A partial-equilibrium model of the electricity market

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Abstract

We present a model of the electricity market which has a simple connection to the rest of the economy, due to a quasi-linear utility function. A key stochastic parameter shifts electricity demand between points of time. The correlation between these demand shifts and the variation of supply from variable renewable electricity (VRE) sources is crucial for the equilibrium price of electricity. Based on these equilibria we compute the value factors of VREs, as well as producer and consumer surpluses. An emerging outcome of the model analysis is that value factors of VREs decrease as their penetration gets higher, which is also commonly found empirically [up to a point].

Keywords: Electricity market equilibrium, demand variability, intermittent supply, economic evaluation.

JEL classification: Q41, Q42.

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1 Introduction

The power supply from Variable Renewable Electricity (VRE) sources, like wind turbines and solar panels, is to a large extent uncontrollable and unpredictable due to variations in wind speed and solar radiation. By contrast, generators like hydro and nuclear power are 'dispatchable', which means that their generation can be turned on and off when called on by the system operator.[1]

While the VREs have low environmental impacts, the intermittence is likely detrimental to their economic value. This is essentially determined by the electricity prices that prevail at times when the VREs are supplying power in large quantities.[2] If, for example, the supply of VREs is high when demand is moderate, the price will be low and thus the value of VRE will be limited. Similarly, the consumer and producer surpluses are affected by an increased penetration of VRE, through its effect on the price.

We formulate a simple microeconomic model[3], with the electricity market at its centre. Assuming a quasi-linear utility function, the electricity market is thus connected to the rest of the economy only through the income that is left over for other goods. We define the residual demand as the difference between total demand and the stochastically produced VRE. This residual demand is directed towards the dispatchable electricity; the VRE has lower marginal costs and thus are delivering first. The dispatchable part of supply is constituted by a merit order of horizontal linear segments, reflecting different marginal costs of different types of generators. The equilibrium price is determined by the intersection between the residual demand and the dispatchable supply curve.

[1]In practice there are two technical constraints: the minimum up and down time; and the max ramp up and down time. See Kirschen and Strbac (2018).

[2]Joskow (2011) provides a lucid discussion about this. The uncertainty of VRE supply makes it necessary to build additional capacity in base supply (or in storing). This is an additional cost to the power system, caused by the increase of VRE; see e.g. Llobet and Padilla (2018) and Holmberg and Ritz (2019).

[3]The benefit of this model can be expressed as follows: It takes a middle ground between small 'static' models like Joskow and Tirole (2007) and Llobet and Padilla (2018) on the one hand, and large numerical models like Gowrisankaran, Reynolds and Samano (2016) and Hirth (2013) (see Hirth (2015) for a survey of such models). It provides a higher transparency than larger and more realistic models, while at the same time laying out more details about all sectors of the industry, unlike the simpler models which tend to focus on marginal (cut-off) conditions.

We use the model to numerically analyze the fictive equilibria of the electricity market at different points of time, with a special focus on the variability of supply from VREs. The equilibria are then used to examine the welfare effects of different scenarios, including the value factor of VRE.

Since electricity can be considered as a time-heterogeneous good, the focus of our paper is on the variability of supply and demand between different points of time. On the supply side, this variability stems from the VREs, as discussed above. On the demand side, a central issue is how flexible consumers are when it comes to reallocating their use of electricity to times when supply is high, for instance by washing clothes and dishes during night. In the widely cited paper by Joskow and Tirole (2007) intertemporal transfers in demand is not allowed: demand at any time depends only on the price faced by the consumer at that time. In this paper we attempt to get around this limitation by use of time-varying subsistence levels of electricity consumption to get some 'time-flexibility' in demand.

We find that the value of VRE indeed depends on the covariation between the variable elements of supply and demand [that were mentioned above]. However, although the value factor of VRE declines if the penetration of variable renewable electricity increases, it does so only within limits. This result differs somewhat from the existing literature, suggesting that the broader concepts of indirect utility and profit/producer surplus are more appropriate when evaluating VREs. The consumer surplus increases monotonously as VRE takes a larger share of electricity supply and prices fall. On the other hand, with falling prices producer surplus monotonously declines as the penetration of VRE increases.

Our paper follows Hirth (2013) quite closely, although the demand side is more explicitly modelled.[4] The electricity sector is here also connected to the rest of the economy, albeit in a simple way. While the work of Hirth is mainly empirical, our purpose here is to increase the understanding of a theoretical model, by using numerical analysis. We plan to apply this model in future empirical work. In the existing literature it seems to be more common to work with a multiplicative demand-shift parameter; see for instance Gowrisankaran, Reynolds and Samano (2016) and Joskow and Tirole (2007). In this paper we put the main focus on the case with an

[4]We replace the quite common linear (and vertical) demand function by a constant-elastic one. This allows for a richer welfare analysis.

additive shift parameter.

A recent paper by Bushnell and Novan (2018) addresses the issues raised here econometrically. They find that the increased installations of VREs have resulted in declining prices over time and worsened the conditions for the traditional power industry. The same result arises in our model.

The paper proceeds as follows. In Section 2 we present the demand side of the model, while the supply side is introduced in Section 3. Section 4 discusses the different equilibria of the model. It also defines the value factor of VRE, as well as the producer and consumer surpluses. Section 5 is devoted to a numerical analysis and Section 6 looks into the incentives to invest.

2 Electricity demand

2.1 Utility and constraint

Consider an electricity market with I individual consumers who use electricity over T points of time (hours). Instantaneous utility is a quasi-linear function of electricity consumption, x, and money for other goods, y. For individual i at time t we have

$$F_i\left(x_{it}, y_{it}\right) = \theta_{it} u_i\left(x_{it} - \gamma_{it}\right) + \mu_i y_{it}.$$

The function u_i is strictly concave, with $\lim_{x_{it} \to \gamma_{it}} u_i' = \infty$. The marginal utility of money is μ_i, while θ_{it} captures how important it is for individual i to consume electricity at time t. Finally, γ_{it} is the subsistence consumption level of electricity. To some extent θ_{it} and γ_{it} are two different ways of capturing the same thing, namely time-variability in demand, but they do so multiplicatively and additively, respectively. To our knowledge, the multiplicative approach is more common in the literature. See for instance Gowrisankaran, Reynolds and Samano (2016) and Joskow and Tirole (2007).

Summing utilities over all T points of time, typically the 8760 hours of a year, the total utility function of consumer i is

$$U_i = \sum_{t=1}^{T} \left[\theta_{it} u_i\left(x_{it} - \gamma_{it}\right) + \mu_i y_{it}\right]. \tag{1}$$

The time horizon is rather short, e.g. a year, so discounting can be ignored. Consumer i has the budget constraint

$$\sum_{t=1}^{T} p_t x_{it} + \sum_{t=1}^{T} y_{it} = m_i,$$

where p_t is the market price for electricity at time t and m_i is the (fixed) income of consumer i, to be used over the T points of time. The money for other goods (y_{it}) can be transferred between points of time.

2.2 Optimal demand

To solve the utility maximization problem of individual i we form the Lagrangean function

$$\mathcal{L}_i = \sum_{t=1}^{T} \left(\theta_{it} u_i \left(x_{it} - \gamma_{it}\right) + \mu_i y_{it}\right) - \lambda_i \left(\sum_{t=1}^{T} p_t x_{it} + \sum_{t=1}^{T} y_{it} - m_i\right),$$

where λ_i is the Lagrange multiplier. An optimal solution satisfies the conditions[5]

$$\theta_{it} u_i' \left(x_{it} - \gamma_{it}\right) = \mu_i p_t, \quad t = 1, 2, \ldots, T.$$

This can be solved for the demand function

$$x_{it} = d_i \left(\mu_i p_t / \theta_{it}\right) + \gamma_{it}, \qquad t = 1, 2, \ldots, T. \tag{2}$$

In this expression we have defined $d_i = (u')^{-1}$, with $d_i' < 0$. Obviously, the demanded quantity of electricity is low if its price is high or if the marginal utility of money for other goods is high. On the other hand, a high θ_{it} implies a higher electricity consumption.

The money for other goods is:

$$\sum_{t=1}^{T} y_{it} = m_i - \sum_{t=1}^{T} p_t d_i \left(\mu_i p_t / \theta_{it}\right) - \sum_{t=1}^{T} p_t \gamma_{it}. \tag{3}$$

2.3 Market demand

Market demand at time t is the sum of the demands from all consumers. We use Equation (2) and get

$$D_t = \sum_{i=1}^{I} x_{it} = \sum_{i=1}^{I} d_i \left(\mu_i p_t / \theta_{it}\right) + \gamma_t,$$

[5] We have the conditions $\mu_i = \lambda_i$ and $\theta_{it} u_i' \left(x_{it} - \gamma_{it}\right) = \lambda_i p_t$, $t = 1, 2, \ldots, T$.

where $\sum_{i=1}^{I} \gamma_{it} \equiv \gamma_t$ is the total amount of subsistence consumption at time t. It may be interesting to note that the sum of γ_{it}:s appear as a simple additive term.

2.4 Example

For concreteness we now introduce a specific form for the utility function u, which will also be used in the numerical simulations below.[6] We assume that the instantaneous utility of electricity is given by the constant-elastic function

$$u_i\left(x_{it} - \gamma_{it}\right) = \frac{\left(x_{it} - \gamma_{it}\right)^{1-\frac{1}{\alpha}} - 1}{1 - \frac{1}{\alpha}}, \qquad \alpha \neq 1. \tag{4}$$

Then the marginal utility is $u'\left(x_{it} - \gamma_{it}\right) = \left(x_{it} - \gamma_{it}\right)^{-\frac{1}{\alpha}}$ and the demand function (2) becomes

$$x_{it} = \left(\frac{\theta_{it}}{\mu_i p_t}\right)^{\alpha} + \gamma_{it}, \qquad t = 1, 2, \ldots, T. \tag{5}$$

As expected, consumption will be positive even if $p \to \infty$, because the subsistence consumption γ_{it} is always necessary. The parameter α is slightly higher than the demand elasticity in absolute value. Borenstein (2008) and Gowrisankaran, Reynolds and Samano (2016) assume that this elasticity is close to -0.1.

For this special case the market demand is

$$D_t = p_t^{-\alpha} \sum_{i=1}^{I} \left(\frac{\theta_{it}}{\mu_i}\right)^{\alpha} + \gamma_t. \tag{6}$$

This can be solved for the inverse market demand:

$$p_t = \left(\frac{\Sigma_{i=1}^{I} \left(\theta_{it}/\mu_i\right)^{\alpha}}{D_t - \gamma_t}\right)^{\frac{1}{\alpha}}. \tag{7}$$

3 Supply

3.1 Dispatchable supply

The dispatchable part of electricity supply is usually described by 'stair cases' that indicate the different marginal costs from various sources. For a simple introductory example, we here follow Lamont (2008) and assume

[6] As an alternative formulation, a linear demand curve is derived in Appendix A.1.

that it consists of baseload (b), intermediate (i) and peak (p) generators. Market power considerations are ignored. The marginal costs from these sources are c_b, c_i and c_p, respectively, where the unit is \$/MWh. One can think of these three lines of production as 'hydro', 'nuclear' and 'thermic' power, respectively.

There are two ways to define the marginal costs. The first is to interpret them as the true marginal cost. This leaves out the fixed costs, which we then have to compare with the producer surpluses. This is the natural approach in the short-run setting of this section. In the long run, however, the fixed costs must be explicitly included. In practice, the daily supply bids include 'mark-ups' that are expected to cover the fixed costs.[7] A complication is that the correct mark-ups depend on how much load a company delivers. We come back to this in the long-run analysis of Section 6. In the present section we take a sunk-cost view of the fixed costs, and thus consider the marginal costs to be the true marginal cost.

The actually delivered quantities at time t are S_{bt}, S_{it} and S_{pt}, respectively (MWh). This is the supplied quantity during one hour and therefore it is also correct to use the unit $\frac{\text{MWh}}{\text{h}}$. The maximum capacities are $\bar{S}_b$, $\bar{S}_i$ and $\bar{S}_p$, respectively, which are measured in MW or $\frac{\text{MWh}}{\text{h}}$. The dispatchable supply function (also called the merit order) is illustrated in Figure 1, where $\bar{S} = \bar{S}_b + \bar{S}_i + \bar{S}_p$. These generators are dispatched when the price for power covers their marginal cost.

Due to maintenance and reparations, these power sources do not run at full capacity all year around. Thus, let $\tilde{S}$ be the installed capacity and $\psi \in [0, 1]$ the capacity factor. Then the maximum capacities are $\psi_b \tilde{S}_b = \bar{S}_b$ and so on. For simplicity we assume that these expressions are constant, meaning that the different suppliers in each category 'take turns' in reparations and maintenance, so that the 'industry' supply is constant. This non-variation in the $\bar{S}_k$:s allows us to put more focus on the time variation in the VRE supply and the subsistence level of electricity consumption.[8][9]

[7]Borenstein (2016) discusses possible solutions to the difficulty of achieving economic efficiency when there is a need to cover fixed costs. He argues that a fixed charge each month, to connect to a utility, is a good way to make possible (social) marginal cost pricing.

[8]Notice that we disregard the reserve capacity. The requirements reduce the total generation capacity and the price is above the marginal cost. See Wangensteen (2012).

[9]Hirth (2013) mentions (p. 223) that short-term price fluctuations are reduced in the Nordic countries, by use of the flexibility in hydro generation. There are no such variations

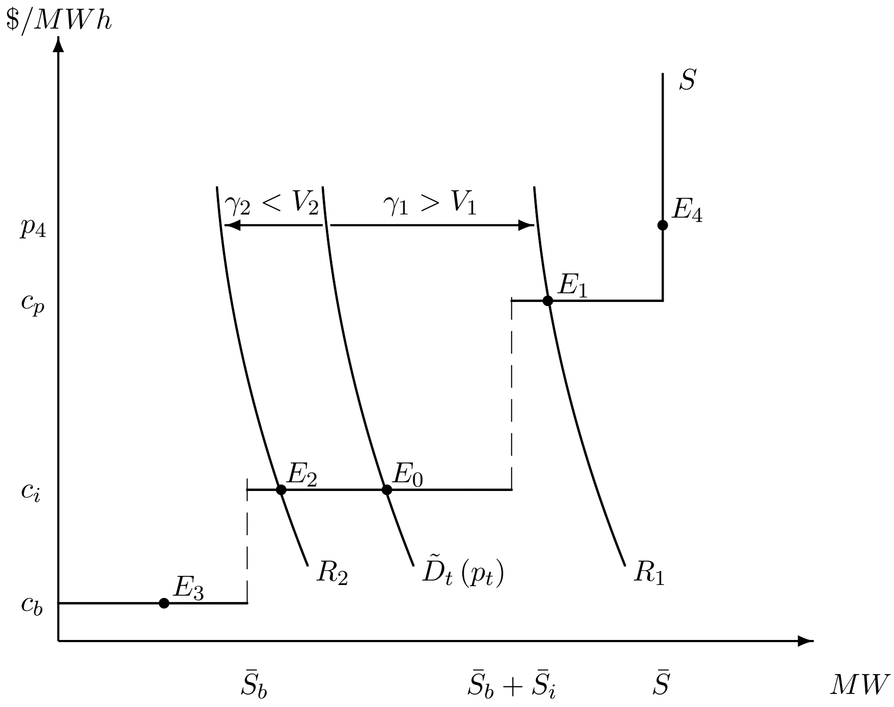

Figure 1: Equilibrium

3.2 VRE

Turning to the Variable Renewable Electricity, there are three technological properties that characterize it: (i) Due to fluctuations in wind and sunlight, the supply is *variable*; (ii) While the trading in electricity takes place in advance, the actual output is *uncertain* until the time of realization; (iii) The *location* of the VRE source, combined with transmission constraints, puts limits to the possibility to deliver power to load centers from distant sites where the weather conditions for VREs are more favourable. Following Hirth (2013), this paper focuses on the impact of variability (also called the 'profile'), since this is the most important factor (in value terms) for VREs.

In the model the intermittent supply of variable renewable electricity is denoted by V_t (MWh). It has a mean output of $\bar{V}$ (MW) and the realized value is just a function of 'weather'. The short-run marginal cost is typically very low and for simplicity we put it equal to zero.

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in the base load here. Our analysis is thus more applicable to thermal power systems, like in Germany, where prices vary more with penetration rates.

4 Equilibrium

4.1 Quantities and Prices

To highlight the variations in V_t and γ_t in the illustration of the equilibrium we define

$$\tilde{D}_t\left(p_t\right) = p_t^{-\alpha} \sum_{i=1}^{I} \left(\frac{\theta_{it}}{\mu_i}\right)^{\alpha},$$

so that $D_t = \tilde{D}_t\left(p_t\right) + \gamma_t$. Then the residual demand is

$$R_t = \tilde{D}_t + \gamma_t - V_t.$$

This is the part of electricity demand that is directed toward the dispatchable supply.

In Figure 1 the $\tilde{D}_t\left(p_t\right)$ curve stays put, (as long as θ_{it} is constant).[10] This curve coincides with the residual demand curve if $\gamma_t = V_t$, leading to the equilibrium E_0. Deviations from the $\tilde{D}_t$ curve are then entirely determined by the magnitude and sign of $\gamma_t - V_t$, which varies every hour. For period 1 the outcomes of the stochastic variables are $\gamma_t = \gamma_1$ and $V_t = V_1$, respectively. Since $\gamma_1 > V_1$, R_1 lies to the right of $\tilde{D}_t$ and the equilibrium is found at E_1. In the second case $\gamma_2 < V_2$ and R_2 is found to the left of $\tilde{D}_t$, going through the equilibrium point E_2. An even higher V_t could result in an equilibrium like E_3.

In the type of equilibria mentioned above, the R curve intersects the dispatchable supply curve at one of its horizontal segments, meaning that the price is equal to c_b, c_i or c_p. Alternatively, the intersection can occur with any vertical segment of the supply curve, like in E_4, where the quantity is given by $\bar{S} = \bar{S}_b + \bar{S}_i + \bar{S}_p$, leading to the price p_4. Other possible quantities in this latter type of equilibrium are 0, $\bar{S}_b$ or $\bar{S}_b + \bar{S}_i$.

If the price is equal to c_b, c_i or c_p, the demanded quantity according to (6) is

$$D_t = \frac{\Sigma_{i=1}^{I}\left(\theta_{it}/\mu_i\right)^{\alpha}}{c_k^{\alpha}} + \gamma_t, \qquad k = b, i, p.$$

and the residual demand is

$$R_t = \frac{\Sigma_{i=1}^{I}\left(\theta_{it}/\mu_i\right)^{\alpha}}{c_k^{\alpha}} + \gamma_t - V_t, \qquad k = b, i, p. \tag{8}$$

[10] If θ_{it} is a constant then $\tilde{D}_t\left(p_t\right)$ can be written as $\tilde{D}\left(p_t\right)$, which clearly implies a curve that does stay put in the figure.

From this expression it is clear that, at given c_k and θ_{it}, any change in the residual demand depends on the change in $\gamma_t - V_t$.

In an equilibrium like E_4, the condition $D_t - V_t = \bar{S}$ is fulfilled. By (6) the equilibrium price then is

$$p_t = \left(\frac{\Sigma_{i=1}^{I} (\theta_{it}/\mu_i)^\alpha}{\bar{S} + V_t - \gamma_t} \right)^{\frac{1}{\alpha}}. \quad (9)$$

The equilibrium price has properties that are entirely expected. For instance, it is inversely related to the supply (net of the subsistence quantity). Recall that $\bar{S}$ can be replaced by 0, $\bar{S}_b$ or $\bar{S}_b + \bar{S}_i$.

We now turn to various ways of evaluating the equilibria, both from the customers' and from the producers' points of view.

4.2 Value factors

Joskow (2011) and others argue that the valuation of electricity generators must take the variation in prices into account. In particular, it is important for the valuation of VRE what the market price is when the VRE produces much electricity. In this spirit, Hirth (2013) uses a value factor, consisting of the ratio between two price 'indexes'. First, there is the time-weighted average system price of electricity from all sources:

$$\bar{p} = \frac{\sum_{t=1}^{T} p_t}{T},$$

i.e. the average market equilibrium price. Second, the average revenue of VRE is

$$p^V = \frac{\sum_{t=1}^{T} p_t V_t}{\sum_{t=1}^{T} V_t}.$$

The numerator is the total revenue over the period, while the denominator is the total generation. Therefore p^V is the 'average price' of VRE. The VRE *value factor* is then defined as the ratio of average VRE revenues to the system price:

$$v^V = \frac{p^V}{\bar{p}}.$$

This definition of the value factor means that VRE is more valuable if it produces much at times when the price is high. In other words, it is beneficial for V_t to be positively correlated with demand γ_t, but only up to a point. If the capacity of VRE is high (compared to γ), it may to some extent work 'against itself', by lowering residual demand during sunny and windy hours. It then shifts the equilibrium down the merit-order curve and thereby lowers the price and its own 'value'. For example, the market equilibrium would often go from E_2 to E_3 in Figure 1 if VRE generation goes from a low to a high value.

It thus appears that the value factor falls with higher penetration and it seems to be a common conjecture in the literature that this process continues monotonously as the penetration of VRE increases.[11] However, our simulations demonstrate that it is not correct. To see why this is so, suppose first that the outcomes of V_t and γ_t are such that the equilibria are almost always found on the middle stair of the supply curve, e.g. at points such as E_2 and E_0 in Figure 1. The difference between $\bar{p}$ and p^V is then negligible and this means that $v^V \approx 1$.

As $\bar{V}$ increases, the equilibrium points on average move leftwards and so the number low-price equilibria increases. Since this is more likely to happen when V_t is high, p^V tends to get lower than $\bar{p}$ and the consequence is that $v^V < 1$. As $\bar{V}$ increases even further, however, low-price equilibria like E_0 come to dominate. Then p^V and converge $\bar{p}$ and v^V increases towards 1.

This is because v^V is an indicator of relative prices: at a general decline of prices, due to a higher penetration of VRE, p^V and $\bar{p}$ follow each other downwards, although not perfectly because of the stairs. The value factor does therefore not show a secular downward trend but rather a cyclical pattern. This is most obvious in our many-stairs simulations.

4.3 Producer surplus

With zero marginal cost, the producer surplus of the VRE producers at time t is $p_t V_t$. For an equilibrium of the type E_3 in Figure 1 there is no producer surplus for the firms that produce dispatchable electricity. If the equilibrium is of the E_2 type, the base load producers make a surplus equal to $(c_i - c_b)\bar{S}_b$. At times when the market equilibrium is like E_1, the producer surplus is $(c_p - c_b)\bar{S}_b + (c_p - c_i)\bar{S}_i$. In cases like E_4, with a price p_4, the producer surplus is $(p_4 - c_b)\bar{S}_b + (p_4 - c_i)\bar{S}_i + (p_4 - c_p)\bar{S}_p$. Similar expressions apply to equilibria on the other vertical segments of the supply curve.

[11] See e.g. Hirth (2013).

4.4 Indirect utility

To the extent that renewable electricity generation lowers the price, it will increase the indirect utility, partly by leaving more money for other goods. It is therefore interesting to compare the v^V and the producer surplus to the indirect utility function in various scenarios. This is a benefit of the approach of this paper, to derive demand from first principles. It provides the possibility to execute a fully-fledged cost-benefit analysis.

The indirect utility functions are derived in Appendix A. Summing over all periods, the indirect utility for one individual is[12]

$$U_i = \mu_i m_i + \frac{\alpha}{1-\alpha}\sum_{t=1}^{T}\theta_{it} - \frac{1}{1-\alpha}\sum_{t=1}^{T}\mu_i^{1-\alpha}\theta_{it}^{\alpha}p_t^{1-\alpha} - \sum_{t=1}^{T}\mu_i p_t \gamma_{it}.$$

Summing this over all individuals, we have:[13]

$$U = \sum_{i=1}^{I} U_i = \sum_{i=1}^{I}\mu_i m_i + \frac{\alpha}{1-\alpha}\sum_{i=1}^{I}\sum_{t=1}^{T}\theta_{it} - \frac{1}{1-\alpha}\sum_{i=1}^{I}\sum_{t=1}^{T}\mu_i^{1-\alpha}\theta_{it}^{\alpha}p_t^{1-\alpha} - \sum_{i=1}^{I}\sum_{t=1}^{T}\mu_i p_t \gamma_{it}.$$

This is the consumer benefit over the time frame running from $t = 1$ to $t = T$. A higher price clearly makes the indirect utility lower.

For equilibria like E_1, E_2 or E_3 in Figure 1, the price is equal to c_p, c_i or c_b. In the case where the demand curve intersects the supply curve along its vertical segment, as in point E_4 in Figure 1, the price is given by (9). A higher (net) supply, $\bar{S} + V_t - \gamma_t$, e.g. due to a higher V_t, then clearly makes the indirect utility higher, by reducing the equilibrium price. In the case with equilibria on the horizontal segments, higher V_t:s can raise the indirect utility by pushing the equilibrium to a lower stair.

5 Numerical simulations

The numerical simulations are based on the equilibrium conditions (8) and (9), where the equilibrium values are dependent on random realizations of

[12]At time t:

$$U_{it} = \frac{\mu_i m_i}{T} + \frac{\alpha}{1-\alpha}\theta_{it} - \frac{1}{1-\alpha}\mu_i^{1-\alpha}\theta_{it}^{\alpha}p_t^{1-\alpha} - \mu_i p_t \gamma_{it}.$$

[13]At time t:

$$U_t = \sum_{i=1}^{I}\frac{\mu_i m_i}{T} + \frac{\alpha}{1-\alpha}\sum_{i=1}^{I}\theta_{it} - \frac{1}{1-\alpha}\sum_{i=1}^{I}\mu_i^{1-\alpha}\theta_{it}^{\alpha}p_t^{1-\alpha} - \sum_{i=1}^{I}\mu_i p_t \gamma_{it}.$$

V_t and γ_t at the different times. In times when the residual demand curve intersects the supply at one of its horizontal segments, equation (8) determines the equilibrium quantity of dispatchable electricity. For convenience, this equation is reproduced here:

$$R_t = \frac{\Sigma_{i=1}^{I} (\theta_{it}/\mu_i)^{\alpha}}{c_k^{\alpha}} + \gamma_t - V_t, \qquad k = b, i, p. \tag{8}$$

When residual demand intersects supply at a vertical segment, the equilibrium price is given by equation (9), which we also reproduce:

$$p_t = \left(\frac{\Sigma_{i=1}^{I} (\theta_{it}/\mu_i)^{\alpha}}{\bar{S}_k + V_t - \gamma_t} \right)^{\frac{1}{\alpha}}, \qquad k = 0, b, i, p, \tag{9}$$

where $\bar{S}_k$ is either 0, $\bar{S}_b$, $\bar{S}_b + \bar{S}_i$ or $\bar{S}$.

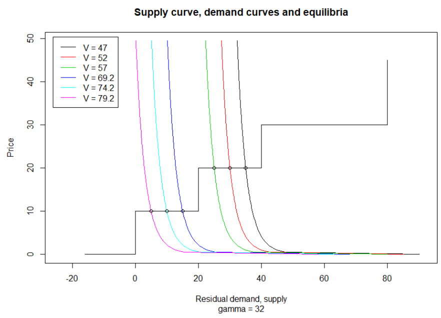
For one round of numerical simulation we draw 8760 observations (one for each hour of a year) of V_t and γ_t from a jointly normal probability distribution. Each draw produces an equilibrium point in one of the seven segments according to equations (8) or (9). From that, the value factor, producer surplus, consumer surplus etc can be computed. This is repeated in several rounds, with different correlations between V_t and γ_t and with different degrees of penetration of VRE, $\bar{V}$. Then the value factors and the other welfare indicators are compared between the rounds, to see the effects of changing VRE penetration and correlation between V_t and γ_t.

More precisely, we address the following aspects in the numerical analysis.

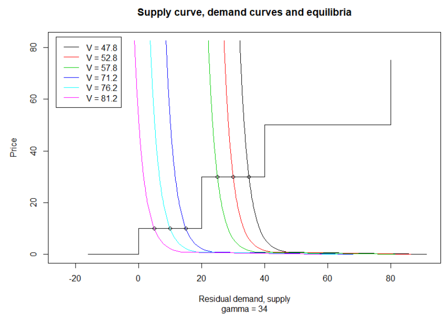
1. The correlation between V and γ decides the extent to which a higher VRE penetration makes the equilibria climb down the 'merit order' at a higher frequency. One could hypothesize that the value factor is high if the correlation is positive, because then γ often takes the price up one stair even when much VRE is produced.

2. Higher steps in the dispatchable supply curve can lead to a steeper decline in the value factor when the penetration increases, as discussed by Hirth (2013). This is because high V:s tend to push the price down much when VRE production is high. However, the results also depend on how wide the steps are, which is connected to the next point.

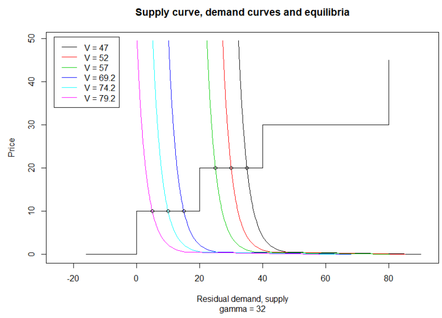
3. If the outcomes of V and γ are such that the equilibria are mainly spread over few steps, or concentrated on one step, there are limits



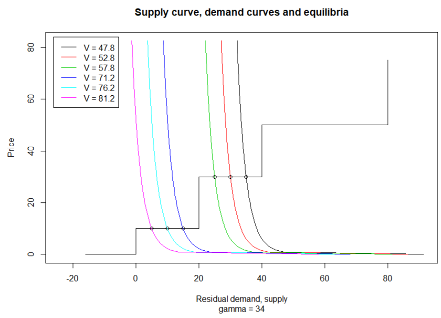
(a) Benchmark



(b) Change: $c_i = 30$ and $c_p = 50$



(c) Change: $\sigma_\gamma = 10$



(d) Change: $\sigma_\gamma = 10$, $c_i = 30$, $c_p = 50$

Figure 2: Dispatchable Supply and Residual Demand

in the price variations, and therefore the value factor will be close to unity. If, on the other hand, the spread covers many steps, to a large extent, then the value factor can deviate more from 1 (in both directions).[14]

4. Is *Utility-plus-profits* increasing with VRE penetration? Or is there a maximum of this sum, that can be interpreted as a kind of welfare maximum, implying a socially optimal level of $\bar{V}$?

5.1 Basic example

In our basic example the dispatchable supply curve is characterized by the following parameters: $c_b = 10$, $c_i = 20$, $c_p = 30$, $\bar{S}_b = 20$, $\bar{S}_i = 20$ and

[14] A higher spread may also reduce the cyclical pattern in the value factor.

$\bar{S}_p = 40$.[15] It is further assumed that $\mu_i = \theta_{it} = 1$, $I = 60$ and $\alpha = 0.06086$. The default values of the standard deviations of V and γ, σ_V and σ_γ, are both equal to 5.[16] The mean value of V, which is taken as a proxy for penetration, is varied and indicated in the figures below, which also indicate the various degrees of correlation between V_t and γ_t, i.e. ρ.

Figure 2 describes the four scenarios that we will consider in this basic example. Figure 2a illustrates the benchmark that was just described. In Figure 2b the stairs are higher: $c_i = 20$ and $c_p = 30$ are replaced by $c_i = 30$ and $c_p = 50$. In Figure 2c the stairs are back to the benchmark levels, while the standard deviation of γ is increased to $\sigma_\gamma = 10$. In Figure 2d both changes appear at the same time.

The residual-demand (R_t) curves at the centers of the two lower steps are placed there by appropriate choices of V and γ. On each side of these there is a curve on a one-standard-deviation distance from the centre.

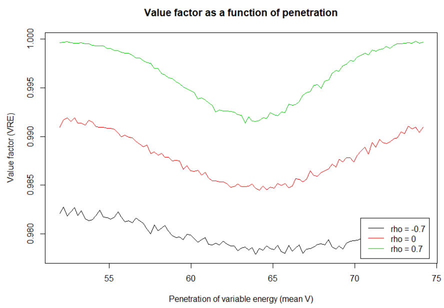
5.1.1 Value factor

We begin our investigation of the effects of the above variations by looking at the value factor, which is displayed in Figure 3. A conjecture in much of the literature is that the value factor of VRE is monotonously decreasing in the penetration rate.[17] The idea is that, when the penetration rate increases, the residual demand shifts left and thus causes lower prices for both VREs and dispatchable generators. In Figure 3a we see this phenomenon, but only up to a point. The level of the curve seems to depend on whether the bulk of the equilibria are on the same stair or not. For the downward sloping part, the addition of more VRE capacity pushes more and more equilibria down to the next stair and thus reduces the revenue of the VRE. As the penetration goes even further, the bulk of equilibria are found at the lower level. There is then less variation in the price and therefore a smaller difference between the mean price of VRE electricity and the price in general. This explains

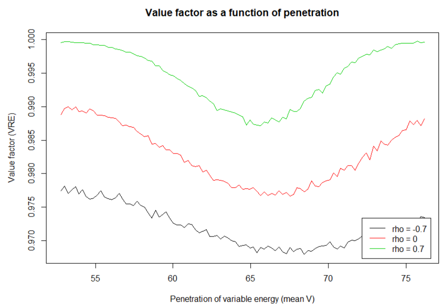
[15] This final step is longer than the others, in order to avoid a high frequency of very high prices (on the final vertical part of the supply curve).

[16] The mean value of γ is a function of the hight of the steps, in order to center the curves on the steps in Figure 2.

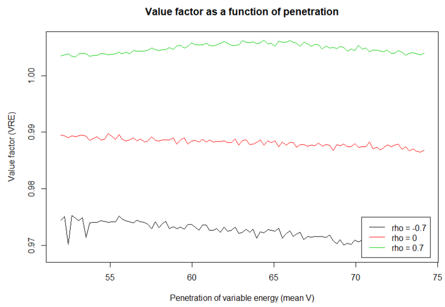
[17] See Göransson and Johnsson (2018), section 5.1. They find that the (wind) value factor declines as the share of wind increases. This trend can be counteracted by load shifting and storage facilities. See also Fripp and Wiser (2008) and Hirth (2013).



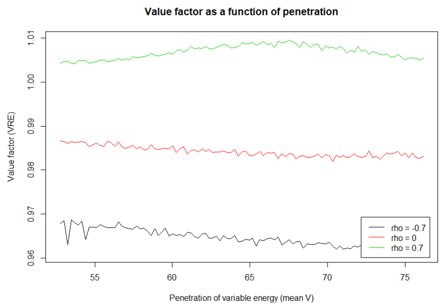
(a) Benchmark



(b) Change: $c_i = 30$ and $c_p = 50$



(c) Change: $\sigma_\gamma = 10$



(d) Change: $\sigma_\gamma = 10$, $c_i = 30$, $c_p = 50$

Figure 3: Value factor as a function of penetration

the increase in the curve at even higher levels of penetration.[18]

The different levels of the three curves in all panels of Figure 3 mean that the value factor decreases as the correlation between V and γ gets lower. A high correlation means that VRE output often is large when demand is high. This increases the possibility to sell renewable electricity at high prices, perhaps even climbing one stair up, thus keeping the value of VRE high.

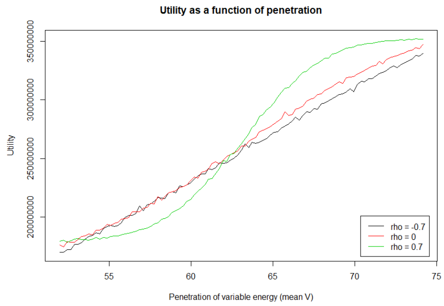
If the stairs of the dispatchable supply curve become higher, as in Figure 3b, the general pattern is maintained but levels are somewhat lower. To the extent that VRE is delivered in large amounts when the price falls down to the next stair it now suffers from a larger price reduction which reduces the value factor.

Going back to the benchmark supply curve, but instead increasing the spread of γ, we get Figure 3c. Here the wave patterns of the curves have disappeared, with the possible exception of the highest curve, for which the correlation is positive. It matters less whether the mean R is close to the next stair or not, when the equilibrium is spread over more stairs anyway. We also note that the values of the curve for the positive correlation are moving up (above 1), while the curve for the negative correlation is moving down (compared to Figure 3a). An interpretation of this may be offered from equilibrium price histogram in Figure 6c.[19] The higher spread increases the frequency of prices that are one level higher or lower. In the case of a positive correlation between V and γ the VRE is more often sold at a high price. This explains why v^V finally climbs above 1. Similarly, at a negative correlation the higher spread results in more instances of low prices when VRE delivers much electricity, which explains the lower level of the curve in this case.

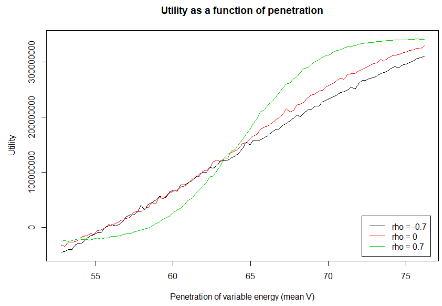
In panel 3d the effect of the wider spread from panel 3c is reinforced by the higher steps. If $\rho > 0$ it is more likely to get a very high price when VRE produces much, thus pushing the value factor a bit higher. A negative correlation, on the other hand increases the probability that VRE produces much when demand is low, thereby pushing the bottom curve a bit lower, compared to panel 3c.

[18]This result that the value factor shows a wave pattern is more visible in the case with many steps on the curve.

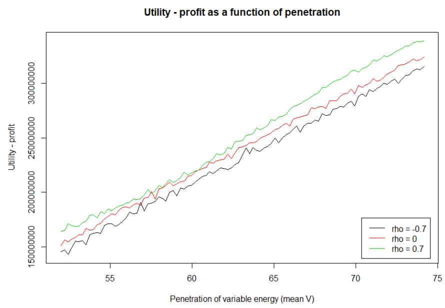
[19]Note that $\rho = 0$ in this figure.



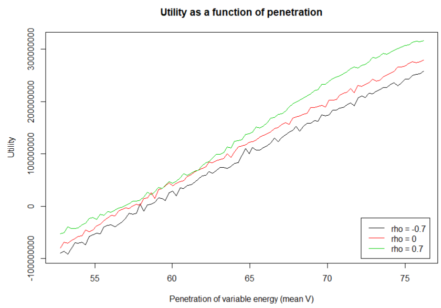
(a) Benchmark



(b) Change: $c_i = 30$ and $c_p = 50$



(c) Change: $\sigma_\gamma = 10$



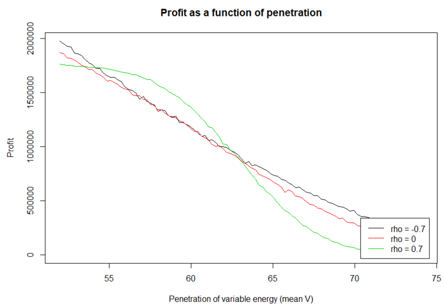
(d) Change: $\sigma_\gamma = 10$, $c_i = 30$, $c_p = 50$

Figure 4: Utility as a function of penetration

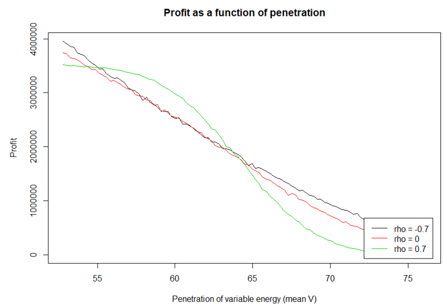
5.1.2 Welfare

Consumers clearly benefit from the entrance of more VRE firms that produce electricity at low (zero) marginal cost. This is illustrated in Figure 4, where utility increases (in all panels) as the addition of more VRE generators lowers the mean equilibrium price. At higher stairs (Figure 4b), utility starts lower, because the price level is generally higher. In Figures 4a and 4b, with low σ_γ, the curves are S-shaped in the positive-correlation cases. These shapes are absent in Figures 4c and 4d, where σ_γ is higher. At a high correlation, the first additional capacity of VRE is not sufficient to push the equilibrium price down to the next level, because demand tends in the opposite direction. This is compensated by a steep increase later.

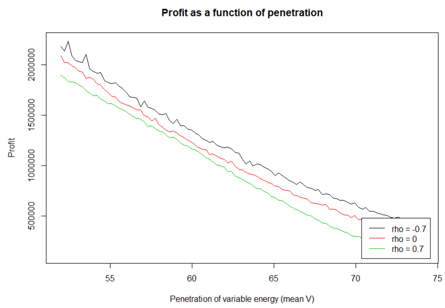
A quite expected result emerges when we turn to the relation between VRE penetration and profit, which is described in Figure 5. In all panels the profits fall monotonously as more VRE generators enter the market. Since the latter have lower marginal costs, they tend to reduce the prices (as well



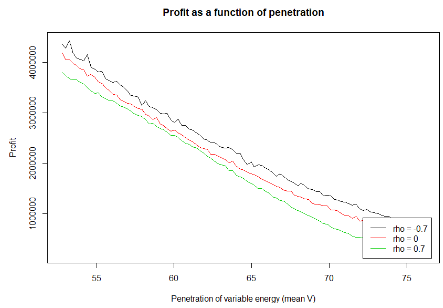
(a) Benchmark



(b) Change: $c_i = 30$ and $c_p = 50$

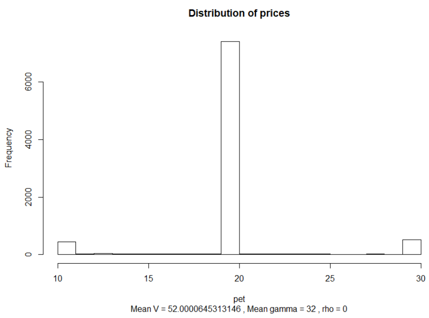


(c) Change: $\sigma_\gamma = 10$

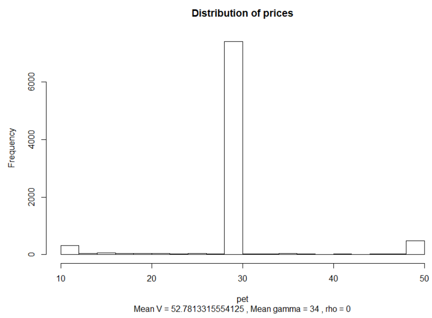


(d) Change: $\sigma_\gamma = 10$, $c_i = 30$, $c_p = 50$

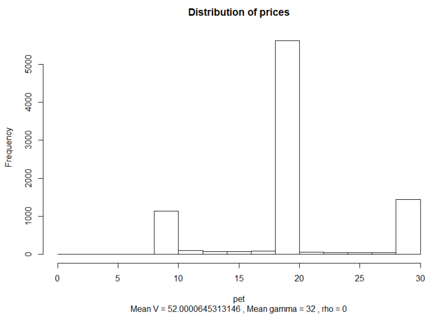
Figure 5: Profit as a function of penetration



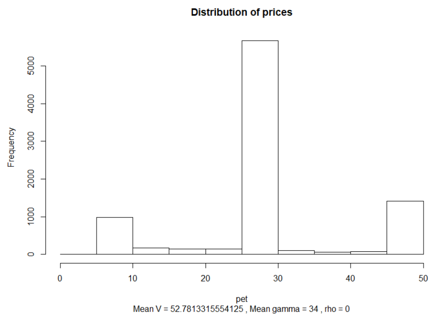
(a) Benchmark



(b) Change: $c_i = 30$ and $c_p = 50$



(c) Change: $\sigma_\gamma = 10$



(d) Change: $\sigma_\gamma = 10$, $c_i = 30$, $c_p = 50$

Figure 6: Distribution of prices

as quantities) for all suppliers, thus reducing profitability in the dispatchable sectors. The fall in profits is quite remarkable in percentage terms.[20]

The most notable lesson from Figure 6 is that the equilibrium price varies more as the variance of γ increases (which of course is entirely expected). The frequency of the higher price is just a little bit higher than for the lower price. This is because the 'basic' residual demand curve is found at the centre of the stair, combined with the zero correlation between γ and V.

[20]They could be compared to the annualized fixed costs and it is not unlikely that the total net value is negative even at moderate degrees of penetration. This illustrates that a social cost of higher VRE penetration lies in the lower profitability of dispatchables that are needed as back-ups at times with little wind and sunshine.

6 Fixed costs

We now introduce fixed costs (\$/MW/year), which were briefly discussed in Section 3.1. They are F_V and F_k for VREs and dispatchables, respectively. There are n_k representative firms in branch k and these fixed costs apply for a firm of the V or k branch. These costs must be recovered by an 'add-on' on bid prices, f_k, which for simplicity is assumed to be constant. This makes for a reinterpretation of the marginal cost, such that it includes a compensation for the fixed cost. Thus, for dispatchables we have

$$c_k = \tilde{c}_k + f_k,$$

where $\tilde{c}_k$ is the true marginal cost. (Electricity is still supplied only if $p_t \geq c_k$.) One way to write the profit function then is

$$\Pi_k = \sum_{t=1}^{T} p_t \cdot s_{kt} - \sum_{t=1}^{T} (\tilde{c}_k + f_k) \cdot s_{kt},$$

where $s_{kt} = S_{kt}/n_k$ is the supply a representative firm. It is thus spread evenly over the firms. The condition for the recovery of fixed cost is

$$F_k = f_k \sum_{t=1}^{T} s_{kt}.$$

The fixed cost is thus spread over all the kilowatt hours delivered during the year. This means that peak utilities, that deliver rather small amounts of electricity over a year, must choose a large f_k (unless F_k is small, which is unlikely).

Similarly, the profit for a representative VRE firm is

$$\Pi_V = \sum_{t=1}^{T} p_t \cdot v_t - \sum_{t=1}^{T} (0 + f_V) \cdot v_t,$$

where $v_t = V_t/n_V$ is the supply of this representative unit. Fixed costs are recovered if

$$F_V = f_V \sum_{t=1}^{T} v_t.$$

As a simple addition to the numerical analysis above, one could start by assuming some given values for f_k and f_V. Then one would uses the quantities delivered by different branches and compute the add-on revenues. These correspond to the maximum tolerable fixed costs.

An alternative possibility is to use the conditions for the recovery of fixed costs as constraint in an optimization problem, for instance like in Gowrisankaran, Reynolds and Samano (2016). The f_k:s would then be a part of the solution.

7 Conclusions

The purpose of this paper has been to present and analyze a simple partial-equilibrium model of the electricity market. The focus of interest has been put on the time variation of demand and the supply from variable renewable electricity sources, and on the effects of these variations on the equilibrium prices at different points of time.

The model is used to analyze the hourly equilibria of a fictive year numerically, by computations of the welfare effects of different scenarios, including the value factors of VRE at different degrees of penetration. The equilibrium analysis is conveniently focused on the relation between two quantities, namely the electricity supply from intermittent sources and the subsistence level of electricity use. We find that the value factor of VRE declines if the penetration of variable renewable electricity increases, but only within limits. This result differs somewhat from the existing literature, suggesting that the broader concepts of indirect utility and profit/producer surplus are more appropriate.

We consider this model as a useful starting point for several interesting extensions. First, it can be applied to the question of integration between separate power systems. What are the costs and benefits of building new and costly transmission lines? Second, it is straightforward to integrate pollution effects from non-renewable generators and analyze the optimal level of penetration in the light of welfare effects of positive environmental effects from VREs.

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A Deriving the indirect utility

As a first step in deriving the indirect utility function we substitute the demand in (5) back into (4):

$$u_i\left(x_{it}-\gamma_{it}\right)=\frac{\alpha}{1-\alpha}-\frac{\alpha}{1-\alpha}\cdot\left(\frac{\mu_i p_t}{\theta_{it}}\right)^{1-\alpha}.$$

This part of the utility function increases if the price gets lower. Substituting this into the 'full' utility function in (1):

$$U_i=\frac{\alpha}{1-\alpha}\sum_{t=1}^{T}\theta_{it}-\frac{\alpha}{1-\alpha}\sum_{t=1}^{T}\theta_{it}\left(\frac{\mu_i p_t}{\theta_{it}}\right)^{1-\alpha}+\mu_i\sum_{t=1}^{T}y_{it}$$

To develop the final term, use (5) in (3):

$$\sum_{t=1}^{T}y_{it}=m_i-\sum_{t=1}^{T}p_t\left(\frac{\theta_{it}}{\mu_i p_t}\right)^{\alpha}-\sum_{t=1}^{T}p_t\gamma_{it}$$

Using this in the utility function:

$$U_i=\mu_i m_i+\frac{\alpha}{1-\alpha}\sum_{t=1}^{T}\theta_{it}-\frac{1}{1-\alpha}\sum_{t=1}^{T}\mu_i^{1-\alpha}\theta_{it}^{\alpha}p_t^{1-\alpha}-\sum_{t=1}^{T}\mu_i p_t\gamma_{it}$$

Finally, we sum over all individuals:

$$U=\sum_{i=1}^{I}U_i=\sum_{i=1}^{I}\mu_i m_i+\frac{\alpha}{1-\alpha}\sum_{i=1}^{I}\sum_{t=1}^{T}\theta_{it}-\frac{1}{1-\alpha}\sum_{i=1}^{I}\sum_{t=1}^{T}\mu_i^{1-\alpha}\theta_{it}^{\alpha}p_t^{1-\alpha}-\sum_{i=1}^{I}\sum_{t=1}^{T}\mu_i p_t\gamma_{it}$$

A.1 Linear demand

An alternative specification is a quadratic utility function of electricity

$$u\left(x_{it}-\gamma_{it}\right)=-\frac{1}{2}\left(a-b\left(x_{it}-\gamma_{it}\right)\right)^2.$$

In this case the marginal utility is $u'\left(x_{it}-\gamma_{it}\right)=ab-b^2\left(x_{it}-\gamma_{it}\right)$, which means that individual demand is

$$x_{it}=\frac{ab+b^2\gamma_{it}}{b^2}-\frac{\mu_i p_t}{b^2\theta_{it}} \tag{10}$$

and market demand is

$$D_t=\frac{a}{b}\cdot I+\gamma_t-\frac{p_t}{b^2}\sum_{i=1}^{I}\frac{\mu_i}{\theta_{it}}. \tag{11}$$