$ is strictly increasing for any q_0 such that $R(q, q_0)$ is not null for all possible q.[53] Then for any report q^*, there is a unique price p^* leading to the zero profit condition $\mathbb{E}_f[U(p^* \cdot R(q, q^*))] = U(C)$. Only such p^* would be consistent with equilibrium behavior: if $p < p^*$, the winning bidder would prefer to deviate to lose the auction. If $p > p^*$, then (for any given tie-breaking rule) at least one bidder would prefer to deviate to be sure to win the auction with probability one. Furthermore, in equilibrium, for any given price p^*, a strategic producer should report an expected quantity belonging to $\operatorname{Arg\,max}_{q_0 \geq 0} \mathbb{E}_f U(p^* \cdot R(q, q_0))$. On the whole, we have thus shown that any equilibrium bid pair (p^S, q^S) satisfies the condition in equation (3).

Let us now show the existence and uniqueness of such consistent equilibrium bid pair (p^S, q^S). Let $H(p) := \max_{q_0 \geq 0} \mathbb{E}_f[U(p \cdot R(q, q_0))]$. From the conditions above, the equilibrium price p must belong to the set of prices p such that $H(p) = U(C)$. To get that there exists a unique equilibrium price p^S, we show below that the function H is increasing. Take q_0 such that $\mathbb{E}_f[R(q, q_0)] > 0$ and any pair (p, p') such that $p' > p > 0$. We have $\mathbb{E}_f[U(p' \cdot R(q, q_0))] > \mathbb{E}_f[U(p \cdot R(q, q_0))]$. We have then $H(p') \geq \mathbb{E}_f[U(p' \cdot R(q, q_0))] > \mathbb{E}_f[U(p \cdot R(q, q_0))]$ for any q_0 with $\mathbb{E}_f[R(q, q_0)] > 0$ and then in particular for an optimal q_0 at the price $p > 0$. We have thus shown that H is increasing on $\mathbb{R}_+$. Furthermore, $\mathbb{E}_f[U(p \cdot R(q, q_0))]$ is equal to $U(0)$ for $p = 0$ and goes to $\lim_{x \to +\infty} U(x)$ when p goes to infinity. Therefore there exists a unique p^S with its associated q^S satisfying the condition in equation (3).

In order to show that $p^S \leq p^{NS}$, we proceed by contradiction. Suppose that on the contrary that $p^S > p^{NS}$. Then we have $\max_{q_0 \geq 0} \mathbb{E}_f[U(p^S \cdot R(q, q_0))] \geq \mathbb{E}_f[U(p^S \cdot R(q, \bar{q}))] > \mathbb{E}_f[U(p^{NS} \cdot R(q, \bar{q}))]$. From (2) (resp. (3)), the last (resp. first) term is equal to $U(C)$ and we have thus raised a contradiction.

If the payment rule is manipulable at the price p^{NS}, then we have $\max_{q_0 \geq 0} \mathbb{E}_f[U(p^{NS} \cdot R(q, q_0))] > \mathbb{E}_f[U(p^{NS} \cdot R(q, \bar{q}))]$. Given (2), then the last term is equal to $U(C)$. If $p^S = p^{NS}$ and given (3), then $\max_{q_0 \geq 0} \mathbb{E}_f[U(p^{NS} \cdot R(q, q_0))] = U(C)$ and we have thus raised a contradiction. We have thus shown that if the payment rule is manipulable at the price p^{NS}, then $p^S < p^{NS}$. Note that Proposition 2 establishes that if producers are risk-neutral, all production-insuring payment rule are manipulable.

[53] Note that this latter case could not be part of an equilibrium bid since the producer's payoff would then be $U(0) < U(C)$.

If the payment rule provides full insurance against production risk to truthful bidders and is homogeneous of degree 1, then we get from proposition 3 that a strategic bidder will not be fully insured against production risk: $\text{Var}_f[R(q, q_0^S)] > \text{Var}_f[R(q, \bar{q})] = 0$. In equilibrium, we have $\mathbb{E}_f[U(p^S R(q, q_0^S))] = U(p^{NS}\bar{q})$ since the payoff of the truthful bidder is certain thanks to full insurance by the payment rule. If bidders are strictly risk-averse then $U(\cdot)$ is strictly concave, and therefore $U(\mathbb{E}_f[p^S R(q, q_0^S)]) > \mathbb{E}_f[U(p^S R(q, q_0^S))]$. Then $U(\cdot)$ being increasing implies that the buyer's expected cost under strategic bidding $\mathbb{E}_f[p^S R(q, q_0^S)]$ is greater than its equivalent under truthful bidding $p^{NS}\bar{q}$.

Example 1: Let us build a production-insuring rule $R(.,.)$ and a distribution f such that the cost to the buyer is larger under truthful reporting than under strategic reporting.

Take $\epsilon \in (0, 1)$. For each $q_0 > 0$, let us define the function $R(., q_0) : \mathbb{R}_+ \to \mathbb{R}_+$ recursively in the following way: for $q \in [\frac{5}{6}q_0, \frac{7}{6}q_0]$, we let $R(q, q_0) := q_0 + (1 - \epsilon) \cdot (q - q_0)$ so that payment is almost equivalent to the linear contract for ϵ small, but with a slightly smaller slope; for $q \in [(\frac{1}{2} + \epsilon)q_0, \frac{5}{6}q_0[$ we let $R(q, q_0) := R(\frac{5}{6}q_0, q_0)$, for $q \in]\frac{7}{6}q_0, (\frac{3}{2} - \epsilon)q_0]$ we let $R(q, q_0) := R(\frac{7}{6}q_0, q_0)$ so that payment is flat in these two intervals; for $q \in [0, \frac{1}{2}q_0[$ and for $q \geq \frac{3}{2}q_0$ we let $R(q, q_0) := q$, then the payment is equivalent to the linear contract on these intervals; finally we define $R(.,.)$ in $[\frac{1}{2}q_0, (\frac{1}{2} + \epsilon)q_0[$ and in $[(\frac{3}{2} - \epsilon)q_0, \frac{3}{2}q_0[$ so that payment is continuous in q: on the first segment $R(q, q_0) := q(\frac{1}{3\epsilon} + \frac{1}{6}) + q_0(\frac{5}{12} - \frac{1}{6\epsilon})$, and on the second segment $R(q, q_0) := q(\frac{1}{3\epsilon} + \frac{1}{6}) + q_0(\frac{5}{4} - \frac{1}{2\epsilon})$.

For the distribution f, take the uniform distribution on $[1-\delta, 1+\delta]$ where $\delta < \frac{1}{6}$. Under truthful reporting, we have that the equilibrium price p^{NS} is characterized by $\int_{1-\delta}^{1+\delta} U(p^{NS} \cdot (1-\epsilon)q) = U(C)$. Under strategic reporting, we have that the producer overestimates its production by reporting $q^* > \bar{q}$ in order to benefit from the payment being largely inflated in lower flat areas.

Through simulations with $\delta = 1/6$, a CRRA utility with $\gamma = 1$ and $\epsilon = 0.01$, we find the optimal reporting of q_0 being 1.6605. For such reporting, the lower bound of the distribution (relative to the average realization $\bar{q}$), $1 - \delta$, is slightly below 1/2 (0.044), while the upper bound is slightly below 5/6 (0.77). Then most of the support of the distribution stands on the flat part of the payment rule, which results in a smaller risk premium. With the firm's cost being 1, the buyer's expected cost drops from 1.0045 when producers àre truthful to 1.0009 when producers are strategic.

Proof of Proposition 6

The equilibrium analysis is analogous to Maskin and Riley (1985): having a low (high) valuation corresponds here to being a truthful (strategic) producer. As in Maskin and Riley (1985), we have in equilibrium that truthful bidders make no profit and bid thus $(p^{NS}, \bar{q})$ and that the bidding strategy of strategic bidders involves no atoms but rather a mixed strategy where the supremum of the price bids p_{max} is equal p^{NS} (if $p_{max} < p^{NS}$, then strategic bidders submitting the price bid around p_{max} would have a strictly profitable deviation by bidding just below p^{NS}).

Let $G(.)$ denote the CDF of the strategic producers price bid and let $\Pi_S(p) := \max_{q_0 \geq 0} \Pi(p, q_0)$. In equilibrium, any price bid p made as part of a mixed strategy must generate the same expected payoff for a strategic producer. Therefore, for any price bid p in the support of G, the distribution G satisfies

$$[1 - \alpha + \alpha(1 - G(p))]^{N-1} \cdot [\Pi_S(p) - U(C)] = (1 - \alpha)^{N-1} \cdot [\Pi_S(p^{NS}) - U(C)]. \tag{7}$$

We obtain then that $G(p) = 1 - \frac{1-\alpha}{\alpha} \left(\sqrt[N-1]{\frac{\Pi_S(p^{NS})-U(C)}{\Pi_S(p)-U(C)}} - 1 \right)$. Let p_{min} denote the infimum of the prices bids (and we have thus that $G(p_{min}) = 0$). For any $\alpha \in (0, 1)$, we have $\Pi_S(p_{min}) - U(C) = (1-\alpha)^{N-1} \cdot [\Pi_S(p^{NS}) - U(C)] > 0 = \Pi_S(p^S) = U(C)$, and then that $\Pi_S(p_{min}) > \Pi_S(p^S)$ which further implies that $p_{min} > p^S$.

The payoff of a producer from an ex ante perspective (i.e. before knowing that he/she is strategic or truthful) is equal to $U(C) + \alpha \cdot [(1 - \alpha)^{N-1} \Pi_S(p^{NS})]$. The rent of all the producers is then equal to $N\alpha \cdot [(1 - \alpha)^{N-1} \Pi_S(p^{NS})]$.

We now consider the buyer's expected cost. It can be written as

$$(1 - \alpha)^N \cdot p^{NS} \cdot \mathbb{E}_f[R(q, \bar{q})] + \int_{p_{min}}^{p_{max}} p \cdot \mathbb{E}_f[R(q, q_0^*(p))] dK(p)$$

where $q_0^*(p) \in \operatorname{Arg\,max}_{q \geq 0} \Pi(p, q)$ and $K(p) := 1 - (1 - \alpha + \alpha(1 - G(p)))^N$ denotes the CDF of the price bid of the winning bidder. We obtain then that the buyer's expected cost is equal to

$$p^{NS} \cdot \mathbb{E}_f[R(q, \bar{q})] + N\alpha \int_{p_{min}}^{p_{max}} [p \cdot \mathbb{E}_f[R(q, q_0^*(p))] - p^{NS} \cdot \mathbb{E}_f[R(q, \bar{q})]] \cdot [1 - \alpha + \alpha(1 - G(p))]^{N-1} dG(p).$$

From (7) and since $U(C) = \mathbb{E}_f[U(p^{NS} \cdot R(q, \bar{q}))]$, the latter expression is equal to

$$p^{NS} \cdot \mathbb{E}_f[R(q, \bar{q})] + N\alpha(1-\alpha)^{N-1}[\Pi_S(p^{NS}) - U(C)] \int_{p_{min}}^{p_{max}} \frac{p \cdot \mathbb{E}_f[R(q, q_0^*(p))] - p^{NS} \cdot \mathbb{E}_f[R(q, \bar{q})]}{\mathbb{E}_f[U(p \cdot R(q, q_0^*(p)))] - \mathbb{E}_f[U(p^{NS} \cdot R(q, \bar{q}))]} \cdot dG(p).$$

Note that the term $p^{NS} \cdot \mathbb{E}_f[R(q, \bar{q})]$ corresponds to the buyer's expected cost with truthful producers.

Furthermore, since U is concave, then:

$$\begin{aligned} \mathbb{E}_f[U(p \cdot R(q, q_0^*(p)))] - U(p^{NS} \cdot \mathbb{E}_f[R(q, \bar{q})]) &\leq U(p \cdot \mathbb{E}_f[R(q, q_0^*(p))]) - U(p^{NS} \cdot \mathbb{E}_f[R(q, \bar{q})]) \\ &\leq U'(p^{NS} \cdot \mathbb{E}_f[R(q, \bar{q})]) \cdot [p \cdot \mathbb{E}_f[R(q, q_0^*(p))] - p^{NS} \cdot \mathbb{E}_f[R(q, \bar{q})]] \end{aligned}$$

and the inequalities stand as equalities if U is linear (or equivalently if producers are risk-neutral).

We conclude after noting G is a CDF on the support $[p_{min}, p_{max}]$ such that $\int_{p_{min}}^{p_{max}} dG(p) = 1$ and that $\Pi_S(p^{NS}) \geq U(C)$ Q.E.D.

Q.E.D.

Appendix 3: Useful properties with CRRA utility functions

For a given payment rule and a given utility function, let us use the notation $S_f(p) := \max_{q_0 \geq 0} \mathbb{E}_f[U(p \cdot R(q, q_0))]$. For a set $S \subset \mathbb{R}$ and $\lambda \in \mathbb{R}$, we let $\lambda \times S := \{x \in \mathbb{R} | \exists s \in S \quad \text{such that} \quad \lambda \cdot s = x\}$.

For a given production distribution f (with the corresponding CDF F) and $\lambda > 0$, we let denote $f_\lambda(.)$ the pdf (with the corresponding CDF F_λ) such that $F_\lambda(q) = F(\lambda \cdot q)$ for any $q \in \mathbb{R}_+$. The distribution f_λ corresponds to a homothetic transformation of the distribution f. The mean of f_λ is then equal to $\frac{\bar{q}}{\lambda}$.

Lemma 7. *Suppose that the utility function U is a CRRA utility function and consider a production distribution f on $\mathbb{R}_+$.*

Then the set $S_f(p)$ does not depend on p for any $p > 0$ and the ratios $\frac{p^{NS} \cdot \mathbb{E}_{f_\lambda}[R(q,\bar{q})]}{C}$, $\frac{p^{S} \cdot \mathbb{E}_{f_\lambda}[R(q,q_0^*(p^S))]}{C}$ and $\frac{p^{NS} \cdot \mathbb{E}_{f_\lambda}[R(q,q_0^*(p^{NS}))]}{C}$ do not depend on C.

If the payment rule R is homogeneous of degree 1, then $S_{f_\lambda}(p) = \frac{1}{\lambda} \times S_f(p)$ for any $p, \lambda > 0$ and the ratios $\frac{p^{NS} \cdot \mathbb{E}_{f_\lambda}[R(q,\bar{q})]}{C}$, $\frac{p^{S} \cdot \mathbb{E}_{f_\lambda}[R(q,q_0^*(p^S))]}{C}$ and $\frac{p^{NS} \cdot \mathbb{E}_{f_\lambda}[R(q,q_0^*(p^{NS}))]}{C}$ do not depend on λ.

Proof of Lemma 7

If U is a CRRA utility function, then $U(p \cdot R(q, q_0)) = p^{1-\gamma} \cdot U(R(q, q_0))$. For any $p > 0$, we have then $S_f(p) = S_f(1)$.

Let us now consider the ratios between the cost for the buyer and the cost for the producer under our various bidding paradigms. From (2) and (3) with a CRRA utility function, the equilibrium prices p^{NS} and p^S are such that the ratios $\frac{p^{NS}}{C}$ and $\frac{p^S}{C}$ do not depend on C. Since $S_f(p)$ does not depend on C, we have that $\mathbb{E}_f[R(q, \bar{q})]$ and $\mathbb{E}_f[R(q, q_0^*(p^S)]$ and $\mathbb{E}_f[R(q, q_0^*(p^{NS})]$ do not depend on C and finally that the three ratios in Lemma 7 do not depend on C.

Consider now that R is homogeneous of degree 1. We have then $\mathbb{E}_{f_\lambda}[U(p \cdot R(q, q_0))] = \int_0^\infty U(p \cdot R(q, q_0)) f_\lambda(q) dq = \int_0^\infty U(p \cdot R(q, q_0)) f(\lambda q) d(\lambda q) = \int_0^\infty U(p \cdot R(\frac{q}{\lambda}, q_0)) f(q) dq = \mathbb{E}_f[U(p \cdot R(\frac{q}{\lambda}, q_0))] = \frac{1}{\lambda^{1-\gamma}} \cdot \mathbb{E}_f[U(p \cdot R(q, \lambda \cdot q_0))]$ where the last equality uses the homogeneity of degree one assumption. Since $\mathbb{E}_{f_lambda}[U(p \cdot R(q, q_0))] = \frac{1}{\lambda^{1-\gamma}} \cdot \mathbb{E}_f[U(p \cdot R(q, \lambda \cdot q_0))]$, we obtain then than $S_f(p) = \lambda \times S_{f_\lambda}(p)$.

If R is homogeneous of degree 1 and using that U is a CRRA utility function, let us show that the equilibrium prices p^{NS} and p^S are linear in λ. Below we explicit in our notation the dependence in λ and in particular use the notation $\bar{q}_\lambda$, $q^*_{0,\lambda}(p)$, p^{NS}_λ and p^S_λ. Note that we have $p^{NS} = p^{NS}_1$ and $p^S = p^S_1$. We have also $\bar{q}_\lambda = \frac{\bar{q}}{\lambda}$ and $q^*_{0,\lambda}(p) = \frac{q^*_0(p)}{\lambda}$ if the sets $S_f(p)$ is a singleton (if $S_f(p)$ is not a singleton, the selection does not play any role and without loss of generality, we can thus pick a selection such that $q^*_{0,\lambda}(p) = \frac{q^*_0(p)}{\lambda}$).

From (2), we have that for any λ:

$$\mathbb{E}_f[U(p^{NS} \cdot R(q,\bar{q}))] = U(C) = \mathbb{E}_{f_\lambda}[U(p^{NS}_\lambda \cdot R(q,\bar{q}_\lambda))] = \mathbb{E}_f[U(p^{NS}_\lambda \cdot R(\frac{q}{\lambda},\frac{\bar{q}}{\lambda}))] = \mathbb{E}_f[U(\frac{p^{NS}_\lambda}{\lambda} \cdot R(q,\bar{q}))].$$

The equality $\mathbb{E}_f[U(p^{NS} \cdot R(q,\bar{q}))] = \mathbb{E}_f[U(\frac{p^{NS}_\lambda}{\lambda} \cdot R(q,\bar{q}))]$ implies then that $p^{NS}_\lambda = \lambda \cdot p^{NS}$.

Similarly, from (3), we have that for any λ:

$$U(C) = \mathbb{E}_{f_\lambda}[U(p^S_\lambda \cdot R(q, q^*_{0,\lambda}(p^S_\lambda)))] = \mathbb{E}_f[U(p^S_\lambda \cdot R(\frac{q}{\lambda}, \frac{q^*_0(p^S_\lambda)}{\lambda}))] = \mathbb{E}_f[U(\frac{p^S_\lambda}{\lambda} \cdot R(q, q^*_0(p^S)))]$$

and

$$U(C) = \mathbb{E}_f[U(p^S \cdot R(q, q^*_0(p^S)))] = \mathbb{E}_f[U(p^S \cdot R(q, q^*_0(p^S_{lambda})))]$$

where the last equality comes from the fact that the optimal report $q^*_0(p)$ does not depend on p. Finally, this implies that $p^S_\lambda = \lambda \cdot p^S$.

We conclude the proof by noting that the buyer's expected cost can be written then in the tree bidding paradigms:

- $p^{NS}_\lambda \cdot \mathbb{E}_{f_\lambda}[R(q,\bar{q}_\lambda)] = p^{NS}_\lambda \cdot \mathbb{E}_f[R(\frac{q}{\lambda},\frac{\bar{q}}{\lambda})] = p^{NS} \cdot \mathbb{E}_f[R(q,\bar{q})]$,
- $p^S_\lambda \cdot \mathbb{E}_{f_\lambda}[R(q, q^*_{0,\lambda}(p^S_\lambda))] = p^S_\lambda \cdot \mathbb{E}_f[R(\frac{q}{\lambda}, \frac{q^*_0(p^S_\lambda)}{\lambda})] = p^S \cdot \mathbb{E}_f[R(q, q^*_0(p^S_\lambda))] = p^S \cdot \mathbb{E}_f[R(q, q^*_0(p^S))]$ (the last equality results from the fact that $q^*_0(p)$ is independent of p),
- $p^{NS}_\lambda \cdot \mathbb{E}_{f_\lambda}[R(q, q^*_{0,\lambda}(p^{NS}_\lambda))] = p^{NS}_\lambda \cdot \mathbb{E}_f[R(\frac{q}{\lambda}, \frac{q^*_0(p^{NS}_\lambda)}{\lambda})] = p^{NS} \cdot \mathbb{E}_f[R(q, q^*_0(p^{NS}_\lambda))] = p^{NS} \cdot \mathbb{E}_f[R(q, q^*_0(p^{NS}))]$ (the last equality results from the fact that $q^*_0(p)$ is independent of p).

Q.E.D.

Remark: under the multi-year contracts used in France and in presence of operating costs, we could extend Lemma 7.

Lemma 8. *Suppose that the utility function U is a CRRA utility function and consider a production distribution f on $\mathbb{R}_+$.*

Then the set $S_f(p)$ does not depend on p for any $p > 0$ and the ratios $\frac{p^{NS} \cdot \mathbb{E}_{f_\lambda}[R(q,\bar{q})]}{C}$, $\frac{p^S \cdot \mathbb{E}_{f_\lambda}[R(q,q^*_0(p^S))]}{C}$ and $\frac{p^{NS} \cdot \mathbb{E}_{f_\lambda}[R(q,q^*_0(p^{NS}))]}{C}$ do not depend on C.

If the payment rule R is homogeneous of degree 1, then $S_f(p) = \lambda \times S_{f_\lambda}(p)$ for any $p, \lambda > 0$ and the ratios $\frac{p^{NS} \cdot \mathbb{E}_{f_\lambda}[R(q, \bar{q})]}{C}$, $\frac{p^{S} \cdot \mathbb{E}_{f_\lambda}[R(q, q_0^*(p^S))]}{C}$ and $\frac{p^{NS} \cdot \mathbb{E}_{f_\lambda}[R(q, q_0^*(p^{NS}))]}{C}$ do not depend on λ.

Appendix 4: Additional Results

Table 2: Buyers Expected Cost ratio to producer's cost - Investment cost as initial wealth

Site	γ	Linear Contract	Non-strategic bidders	All strategic bidders	One strategic bidder
Courseulles	0	1.000	1.000	1.000	1.035
	1	1.003	1.001	1.004	1.036
	3	1.009	1.003	1.010	1.038
	5	1.016	1.006	1.016	1.038
	10	1.033	1.014	1.028	1.038
Fécamp	0	1.000	1.000	1.000	1.036
	1	1.003	1.001	1.003	1.037
	3	1.009	1.003	1.010	1.038
	5	1.015	1.006	1.016	1.039
	10	1.032	1.013	1.028	1.039
Le Tréport	0	1.000	1.000	1.000	1.033
	1	1.003	1.001	1.004	1.034
	3	1.010	1.004	1.011	1.036
	5	1.017	1.008	1.018	1.038
	10	1.037	1.019	1.033	1.040
Saint-Nazaire	0	1.000	1.000	1.000	1.036
	1	1.003	1.001	1.004	1.037
	3	1.009	1.003	1.010	1.038
	5	1.016	1.006	1.016	1.039
	10	1.033	1.014	1.028	1.039
Noirmoutier	0	1.000	1.000	1.000	1.035
	1	1.004	1.001	1.004	1.036
	3	1.011	1.004	1.012	1.038
	5	1.019	1.007	1.019	1.039
	10	1.039	1.019	1.032	1.038

Table 3: Buyers Expected Cost ratio to producer's cost - Total Net Present Cost as initial wealth

Site	γ	Linear Contract	Non-strategic bidders	All strategic bidders	One strategic bidder
Courseulles	0	1.000	1.000	1.000	1.035
	1	1.002	1.001	1.002	1.036
	3	1.006	1.002	1.007	1.037
	5	1.010	1.004	1.011	1.038
	10	1.021	1.008	1.020	1.038
Fécamp	0	1.000	1.000	1.000	1.036
	1	1.002	1.001	1.002	1.036
	3	1.006	1.002	1.007	1.037
	5	1.010	1.004	1.011	1.038
	10	1.021	1.008	1.020	1.039
Le Tréport	0	1.000	1.000	1.000	1.033
	1	1.002	1.001	1.002	1.033
	3	1.006	1.003	1.007	1.035
	5	1.011	1.004	1.011	1.036
	10	1.022	1.009	1.021	1.038
Saint-Nazaire	0	1.000	1.000	1.000	1.037
	1	1.002	1.001	1.003	1.036
	3	1.007	1.003	1.008	1.038
	5	1.012	1.004	1.012	1.039
	10	1.024	1.009	1.023	1.039
Noirmoutier	0	1.000	1.000	1.000	1.035
	1	1.002	1.001	1.002	1.035
	3	1.006	1.002	1.007	1.037
	5	1.011	1.004	1.011	1.038
	10	1.022	1.009	1.021	1.038