Electric Bus Fleet Mileage Maximization with a Given Schedule using Integer Programming

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Overview

Local public transport in Krefeld is provided by SWK MOBIL GmbH, a municipal company, using buses and trams. SWK is discussing the purchase of electric buses. Economic analyses conclude that electric buses are currently uneconomical compared to diesel buses, although mileage-related costs are lower. According to former studies, higher capital expenditures cannot be compensated. [Hondius14, Seeliger16]

In previous works, a heuristic was used to check under which conditions electric standard buses (12 meters) and electric articulated buses (18 meters) can be operated cost-efficient in Krefeld [Gennat20]. Hereby, cost-efficient means same or lower total costs of ownership compared to diesel-powered buses. For this purpose, it was investigated how a combination of given cycles can be used to maximize electrical operating kilometers in order to benefit from lower mileage-dependent costs. A cycle starts with depot exit and ends with depot entrance. Also, it consists of several trips that are driven by a single bus. [Schnieder15, Huisman04]

In this contribution optimal mileage of electrical bus fleet is computed by integer linear programming. This result and its computing time are compared with the heuristic from [Gennat20].

Scheduling software used by SWK organizes and stores cycle information according to standard interface ÖPNV-Datenmodell 5.0 [VDV452]. Cycle data are generated for all different day types. Travel times, stops, routes and line numbers are available for each cycle. As an example Figure 1 shows 18-meter bus cycles on a Monday.

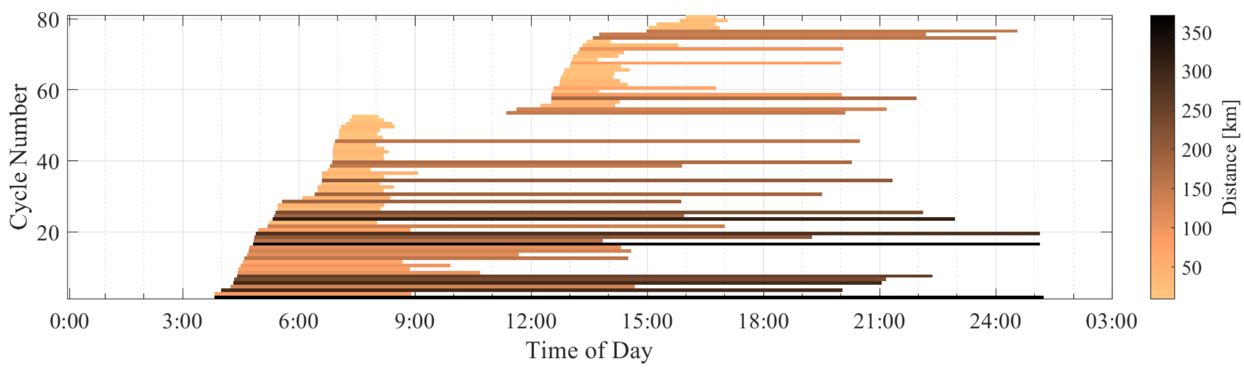

Figure 1: 18-Meter Bus Cycles on a Monday

Problem Formulation and Constraints

First of all, cycles with ten electric articulated buses (n=10) are investigated. Besides, only one single day will be considered at first. Outside temperature curves and thus energy requirements per cycle and number of cycles to be made are day-dependent. Since one day is considered in each case, cycle selection must be carried out anew for each day of the year with respective outside temperatures and day-specific schedules.

Table 1: Cycles of Articulated Buses with Respective Energy Requirements and Routes (Example of one Day)

i	t_{start} [min]	t_{end} [min]	s [km]	W [kWh]
1	49	1333	371.490	739.3678
2	50	353	87.808	174.6653
3	60	1022	291.475	570.9325
…	…	…	…	…

The use of electric buses $b \in B$ is examined, whereby the number of buses is noted with ($|B| = 10$). Day of operation is discretized by minute with $t \in \{1, \ldots, 1440\}$ and starts at 03:00 a.m. Existing cycle $u \in U$ with a total number ($|U| = 81$) may not be changed. A round begins with a depot exit and ends with a depot entrance. It includes empty runs, turning times and transport times. [Schnieder15].

Fig. 1 shows an example of an articulated bus cycle on a Monday as a path-time diagram. In order to avoid time overlaps between days, the beginning of an operating day is set to 3:00 a.m.

Key figures of rounds are times of exit t_{start} and re-entry t_{end}into depot, electrical energy requirement W and distance s. Individual cycles u can be described by the structure $u_i = (t_{start,i}, t_{end,i}, W_i, s_i)$.

For mathematical representation, decision matrix $x \in [0,1]^{u \cdot b}$ is introduced, which is shown below as an example for three electric buses and four cycles.

$$x = \begin{array}{cc} \begin{array}{ccc} b=1 & b=2 & b=3 \end{array} & \\ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} & \begin{array}{c} u=1 \\ u=2 \\ u=3 \\ u=4 \end{array} \end{array} \tag{1}$$

$$|e_b| = \sum_u x_{u,b} = \begin{pmatrix} 2 & 1 & 1 \end{pmatrix} \tag{2}$$

A bus can run several missions $e \in E$ one after other on one day. As an example, bus b runs cycles 1,4 and 84 in a row.

$$E_b = \begin{pmatrix} E_{b,1} \\ E_{b,4} \\ E_{b,84} \end{pmatrix} = \begin{pmatrix} u_1 \\ u_4 \\ u_{84} \end{pmatrix} \tag{3}$$

Heuristic for Determining a Lower Limit

Due to time overlaps, buses cannot run an unlimited number of rounds on one day in succession. With a given timetable and cycle plan, a maximum of $n_{e,max} = 3$ bus deployments per day are possible on Mondays. Total number of all possible combinations is calculated with

$$n_{combin} = \prod_{j=1}^{|B|} \sum_{i=1}^{n_{e,max}} \binom{|U| - j + 1}{i}. \tag{4}$$

Equation (4) returns more than $\mathrm{n_{combin}} = 10^{48}$ combinations for ten articulated buses. Thus, combinatorial methods cannot solve the optimization task in a sufficiently short time. For this reason, a heuristic is being developed that provides a lower limit for the maximum of electrically driven kilometers to be determined in a short time. For a single electric bus, the actual maximum and not only a lower limit can be determined with the heuristic introduced here.

Considering a whole year lower limits of the maximum of electrical operating kilometers are determined under consideration of climatic conditions (see: [VDV236]), day-specific timetables and depending on respective battery specifications for each individual day of year.

Combination of Suitable Cycles

A heuristic was developed to find combination of rounds with the highest mileage per day. This heuristic is shown in Figure 2 as a Nassi-Shneiderman diagram.

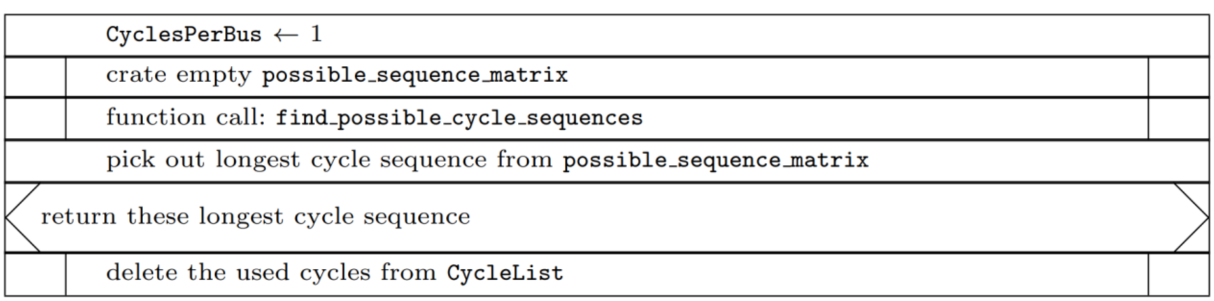

Figure 2: Function: longest_cycles_sequence

At first, the heuristic uses the find_possible_cycle_sequences function Figure 3 to determine all possible cycle combinations for each day and bus. This function checks whether presented constraints are met. It also determines possible loading time between cycles. In a second step, the combination with highest electrical mileage is selected.

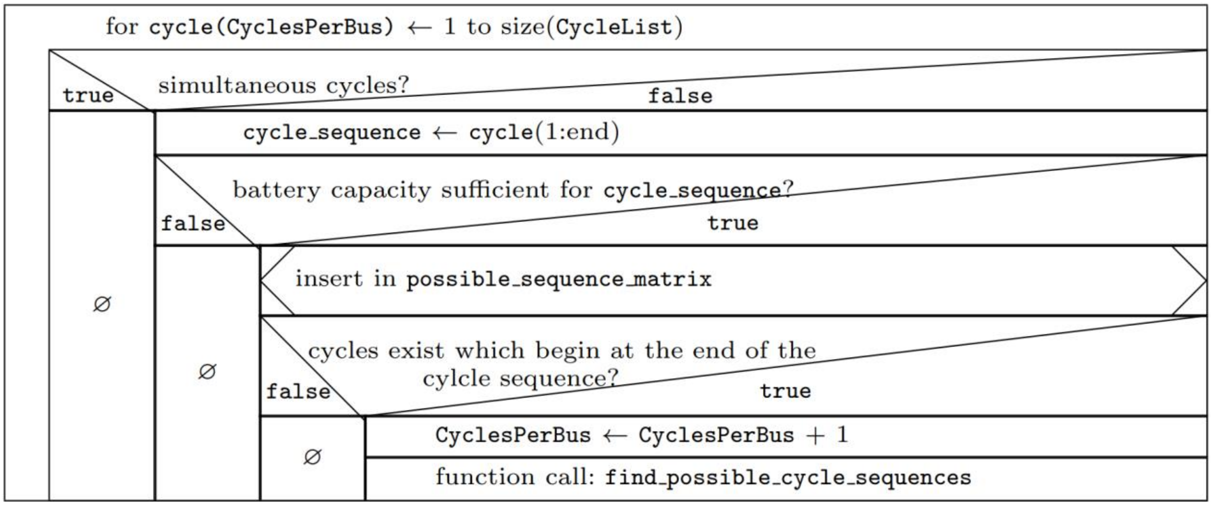

Figure 3: Funktion: find_possible_cycle_sequences

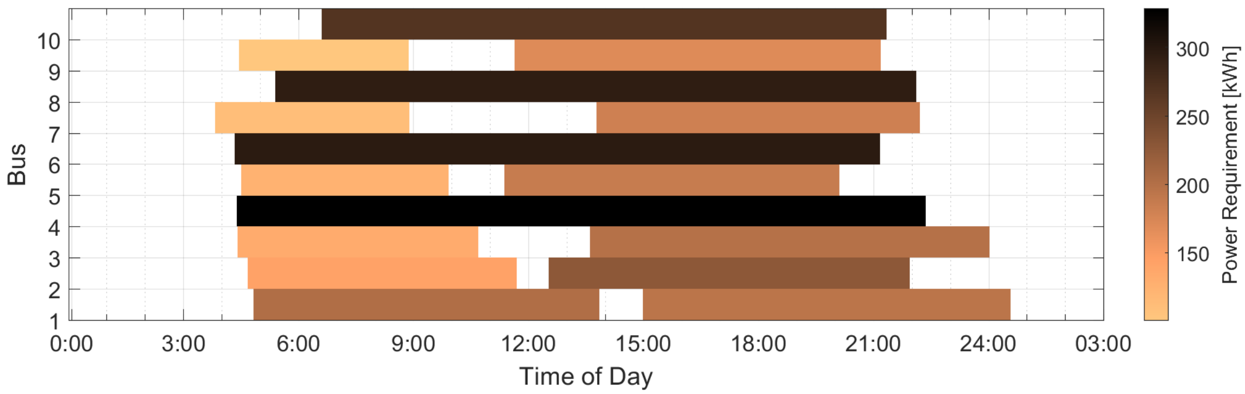

Figure 4: Maximization of Electric Daily Mileage by Combining Cycles

A possible result is shown in Figure 4 for combination of most suitable articulated bus cycles as an example for one day.

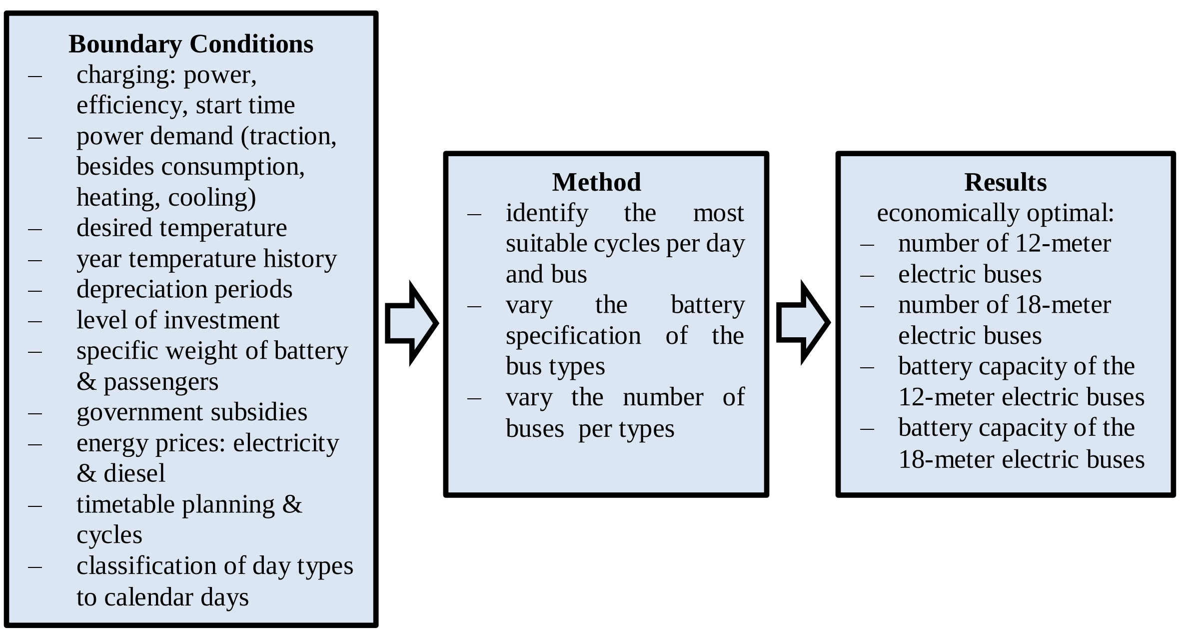

Figure 5: Model Parameters

Extension of the Heuristic for Economic Analysis

So far, mileage of each day and bus has been maximized. An extension is necessary to cover an entire year. Central parameters of the extended heuristic are shown in Figure 5.

For investigation of economic efficiency under given boundary conditions, the represented heuristic was extended with a superordinate function (see Figure 6). This extended heuristic determines minimum costs and maximum electric mileage for an electric bus fleet.

For a single bus, this procedure calculates the best solution. For several buses an optimal solution cannot be guaranteed, because heuristics cannot guarantee to find a global optimum (see [Gennat20]).

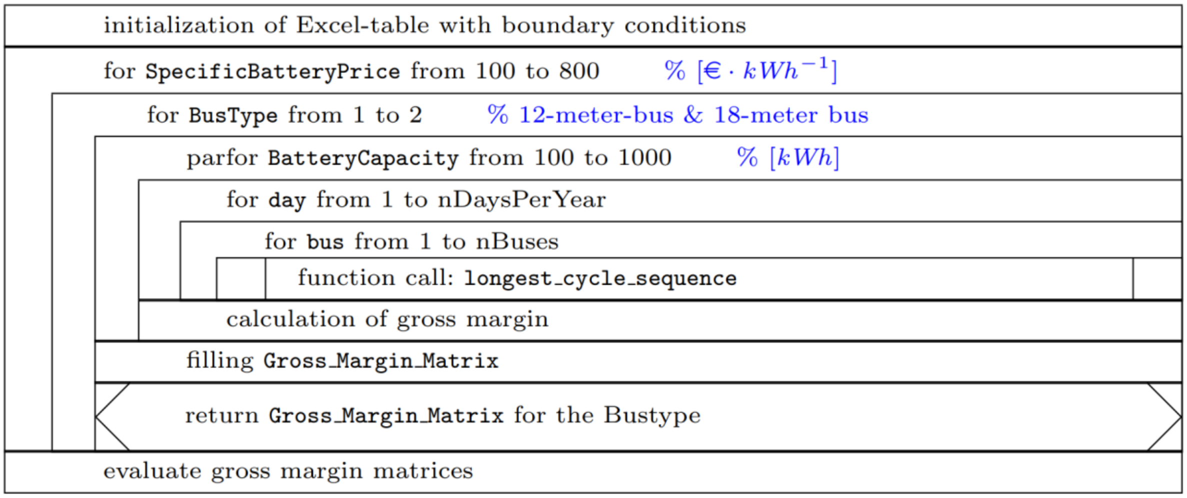

Figure 6: Structure of the Extended Heuristic

Possible daily mileage per bus decreases with number of buses. During summer holidays, a maximum of 16 articulated electric buses can be operated on weekdays, and only five on Sundays, depending on weather conditions. An economic optimum is therefore expected with a small number of electric buses.

Results of Maximizing Electrical Operating Kilometers

Even with maximization of electrically driven kilometers, an economically reasonable acquisition of electric buses is not possible at market prices in 2019, even with an assumed subsidy rate of 40 percent. Figure 7 shows the gross margin as a function of the specific battery price and battery capacity for five articulated electric buses. A break-even is reached when specific prices for traction batteries fall below 480 Euros per kilowatt hour. At these battery prices of around 480 Euros per kilowatt hour, the best economic result is achieved with three articulated electric buses. The result differs only slightly from that achieved with five electric buses.

For an economical use of electric solo buses, a battery price of 450 Euros per kilowatt hour must not be exceeded. There are more articulated bus cycles in Krefeld than solo bus cycles, so there is greater potential for combining articulated bus cycles.

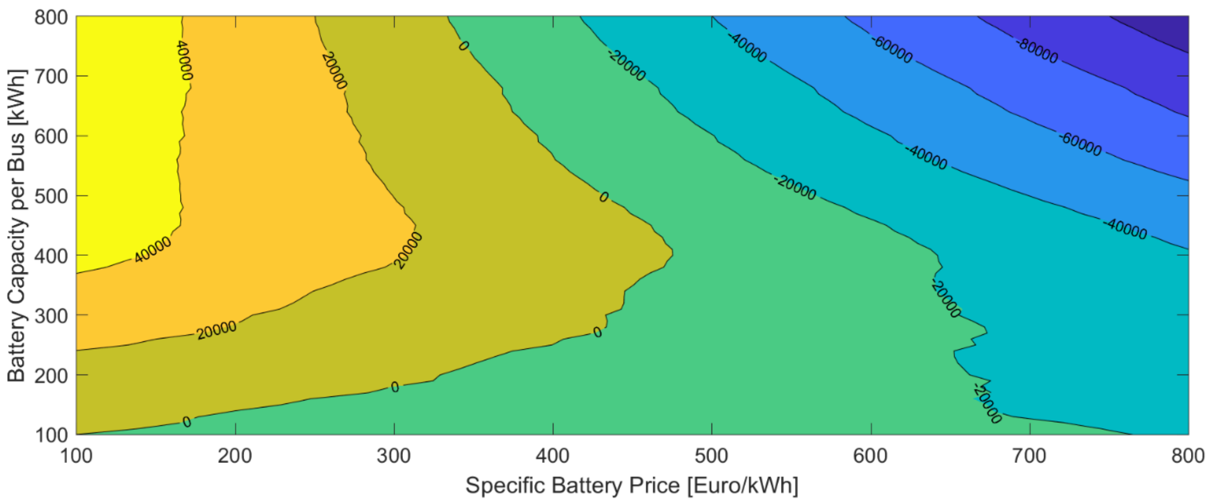

Figure 7: Gross Margins for a Procurement of Five Electric Articulated Buses

Integer Linear Programming to Determine the Guaranteed Maximum

To compare computing times and resolved kilometers, the problem is formulated as an integer linear programming problem in the following. Referring to traveling salesman problems [Briskorn19], a similar approach is used. This problem is solved by commercial solver Gurobi [Gurobi20] and its results are compared with those of the heuristic from [Gennat20]. The objective function is formulated with

$$J = \max_x c^T x, \qquad A \in [0,1]^{m \times n}, b \in \mathbb{R}^m, c \in \mathbb{R}^n, x \in [0,1]^n, \qquad \text{subject to } A \cdot x \leq b \tag{5}$$

whereby n represents the number of buses times the number of cycles. Binary decision vector is noted as x with length $|x| = |U| \cdot |B|$, and c contains the lengths of individual cycles for all electric buses. Inequality constraints of this integer linear program can be passed to the solver as A and b.

Here, for each bus, components of binary decision vector, whether or not a cycle is to be driven, are specified in the form

$$x^T = \begin{array}{ccc} b=1 & b=2 & b=3 \\ (x_{u=1}\, x_{u=2} \cdots x_{u=|U|} & x_{u=1}\, x_{u=2} \cdots x_{u=|U|} & x_{u=1}\, x_{u=2} \cdots x_{u=|U|}) \end{array} \tag{6}$$

are connected. Vector c ($|c| = |x|$) contains the route lengths of individual cycles for all electric buses and is noted with

$$c^T = \begin{array}{ccc} b=1 & b=2 & b=3 \\ (s_{u=1}\, s_{u=2} \cdots s_{u=|U|} & s_{u=1}\, s_{u=2} \cdots s_{u=|U|} & s_{u=1}\, s_{u=2} \cdots s_{u=|U|}). \end{array} \tag{7}$$

A maximum of one electric bus may drive on cycle u. This constraint is specified with

$$\sum_{b=1}^{|B|} x_{(b-1)\,|U|+u} \leq 1 \quad \forall\, u \in U. \tag{8}$$

If the sum for a cycle u is zero, this cycle is driven by a diesel bus. For the IP, this constraint is set up for three buses and four cycles with

$$A_1 = \begin{array}{c} \begin{array}{ccc} b=1 & b=2 & b=3 \end{array} \\ \begin{pmatrix} 1&0&0&0&1&0&0&0&1&0&0&0 \\ 0&1&0&0&0&1&0&0&0&1&0&0 \\ 0&0&1&0&0&0&1&0&0&0&1&0 \\ 0&0&0&1&0&0&0&1&0&0&0&1 \end{pmatrix} \end{array} \quad d_1 = \begin{pmatrix} 1\\1\\1\\1 \end{pmatrix}. \tag{9}$$

A further condition is that a bus cannot make several cycles simultaneously. Every bus with more than one cycle trip starts at the earliest after arrival plus time Δt. During this time the bus is shunted and a driver change can take place.

$$t_{start}, e_{i,b} \geq \quad t_{end}, e_{i-1,b} + \Delta t \quad \forall\, i \in [2, |e_b|] \quad \forall\, b \in B. \tag{10}$$

This constraint with three buses is implemented by

$$A_2 = \begin{array}{c} \begin{array}{ccc} b=1 & b=2 & b=3 \end{array} \\ \begin{pmatrix} 1&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&1&0 \end{pmatrix} \end{array} \quad d_2 = \begin{pmatrix} 1\\1\\1 \end{pmatrix}. \tag{11}$$

In this example, cycle 1 and 3 overlap in time.

In the urban area of Krefeld, there are no large differences in altitude, so route-specific traction energy requirements $W_{traction}$ can be assumed to be constant. Time-specific power requirements consist of besides consumption $P_{besides}$ and cooling $P_{cooling}$ or heating $P_{heating}$.

In the time between two operations, an electric bus can be charged in the depot with the charging current $P_{charging}$. The usable energy capacity of traction battery C_{Bat} is the same for all electric buses. The charge level of traction batteries of individual trolleybuses is described by function $SoC_b(t, E)$. For each time unit considered, SoC_b is terminated with

$$SoC_{b,t} = \quad SoC_{b,t-1} + \frac{(P_{charging} - P_{besides} - P_{cooling} - P_{heating}) \cdot 60s - W_{traction} \cdot s_t}{C_{Bat}} \tag{12}$$

where s_t is the distance covered in the minute before t.

Traction batteries must never be discharged to the extent that reserve capacity must be used. Batteries can be charged to a maximum of 100%. With reserve capacity δ, the energy content of which is not to be taken into account in timetable, can now be set as a further condition with

$$\delta \leq \quad SoC_{b,t} \leq 1 \quad \forall\, b \in B, \; t \in \{1, \dots, 1440\}. \tag{13}$$

This constraint is noted with

$$\begin{array}{cccc} & b=1 & b=2 & b=3 \\ A_3 = & \multicolumn{3}{c}{\begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}} \end{array}, \quad d_3 = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}, \tag{14}$$

where in this example combination of cycles 1, 2 and 4 are excluded for all buses. The right side of the inequality condition is for exclusion of a triple combination $d_{3,1} = 2$ to not prevent the combination of cycles 1 and 4 in this example. Other non-combinable cycle pairs are defined in the same way and appended to A at the bottom.

Since it is possible to determine the maximum number of rounds per bus and day ($n_{e,max}$), solution space can be defined with

$$\begin{array}{cccc} & b=1 & b=2 & b=3 \\ A_4 = & \multicolumn{3}{c}{\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}} \end{array}, \quad d_4 = \begin{pmatrix} \mathrm{n_{e,max}} \\ \mathrm{n_{e,max}} \\ \mathrm{n_{e,max}} \end{pmatrix}. \tag{15}$$

Matrices A_1 to A_4 are lined up vertically and passed to the solver as matrix A. The same applies to vector d.

Comparison and Evaluation

Results of the heuristic, which only provides lower bounds, are compared with two IP solvers Intlinprog [Intlinprog20] from Matlab's optimization Toolbox and Gurobi [Gurobi20]. As an example, a Monday is selected on which $|U| = 80$ different cycles can be assigned to $|B| = 10$ buses. Battery size is assumed to be $C_{Bat} = 250\ kWh$ and charging power in the depot is assumed to be $P_{charging} = 80\ kW$. An additional time of $\Delta t = 4\ min$ is given for bus shunting and driver change.

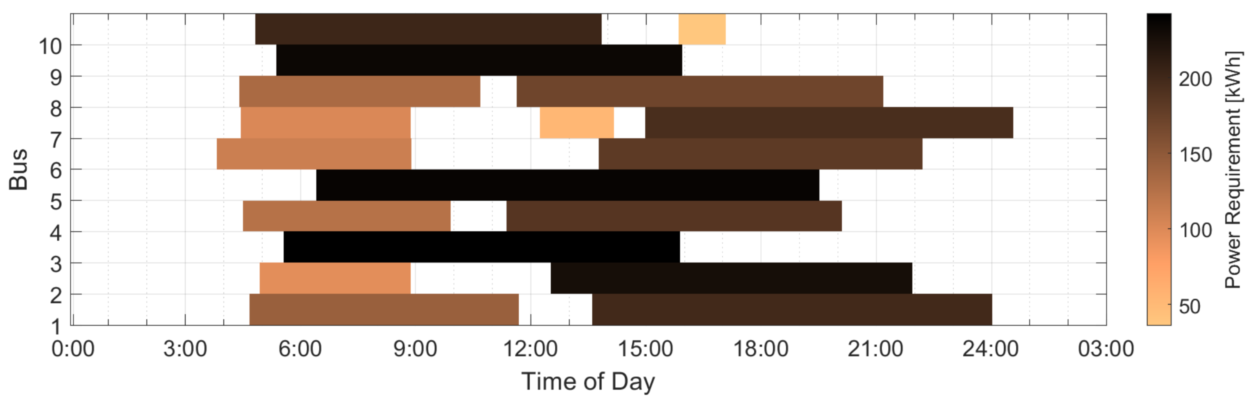

Figure 8: Combination of Cycles with Gurobi

Results of Intlinprog and Gurobi are the same and provide the optimal result. An exemplary result for one day is shown in Figure 8. Total distances travelled electrically per day are shown in Table 2.

Table 2: Results of the Solvers

Solver	Determined Total Distance [km]	Required Computing Time [s]
Heuristic	2271.70	0.05
Gurobi	2277.28	1.48
Intlinprog	2277.28	49,587.23

The lower limit of the heuristic deviates from optimal result by about 5.6 kilometers. This is a deviation of 0.25 percent. Due to the high computing time, Intlinprog will not be considered in the following. Figure 9 compares the heuristic and the linear solver for different battery capacities from 100 to 800 kWh in 25 kWh steps for ten articulated buses. For most battery capacities the heuristic provides the correct result. The maximum deviation is 5.6 kilometers.

Figure 10 compares the heuristic and the solver for 1 to 25 electric buses with a battery capacity of 250 kilowatt hours. In these comparisons, only the required traction energy is considered, energy requirements for air conditioning, heating, ventilation, lighting, etc. are not considered here.

The solver or algorithm is called several million times by the main routine, which iterates through all days, bus types, battery sizes and prices (see Figure 6). Due to the short computing time of one run of the heuristic, total computing time with parallelization on a total of twelve cores of two X5690 CPUs and about 74 gigabytes of RAM is about two days. With the linear solver, years would be needed. Therefore the inaccuracy of about 0.25 percent of the heuristic is accepted in favor of the computing time.

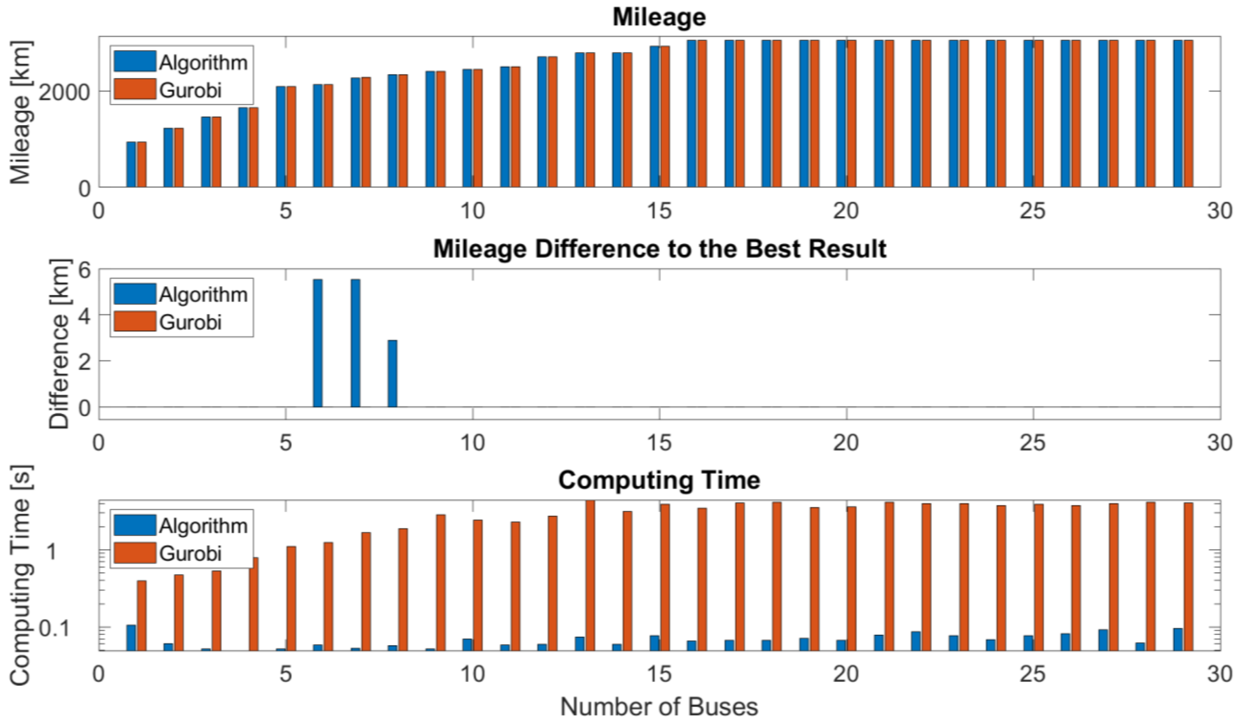

Figure 9: Comparison of Solver and Algorithm with Different Battery Capacities

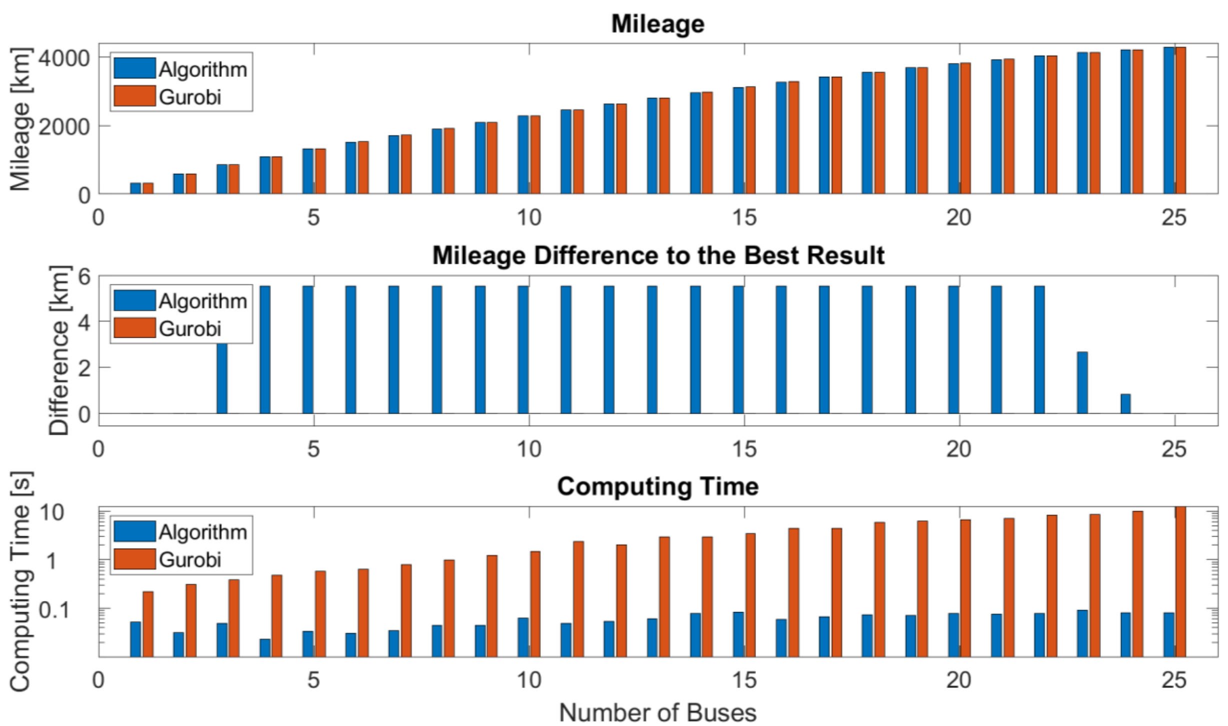

Figure 10: Comparison of Solver and Algorithm With Different Number of Electric Buses (250 kWh)

Conclusions and Outlook

With the help of integer linear programming, optimal cycles allocations can be determined. The quality of the heuristic algorithm can be evaluated by comparing it with the optimal result. However, the computing time of the integer linear programming is several orders of magnitude higher than that of the algorithm. For this reason, the algorithm is used for this computationally intensive task. Based on the lower bounds determined by the heuristic, economic efficiency of a procurement of electric buses for Krefeld can be re-evaluated. Prices of traction batteries for articulated buses must drop below 480 Euros per kilowatt hour in order to operate an electric bus fleet of three electric buses economically.

With three electric buses, a total of 243,000 km per year can be covered electrically, which is significantly above average values of about 60,000 km per bus and year. Using emission factors of German electricity mix (474 g CO_2/kWh [UBA19]) and diesel (2,650 g CO_2/litre [WD19]), 72 tonnes of CO_2 per year can be avoided. This value will improve continuously as CO_2 emission factors in the German electricity mix continue to fall.

Although prices for battery cells have fallen [Ahlswede19], prices for battery packs are between 600 and 1000 Euros per kilowatt hour in recent years [Kunith17, Lehner15, Seeliger16]. An economic operation of electric buses is therefore not yet possible in Krefeld. It is assumed that these results can be transferred to cities with a similar number of cycles. Therefore, under current conditions, electric buses cannot yet be used economically in such cities either (see [Schwarze15, van Liemt18]).

If it is politically desired to internalize external effects such as CO_2 or noise emissions, framework conditions must change. These could be a taxation of fossil fuels or higher financial support of electric buses. These changes would have a high impact on the initial conditions of this study.

Neither actual timetables nor cycles were changed in this study. Only existing cycles were assigned to electric buses on a daily basis. In further investigations further potentials can be identified by breaking up and optimizing cycles.

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