Managing intermittency in the electricity market ☆

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Abstract

We analyze the integration of intermittent renewables-based technologies into an electricity mix comprising of conventional energy. Intermittency is modeled by a contingent electricity market and we introduce demand-side flexibility through the retailing structure. Retailers propose diversified electricity contracts at different prices allowing consumers to choose their optimal electricity consumption. These contracts are modeled by a set of state-contingent electricity delivery contracts. We show existence and uniqueness of a competitive equilibrium of the contingent wholesale and retail markets. We provide a welfare analysis and only obtain constraint efficiency due to a limited number of delivery contracts. Finally, we discuss the conditions under which changing the set of delivery contracts improves penetration of renewables and increases welfare. This provides useful policy insights for managing intermittency and achieving renewable capacity objectives.

Keywords: electricity market, renewables, intermittency, demand flexibility
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1. Introduction

The integration of renewables into the electricity mix is widely accepted as having a significant role to play in decarbonizing the electricity industry. Most particularly, the International Energy Agency reports on the deployment of energy sources as wind and solar helping in increasing renewables-based electricity. Still, their share in the global electricity mix remains quite modest, 7% in 2018 (IEA (2019)). A persisting obstacle to the adoption of these renewables lies in fact in their variable and uncertain nature, collectively referred as *intermittent* (see Perez-Arriaga and Batlle (2012), Verzijlbergh et al. (2017)).

Electricity from renewables varies significantly with natural and uncontrollable conditions[1]. These render the production process from renewable technologies not only intermittent but also inflexible, technically termed as *non-dispatchable*[2]. Hence, the integration of renewable electricity adds a new source of intermittency on the grid; demand intermittency being a long-existing phenomenon already managed with investment in dispatchable[2] power plants mainly (IEA (2011)). So, intermittency from renewables challenges the imperative of the electricity industry to constantly balance supply and demand. Disruptions in this balance have both technical and economical impacts[3]. Contrary to demand intermittency, this new source of intermittency still needs to be tackled.

In the above context, introducing flexibility in the electricity market is recognized as a solution to managing renewables intermittency (Cochran et al. (2014), EURELECTRIC (2014), IEA (2011), IEA-ISGAN (2019)). Now, flexibility can be implemented upstream of the market so that supply follows demand. For example, through existing or new flexible power plants, storage capacities (Benitez et al. (2008), Green and Vasilakos (2012), Pommeret and Schubert (2019), Sioshansi (2011)) and interconnection (Abrell and Rausch (2016), Yang (2020)). It can also be developed downstream through demand-side flexibility, requiring demand to follow supply. In this paper, we focus on the latter.

The main objective of our work is to provide an approach to manage intermittent supply by means of demand flexibility in the retail market. The importance of demand flexibility was highlighted with the question of optimal investment in production capacity to meet intermittent demand. Borenstein and Holland (2005) and Joskow and Tirole (2007)

[1]As examples, electricity production from wind turbines fluctuates with wind speed and direction and that from solar photovoltaic with radiation intensity (Crawley (2013)).

[2]Renewable technologies such as wind turbines and solar photovoltaic are non-dispatchable as their output cannot be turned on, off or adjusted according to variations in electricity demand. In contrast, conventional generators such as coal, nuclear, hydro and gas power plants are controllable and dispatchable. See for instance IEA-ETSAP and IRENA (2015) on dispatchable and non-dispatchable technologies.

[3]Mismatches between supply and demand can lead to, for example, frequency fluctuations which in turn cause brown-outs or blacks (see Passey et al. (2011) for instance). These result in heavy economic losses such as manufacturing and sales losses, interruption of services, etc. See, for example, Küfeoğlu and Lehtonen (2015) on the economical consequences of power outages on service sector customers.

study one weakness of the electricity market to derive optimal investment programs. They point out the disconnection between wholesale prices that vary with electricity provision and retail tariffs such as the flat tariff which do not reflect these changes[4]. Consumers are therefore unaware of varying wholesale market conditions including those where expensive power plants are run to meet peak demands. In a framework where demand is intermittent, Borenstein and Holland (2005) and Joskow and Tirole (2007) suggest the implementation of time-varying tariffs such as Real-Time-Pricing (RTP)[5] to improve efficiency of the market in terms of capacity investment. In practice, time-varying tariffs that have been developed also include Time-of-Use (ToU), Critical Peak Pricing (CPP) and Variable CPP. These tariffs are abled to shape demand thereby reducing peaks[6] and investments in expensive peaking power plants used only for a few hours during a year (IRENA (2019)). Since these retail contracts can stabilize demand intermittency, we now propose to tap into them or use more sophisticated ones to adjust demand according to intermittent supply.

Can retail contracts be designed to unlock demand flexibility and ease the integration of intermittent renewable technologies? As part of their work, Ambec and Crampes (2012) and Rouillon (2015) address this question and find the optimal investment in renewable capacity in a framework with intermittent supply. While Ambec and Crampes (2012) consider that consumers can use either a flat retail tariff or one that varies with the availability of the intermittent source of energy (by anology RTP), Rouillon (2015) considers that there is a mix of both consumers. Our work differs in that matter as we want to take into account more diversified retail contracts than the polar flat-tariff and RTP contracts.

To investigate the above question, we have in mind a theoretical framework of capacity investment and electricity production with two types of energy sources: an intermittent and non-dispatchable source such as wind or solar and a non-intermittent and dispatchable source such as nuclear or fossil fuel. We propose that electricity production due to the integration of the intermittent renewable technology depends on conditions such as weather (e.g., "with" or "without" the intermittent source) or times of day (night, dawn, daytime and dusk). We refer to these conditions as states of nature. Electricity production is therefore state-contingent. We consider that the wholesale market is a contingent one where contingent electricity is traded at contingent prices. We further propose that retailers introduce demand flexibility in the retail market through diversified

[4]Retailers buy electricity at wholesale prices which vary with electricity provision (prices are low when power plants with low marginal cost of production are run and vice-versa.), with the commitment to supply electricity reliably to consumers for any level of demand and at fixed tariffs. The widely spread retail tariff is the flat tariff whereby consumers are charged the same price per unit of electricity all day.

[5]An extensive literature on time-varying tariffs can be found in the work of Borenstein et al. (2002).

[6]In France, for example, the majority of households have retail contracts with "off-peak/peak" hours tariffs where they pay a lower price during off-peak demand hours and a higher price during peak demand hours. This retail contract has been able to incentivize consumers to use electric water heaters to heat water during off-peak hours rather than during peak hours, thereby alleviating the peak demand of electricity due to water heating.

electricity delivery contracts supplied at different prices. These allow consumers to choose their optimal electricity consumption based on their flexibility. The diversity of the delivery contracts is depicted through what we call *base state-contingent electricity delivery contracts* whose structure is similar to the asset structure in the incomplete market theory (see for instance Magill and Quinzii (2002)). These base delivery contracts can well generate contracts with flat and time-varying tariffs but one can also think of more complex contracts depending on weather conditions or on the pressure on the wholesale market. In this paper, we provide a general model for the structure of the base state-contingent electricity delivery contracts.

Considering a competitive setting, we show existence and uniqueness of the equilibrium of the state-contingent wholesale and retail markets in which the optimal investment in intermittent renewable capacity is endogenous. Assuming that the base state-contingent electricity delivery contracts structure is not rich enough to generate delivery contracts allowing perfect adjustment of demand to variations in supply, we find that we are in a situation of missing markets. This lack of richness can be explained, for instance, by variations in conditions and thereby electricity supply at a level of granularity which is too fine to incite response from consumers. We nevertheless show that the electricity market equilibrium and social welfare are constraint efficient. The constraint is induced by the limited number of the base delivery contracts which constrains electricity allocations. We are also able to determine the conditions under which changing the base delivery contracts improves (i) welfare, (ii) the degree of integration of the renewable capacity and (iii) both. Ultimately, we find that it is impossible to find a change in the delivery contracts structure which both increases investment in renewable capacity and reduces the production of conventional electricity in each state of nature.

The rest of the article is organized as follows. Section 2 presents the theoretical framework and the main assumptions. In section 3, we describe the electricity contract markets and derive some useful properties of electricity demand. Section 4 studies state-contingent electricity supply and describes market equilibrium. Section 5 discusses the issue of constraint welfare and missing markets. In section 6, we analyze the impact of changing the state-contingent delivery contracts on social welfare, investment in intermittent renewable capacity and conventional production. Section 7 concludes. Technical proves are relegated to the appendix.

2. The main assumptions

Our paper principally aims at addressing two crucial features of renewables: intermittency and decision on optimal investment in capacity. For this purpose, we use a static framework. However, intermittency is not only a matter of physical conditions on which renewables depend but also how these change over time[7]. As such, there is a dynamic effect in electricity production from renewables which we capture by a set of states of

[7]For example, electricity production from wind turbines depends on wind speed which varies on all time-scales, from sub-seconds to decades (Widn et al. (2015)).

nature $s \in \{1, \dots, S\}$. Since electricity production is state-contingent and the wholesale market being organized in each state, we denote by $p(s)$ the contingent electricity price in state s and by $\mathbf{p} = (p_1, \dots, p_S) \in \mathbb{R}^S$ the vector of all state-contingent prices. This choice allows us not to explicitly introduce probabilities nor to introduce a Von Neumann Morgenstern setting.

Another important feature of this framework is with respect to decision-making. Intermittent producers behave *ex ante*, i.e before the realization of the states of nature. They choose how much to invest in renewables-based capacities by anticipating their future returns. Likewise, conventional producers choose their dispatchable production strategy beforehand. Retailers and consumers *ex ante* exchange retail contracts based respectively on their expected profits and utility for *ex post* electricity deliveries, that is deliveries based on realization of the states of nature.

Keeping these two features in mind, we can now move to describing each agent of the electricity market.

The *intermediary retailing structure* between electricity production and consumption is one of the novelties of the model. In fact, we assume that consumers do not have direct access to the electricity wholesale market. They buy *delivery contracts* from competitive retailers. These delivery contracts are constructed by using *base state-contingent delivery contracts*. To illustrate the idea behind these, let us take for instance a Time-Of-Use retail contract with an off-peak period during the night and peak periods during the day and dusk. We can depict three states of nature: night (s_1), day (s_2) and dusk (s_3). Say that this contract can be broken down into two base state-contingent delivery contracts: an off-peak contract k_1 at price q_1 which delivers a_1 units of electricity in s_1 and a peak contract k_2 at price q_2 which delivers a_2 and a_3 units of electricity in s_2 and s_3 respectively. If retailers trade these base delivery contracts, consumers can now buy independently quantities θ_1 and θ_2 of k_1 and k_2 respectively to have the desired electricity consumption for each state of nature. Now retailers can also trade linear combinations of the base delivery contracts. For example, instead of two delivery contracts, they can trade a single one, k_3, with electricity deliveries a_1, a_2 and a_3 at price q_3. Consumers will buy quantity θ_3 of this single contract.

We now materialize the idea that existing or more sophisticated retail contracts can be constructed from base state-contingent electricity delivery contracts. We provide a general model for the latter and for simplicity we assume that trade is done on the base state-contingent delivery contracts (even if, as we have seen in our example, trade can also be done on linear combinations of the base contracts). We say that the holder of one unit delivery contract k has the right to a random electricity consumption of $\mathbf{a}_k = (a_{1k}, \dots, a_{Sk})$ where $a_{sk} \geq 0$ denotes electricity delivered in state s. We introduce a set $K = \{1, \dots, K\}$ of unit contracts $\mathbf{a}_k$ which are traded on K competitive markets at price $\mathbf{q} = (q_1, \dots, q_K)$ where q_k stands for the price of one unit of contract k. We summarize the electricity delivery in the (S, K) matrix $\mathbf{A} = [\mathbf{a}_k]_{k=1}^K \in \mathbb{R}_+^{SK}$. So, if $\boldsymbol{\theta} = (\theta_1, \dots, \theta_K) \in \mathbb{R}^K$ denotes a portfolio of contracts, $\mathbf{A}\boldsymbol{\theta} \in \mathbb{R}^S$ describes the random electricity flow induced by portfolio $\boldsymbol{\theta}$. We also introduce some assumptions on $\mathbf{A}$. We first say that $K < S$. When $K = S$, there are as many delivery contracts as states of nature which provide perfect information on wholesale market conditions and allow effective adjustment of demand to fluctuating supply. Here we make the analogy to Real Time Pricing contract which can be represented by S delivery contracts. But the interesting question to be asked is what if $\mathbf{A}_{SK}$ is not rich enough to provide a perfect adjustment of demand to intermittent supply. Indeed, while the supply side is characterized by variations in production with a high level of granularity, consumers, on the other hand, may be sensitive to smoother variations depicted by $K < S$ contracts, a condition of missing markets. We are interested precisely in such a situation. Secondly we say that $\mathbf{A}$ is of full rank, here $rank(\mathbf{A}) = K$. This simply means there is no redundant contract, i.e. a contract which can be obtained by a portfolio of the other contracts. Finally we assume $\forall s$, $\exists k$, $a_{sk} > 0$, i.e. there always exists an asset which delivers electricity in a given state.

The *retailers,* in this setting, work as intermediaries. They sell, *ex ante,* delivery contracts $\boldsymbol{\theta}_r$ at price $\mathbf{q}$ and, *ex post*, provide the required amount of electricity to the contract owner, electricity that they buy on the wholesale market. In other words, if a retailer sells a portfolio $\boldsymbol{\theta}_r$, his return is of $\mathbf{q}'\boldsymbol{\theta}_r$ while his expected cost due to electricity delivery is of $\mathbf{p}'\mathbf{A}\boldsymbol{\theta}_r$. His supply follows a competitive profit maximization strategy. Finally, let us observe that the transformation process from contract to electricity delivery is linear. We can therefore replace the set of all retailers by a representative one.

The *renewable energy technology*, in addition to being intermittent, is *non-dispatchable.* To ensure non-dispatchability, we introduce an *ex ante* capacity choice $\kappa \in \mathbb{R}_+$ and model intermittent electricity productivity as the random variable $\mathbf{g} = (g_1, \ldots, g_S) \in \mathbb{R}^S_{++}$ which describes the state contingent production per unit of capacity. The cost of investing in capacity κ of the technology is given by $\mathcal{K}(\kappa)$ and we assume that $\mathcal{K}(0) = 0, \partial\mathcal{K}(\kappa) > 0$ and $\partial^2\mathcal{K}(\kappa) > 0$ as usually described in literature (e.g.Rouillon (2015))[8]. Renewable electricity production is equal to $\kappa\mathbf{g}$ and without loss of generality, we normalize short-run marginal cost of production to zero[9]. We finally assume that the optimal investment in capacity and therefore the state contingent production of electricity follows from a competitive profit maximizing behavior.

The *conventional energy technology* is an existing and fully established one. Its purpose is to act as a back-up capacity of dispatchable electricity generation compensating for fluctuations from the intermittent renewable energy technology. It ensures reliable electricity provision in the absence of demand flexibility. We assume that there is no capacity constraint since electricity production for all $s \in S$ does not exceed installed capacity. As the intermittent renewable technology, electricity production from the conventional technology is state contingent too. We denote it by $\mathbf{y} = (y_1, \ldots, y_S) \in \mathbb{R}^S_+$. The imple-

[8]The strictly convex capacity investment cost function can be viewed as investment starting at the most productive site, e.g., in terms of weather conditions.

[9]The resource is "free"and variable costs such as operation and maintenance costs for solar and wind technologies tend to be typically lower than those of conventional technologies (IRENA (2018), Lazard (2018)).

mentation of such a random production strategy is assumed to have an *ex ante* production cost $\mathcal{C}(\mathbf{y})$. However to capture the idea that the electricity generation is dispatchable, we assume that this cost is additively separable state by state, i.e. $\mathcal{C}(\mathbf{y}) = \sum_{s=1}^{S} c_s(y_s)$. Moreover, we say that in each state inactivity is allowed and the production cost is increasing at an increasing rate, i.e. $c_s(0) = 0$, $\partial c_s(y_s) > 0$ and $\partial^2 c_s(y_s) > 0$ [10] and assume, as for the renewable energy sector, that the optimal production plan is derived from a competitive profit maximizing behavior.

The *consumption decision* is derived from utility maximization of a competitive representative agent. Within our partial equilibrium setting, we assume that this consumer derives his *ex ante* utility from his random electricity consumption $\mathbf{x} = (x_1, \ldots, x_S) \in \mathbb{R}_+^S$ and from money $m \in \mathbb{R}$ that he does not spend for the consumption of this good. We assume, as usually in a partial equilibrium setting, that his utility is linear in money, i.e. $U(\mathbf{x}, m) = \mathcal{U}(\mathbf{x}) + m$ and that the consumer owns a global amount of money, $m_0 > 0$. The utility associated to random electricity consumption is a continuously differentiable function $\mathcal{U}(\mathbf{x})$ which is is increasing and strictly concave, i.e. $\partial\mathcal{U}(\mathbf{x}) \gg 0$ and $\partial^2\mathcal{U}(\mathbf{x})$ is negative definite. We even assume that electricity is a desired good in each state of nature in the sense that for any sequence $\mathbf{x}_n$ of random electricity consumption with the property their exists a state s for which the consumption becomes zero, the global marginal utility, measured as norm of the gradient becomes large and reciprocally. More formally we say that $\forall \mathbf{x}_n \to x_0$, with some $x_{0,s} = 0$, $\lim_n \|\partial\mathcal{U}(\mathbf{x}_n)\| \to \infty$. Finally, let us remember that the consumer has no direct access to the electricity wholesale market but only to the set of contracts proposed by the retailer. His objective is therefore to choose, at price $\mathbf{q} \in \mathbb{R}_+^K$, a portfolio $\boldsymbol{\theta}_c \in \mathbb{R}^K$ of contracts which induces an electricity consumption of $\mathbf{x} = \mathbf{A}\boldsymbol{\theta}_c$ and which maximizes his utility under a standard budget constraint.

To summarize this discussion, let us now propose a definition of an equilibrium.

Definition 1. *An equilibrium of the contingent electricity markets and the different delivery contract markets is given by a vector* $(\boldsymbol{\theta}_c^*, m^*, \boldsymbol{\theta}_r^*, \mathbf{y}^*, \kappa^*, \mathbf{p}^*, \mathbf{q}^*) \in \mathbb{R}^{K+1} \times \mathbb{R}^K \times \mathbb{R}_+^S \times \mathbb{R}_+ \times \mathbb{R}_+^S \times \mathbb{R}_+^K$ *such that:*

(i) the representative consumer maximizes his utility i.e.

$$(\boldsymbol{\theta}_c^*, m^*) \in \underset{(\theta_c, m) \in \mathbb{R}^{K+1}}{\arg\max} \quad \mathcal{U}(\mathbf{A}\boldsymbol{\theta}_c) + m \quad s.t. \quad \begin{cases} \mathbf{A}\boldsymbol{\theta}_c \geq 0 \\ (\mathbf{q}^*)' \boldsymbol{\theta}_c + m = m_0 \end{cases} \tag{1}$$

(ii) the retailer maximizes profit, i.e.

$$\boldsymbol{\theta}_r^* \in \underset{\theta_r \in \mathbb{R}^K}{\arg\max} \quad \left((\mathbf{q}^*)' - (\mathbf{p}^*)' \mathbf{A}\right) \boldsymbol{\theta}_r \tag{2}$$

[10] The strictly convex production cost function can be explained by a variety of conventional power plants with exogenous production capacities and which are ordered by "merit", i.e., by increasing order of their marginal costs of production.

(iii) the conventional production plan maximizes profit, i.e.

$$\mathbf{y}^* \in \underset{\mathbf{y}\in\mathbb{R}^S_+}{\arg\max} \quad (\mathbf{p}^*)'\,\mathbf{y} - \textstyle\sum_{s=1}^{S} c_s\,(y_s) \tag{3}$$

(iv) the investment in the renewable technology is profit maximizing, i.e.

$$\kappa^* \in \underset{\kappa\in\mathbb{R}_+}{\arg\max} \quad \kappa\,(\mathbf{p}^*)'\,\mathbf{g} - \mathcal{K}(\kappa) \tag{4}$$

(v) the contract and contingent electricity markets clear, i.e.

$$\boldsymbol{\theta}_r^* = \boldsymbol{\theta}_c^* \quad \textit{and} \quad \mathbf{y}^* + \kappa^*\mathbf{g} = \mathbf{A}\boldsymbol{\theta}_r^* \tag{5}$$

3. Contract markets and the demand for electricity

Let us first remember that electricity delivery is indirectly obtained through our base contracts which work like financial assets. The existence of an equilibrium therefore requires a no-arbitrage condition. This means here, loosely speaking, that there exists no portfolio of contract which costs nothing and furnishes electricity in at least one state otherwise everybody would purchase this portfolio which is in contradiction with market clearing. To construct this no-arbitrage condition, let us define the $(S+1, K)$ matrix $\mathbf{W}$ given by $\mathbf{W} = \begin{bmatrix} -\mathbf{q}' \\ \mathbf{A} \end{bmatrix}$. If there exists a portfolio $\boldsymbol{\theta} \in \mathbb{R}^K$ with the property that $\mathbf{W}\boldsymbol{\theta} \geq \mathbf{0}$ with at least one strict inequality, we can conclude that there exists an arbitrage portfolio because this one pays back either money or electricity in at least one state $s_0 \in S$ without generating a cost in terms of money or electricity in the other states. The existence of an equilibrium therefore requires some additional restrictions. This is where Farkas' Lemma enters the story which claims that:

$$\nexists\boldsymbol{\theta}\in\mathbb{R}^K,\ \mathbf{W}\boldsymbol{\theta}\geq \mathbf{0} \text{ (at least one strict) } \Leftrightarrow \exists\boldsymbol{\beta}\in\mathbb{R}^{S+1},\ \boldsymbol{\beta}\gg \mathbf{0} \text{ and } \boldsymbol{\beta}'\mathbf{W} = \mathbf{0} \tag{6}$$

Moreover if such a $\boldsymbol{\beta}$ exists, the same must be true for the vector $\begin{pmatrix} 1 \\ \boldsymbol{\beta}_1 \end{pmatrix}$ where $\boldsymbol{\beta}_1\in\mathbb{R}^S_{++}$ is composed of the S last components of $\boldsymbol{\beta}$ divided by its first one. Owing to the block decomposition of $\mathbf{W}$, the no-arbitrage condition becomes:

$$\exists\boldsymbol{\beta}_1\in\mathbb{R}^S,\ \boldsymbol{\beta}_1\gg \mathbf{0} \text{ and } \mathbf{q} = \mathbf{A}'\boldsymbol{\beta}_1 \tag{7}$$

If we now move to the study of the consumer's demand for contracts, we need to restrict the set of asset prices to the open set Q given by :

$$\mathbf{q} \in Q = \left\{\mathbf{q} \in\mathbb{R}^K : \mathbf{q} = \mathbf{A}'\boldsymbol{\beta}_1 \text{ with } \boldsymbol{\beta}_1\in\mathbb{R}^S_{++}\right\} \tag{8}$$

Moreover, from Eq.(1) of definition 1, the demand for contracts, $\boldsymbol{\theta}_c(\mathbf{q})$, can be rewritten as:

$$\boldsymbol{\theta}_c(\mathbf{q}) = \underset{\boldsymbol{\theta}\in\mathbb{R}^K}{\arg\max} \quad \mathcal{U}\,(\mathbf{A}\boldsymbol{\theta}) - \mathbf{q}'\boldsymbol{\theta} \text{ s.t. } \mathbf{A}\boldsymbol{\theta} \geq \mathbf{0} \tag{9}$$

The existence of a solution of this unbounded problem is not really an issue if the no-arbitrage condition is satisfied (see proof of proposition 1). We can even claim that the constraint $\mathbf{A}\boldsymbol{\theta}\geq \mathbf{0}$ is never binding since we have assumed that the marginal utility becomes "large"as the consumption in one state goes to zero. Finally since $\mathcal{U}$ is strictly concave, we can say that the demand for contracts solves the following first order condition:

$$\mathbf{A}'\partial\mathcal{U}\left(\mathbf{A}\boldsymbol{\theta}\right)-\mathbf{q}=\mathbf{0} \tag{10}$$

Moreover by applying the Implicit Function Theorem to the above equation, one can compute the Jacobian of $\boldsymbol{\theta}_c(\mathbf{q})$ and show that this matrix is symmetric and negative definite. This means (i) the effect of the change of the price of contract k on the demand of contract k' is the same as the change of the price of contract k' on the demand of contract k and (ii) the demand for a contract is decreasing with its own price. We can even characterize the boundary behaviors of the contract demand. The first follows from the no-arbitrage condition. In fact, if we take a sequence $\mathbf{q}_n$ of contract prices free of arbitrage which converges to an arbitrage price, we can even expect that the sequence of optimal portfolio choice $\boldsymbol{\theta}_c(\mathbf{q}_n)$ solving Eq.(1) of definition 1 is unbounded, i.e. $\|\boldsymbol{\theta}(\mathbf{q}_n)\|\to\infty$, simply because at the limit the consumer is willing to buy an infinite amount of the arbitrage portfolio. The second follows from our boundary condition on the marginal utility associated to the first order condition (see Eq.(10)). It says that if some contract prices q_k become very high, i.e. $\|\mathbf{q}_n\|\to\infty$, the same must be true, by Eq.(10), for the marginal utility which induces that the electricity consumption in some states become zero. The next proposition summarizes these results.

Proposition 1. *The demand of contract is a differentiable function $\boldsymbol{\theta}_c : Q\to\mathbb{R}^K$ with the property that:*
(i) $\partial\boldsymbol{\theta}_c(\mathbf{q})=\left(\mathbf{A}'\partial^2\mathcal{U}\left(\mathbf{A}\boldsymbol{\theta}_c(\mathbf{q})\right)\mathbf{A}\right)^{-1}$ is a symmetric and negative definite matrix
(ii) If $\forall n,\ \mathbf{q}_n\in Q$ and $\mathbf{q}_n\to\mathbf{q}_0$ with $\mathbf{q}_0\in bd\left(Q\right)$ then $\|\boldsymbol{\theta}_c(\mathbf{q}_n)\|\to\infty$
(iii) If $\forall n,\ \mathbf{q}_n\in Q$ and $\|\mathbf{q}_n\|\to\infty$ then $\exists s_0\in S,\ \left(\mathbf{A}\boldsymbol{\theta}_c(\mathbf{q}_n)\right)_{s_0}\to 0$

Let us now move to the contract supply. From definition 1, we know that the portfolio supplied by the representative retailer solves $\max_{\boldsymbol{\theta}\in\mathbb{R}^K}\left(\mathbf{q}'-\mathbf{p}'\mathbf{A}\right)\boldsymbol{\theta}$. Since this program is linear in $\boldsymbol{\theta}$, we can immediately assert that a finite solution exists if and only if $\mathbf{q}=\mathbf{A}'\mathbf{p}$ Moreover at that portfolio price, his profit is zero and he is indifferent between any kind of supply $\boldsymbol{\theta}_r$. Under market clearing, he therefore supplies exactly what is asked by the consumer, i.e. the equilibrium contract markets quantity $\boldsymbol{\theta}\left(\mathbf{p}\right)=\boldsymbol{\theta}_c(\mathbf{A}'\mathbf{p})$ which is now a function of the state contingent electricity prices. But trading an amount of $\boldsymbol{\theta}\left(\mathbf{p}\right)$ contracts also engages the retailer to provide the consumer with the required amount of electricity in each state of nature. This state contingent demand of electricity $\mathbf{D}:\mathbb{R}^S_{++}\to\mathbb{R}^S$ is given by:

$$\mathbf{D}\left(\mathbf{p}\right)=\mathbf{A}\boldsymbol{\theta}_c(\mathbf{A}'\mathbf{p}) \tag{11}$$

This demand for electricity is therefore directly deduced from the consumer's contract demand and hires several properties induced by proposition 1. First of all, we notice that this demand for electricity is only defined for strictly positive prices, $\mathbf{p} \in \mathbb{R}^S_{++}$, otherwise the no arbitrage condition given by Eq.(7) is violated. In the same vein, if at least one of the contingent prices becomes zero, i.e. $\mathbf{p} \to \mathbf{p}_0$ with some $\mathbf{p}_{0,s} = 0$, the contract price vector $\mathbf{q}$ is no more free of arbitrage. The consumer will therefore take advantage of this opportunity to increase his electricity consumption by an adequate portfolio choice. One can therefore expect that the retail electricity demand which fulfills these contracts becomes large, i.e. $\|\mathbf{D}(\mathbf{p})\| \to \infty$. *A contrario*, if some contingent prices become large, i.e. $\|\mathbf{p}\| \to \infty$, we know since $\mathbf{A} \geq 0$ and $\forall s$, $\exists k$, $a_{sk} > 0$ that the contract prices must become large, $\|\mathbf{q}\| \to \infty$. It immediately follows from (iii) of proposition 1 that the electricity consumption, and therefore the demand of the retailer must go to zero in at least one state, i.e. $\exists s_0 \in S$, $(\mathbf{D}(\mathbf{p}_n))_{s_0} \to 0$. We can finally deduce the Jacobian of $\mathbf{D}(\mathbf{p})$ from that of $\boldsymbol{\theta}_c(\mathbf{q})$. This one is :

$$\partial \mathbf{D}(\mathbf{p}) = \mathbf{A} \partial \boldsymbol{\theta}_c(\mathbf{A}'\mathbf{p})\mathbf{A}' \tag{12}$$

This matrix is again symmetric and induces the same interpretation as earlier but it is only negative semi-definite. The explanation for this is quite obvious. If one considers a state contingent price change which does not affect the contract prices, i.e. a price change in the $\ker(\mathbf{A}')$ of dimension $S - K$, the contract demand does not change and therefore the electricity demand of the retailer as well. The next proposition summarizes this discussion.

Proposition 2. *The state contingent demand of electricity is a differentiable function* $\mathbf{D} : \mathbb{R}^S_{++} \to \mathbb{R}^S$ *which under the no-arbitrage condition is defined for strictly positive state contingent electricity prices* $\mathbf{p}$ *with the property that:*

(i) $\partial \mathbf{D}(\mathbf{p}) = \mathbf{A} \partial \boldsymbol{\theta}_c(\mathbf{A}'\mathbf{p})\mathbf{A}'$ *is a symmetric and negative semi-definite matrix*

(ii) If $\forall n$, $\mathbf{p}_n \in \mathbb{R}^S_{++}$ *and* $\mathbf{p}_n \to \mathbf{p}_0$ *with some* $\mathbf{p}_{0,s} = 0$ *then* $\|\mathbf{D}(\mathbf{p}_n)\| \to \infty$

(iii) If $\forall n$, $\mathbf{p}_n \in \mathbb{R}^S_{++}$ *and* $\|\mathbf{p}_n\| \to \infty$ *then* $\exists s_0 \in S$, $(\mathbf{D}(\mathbf{p}_n))_{s_0} \to 0$

4. The state contingent electricity supply and the market equilibrium

The production level of the conventional sector is obtained by solving the profit maximization program given by Eq.(3) of definition 1. This sector has the ability to adjust its production plan in each state since the contingent cost is additively separable. It follows that in each state the optimal production level simply equates the state contingent price to the marginal cost in state s:

$$\forall s \in S,\ p_s = \partial c_s(y_s) \Leftrightarrow y_s = (\partial c_s)^{-1}(p_s) \tag{13}$$

The contingent supply of the conventional sector is therefore given by $\mathbf{Y} : \mathbb{R}^S_{++} \to \mathbb{R}^S_{+}$ with $\mathbf{Y}(\mathbf{p}) = \left((\partial c_s)^{-1}(p_s)\right)_{s \in S}$. Under our assumption on the different cost function $c_s(y)$, we can claim as usual that the supply in each state, $y_s(p_s)$, is increasing with the state contingent price because

$$\frac{dy_s(p_s)}{dp_s} = \frac{1}{\partial^2 c\left((\partial c_s)^{-1}(p_s)\right)} > 0 \tag{14}$$

and satisfies usual boundary conditions: for a low price the supply goes to zero, i.e. $\lim_{p_s \to 0} y_s(p_s) = 0$ while for a large price, supply becomes infinite, i.e. $\lim_{p_s \to +\infty} y_s(p_s) = +\infty$. Moreover, since the production decision is taken independently state by state, we can see that the Jacobian of this contingent supply, $\partial \mathbf{Y}(\mathbf{p})$, is a diagonal matrix, $\mathcal{D}$, the s^{th} diagonal term being $\frac{dy_s(p_s)}{dp_s} > 0$. Since this quantity is positive, we can even say that $\partial \mathbf{Y}(\mathbf{p})$ is positive definite.

Contrary to the conventional sector, the intermittent renewable energy sector is not able to adjust its production state by state. It chooses, *ex ante*, a production capacity which provides a random production level $\mathbf{g}$ per unit of installed capacity. This capacity choice is obtained by solving the profit maximization program given by Eq.(4) of definition 1. Since $\mathbf{p}$ are state contingent prices, the optimal capacity choice will be simply obtained by equating the expected additional expected return, $\mathbf{p}'\mathbf{g}$ of a new unit of capacity, to its marginal cost, i.e.

$$\partial \mathcal{K}(\kappa) = \mathbf{p}'\mathbf{g} \Leftrightarrow \kappa = (\partial \mathcal{K})^{-1} (\mathbf{p}'\mathbf{g}) \tag{15}$$

The state contingent supply of the intermittent renewable energy sector, $\mathbf{I} : \mathbb{R}^S_{++} \to \mathbb{R}^S_+$, being of :

$$\mathbf{I}(\mathbf{p}) = \left((\partial \mathcal{K})^{-1} (\mathbf{p}'\mathbf{g}) \right) \mathbf{g} \tag{16}$$

This contingent production is therefore not very reactive to price changes since the capacity choice is only responsive to the change of the expected returns. This fact can be easily observed by computing the Jacobian of $\mathbf{I}(\mathbf{p})$ which is given by :

$$\partial \mathbf{I}(\mathbf{p}) = \frac{1}{\partial^2 \mathcal{K}\left((\partial \mathcal{K})^{-1} (\mathbf{g}'\mathbf{p}) \right)} \mathbf{g}'\mathbf{g} \tag{17}$$

a matrix of rank 1 since every price change which is orthogonal to the random production plan $\mathbf{g}$ does not modify the investment and this set of prices is typically a subset of dimension $S - 1$. But we can nevertheless observe that this matrix remains semi-positive definite since $\partial^2 \mathcal{K} > 0$. More precisely :

$$\forall \mathbf{v} \neq 0,\ \mathbf{v}' \partial \mathbf{I}(\mathbf{p}) \mathbf{v} = \frac{1}{\partial^2 \mathcal{K}\left((\partial \mathcal{K})^{-1} (\mathbf{g}'\mathbf{p}) \right)} (\mathbf{g}'\mathbf{v})^2 \geq 0 \tag{18}$$

But this state contingent intermittent production has another interesting property. In fact, as long as one assumes that the production of an installed unit is always strictly positive and even very small, i.e. $\mathbf{g} \gg \mathbf{0}$, the production level becomes, under our standard assumptions on $\mathcal{K}$, infinite in each state when the prices $\mathbf{p} \in \mathbb{R}^S_{++}$ become large, i.e. $\|\mathbf{p}\| \to \infty$.

At this point, we can now construct the contingent electricity supply $\mathbf{S}(\mathbf{p}) = \mathbf{Y}(\mathbf{p}) + \mathbf{I}(\mathbf{p})$ and summarize our results in the next proposition.

Proposition 3. *The contingent electricity supply* $\mathbf{S} : \mathbb{R}^S_{++} \to \mathbb{R}^S_+$*, given by* $\mathbf{S}(\mathbf{p}) = \mathbf{Y}(\mathbf{p}) + \mathbf{I}(\mathbf{p})$*, is a differentiable function with the property that:*

(i) $\partial \mathbf{S}(\mathbf{p}) = \mathcal{D} + \frac{1}{\partial^2 \mathcal{K}\left((\partial \mathcal{K})^{-1} (\mathbf{g}'\mathbf{p}) \right)} \mathbf{g}'\mathbf{g}$ *with* $\mathcal{D}$ *a diagonal matrix of the generic term* $\frac{1}{\partial^2 c\left((\partial c_s)^{-1} (p_s) \right)}$

(ii) $\partial\mathbf{S}(\mathbf{p})$ *is positive definite*
(iii) If $\forall n$, $\mathbf{p}_n \in \mathbb{R}^S_{++}$ *and* $\|\mathbf{p}_n\| \to \infty$ *then* $\forall s \in S$, $(\mathbf{S}(\mathbf{p}))_s \to +\infty$
(iv) If $\forall n$, $\mathbf{p}_n \in \mathbb{R}^S_{++}$ *and* $\mathbf{p}_n \to \mathbf{p}_0$ *with some* $\mathbf{p}_{0,s} = 0$, $\mathbf{S}(\mathbf{p}_n) \to \mathbf{S}(\mathbf{p}_0) \geq \mathbf{0}$ *and finite*

We now can move to market clearing as given by Eq.(5) of definition 1 and first consider the state contingent electricity markets. From proposition 2, we conclude that state contingent demand of electricity as depicted by the function $\mathbf{D}(\mathbf{p})$ is monotonically decreasing while supply, given by the function $\mathbf{S}(\mathbf{p})$ from proposition 3, is monotonically increasing. Subject to the boundary behaviors of $\mathbf{D}(\mathbf{p})$ and $\mathbf{S}(\mathbf{p})$, we can say that there exists a unique set of prices $\mathbf{p}^* \in \mathbb{R}^S_{++}$ that brings demand and supply on the different state contingent electricity markets into equilibrium. Finally, knowing the relationship between free-arbitrage contract prices $\mathbf{q}$ and $\mathbf{p}$, we deduce that the different contract markets are cleared at price $\mathbf{q}^* = \mathbf{A}'\mathbf{p}^* \in \mathbb{R}^K_{++}$.

Proposition 4. *There exists a unique contingent price vector* $\mathbf{p}^* \in \mathbb{R}^S_{++}$ *which clears the different state contingent electricity markets and an associated electricity delivery contract price vector* $\mathbf{q}^* = \mathbf{A}'\mathbf{p}^* \in \mathbb{R}^K_{++}$ *which is free of arbitrage and clears the different contract markets.*

5. Equilibrium and Welfare: the missing market issue

Up to now, we have shown that one can consider an equilibrium structure in which sophisticated state-contingent delivery contracts may exist. The basic questions which remain open concern the richness of the available base contracts and the effect of their change on the equilibrium of the electricity market. Our aim here and in the next section is to point out that the contract structure matters as long as this one is unable to provide an effective adjustment of the demand to the fluctuations of the supply. In our model, this directly follows from the fact that the number of base contracts, K, is smaller than S, the number of states of nature. This restricts the potential contingent electricity consumptions to the linear subspace generated by the columns of $\mathbf{A}$, i.e. $\mathbf{x} \in span(\mathbf{A})$. There are, in some sense, missing markets (with $rank(\mathbf{A}) < S$).

To gain a better understanding of this problem, let us come back to the consumer's program ((i) of definition 1). His portfolio choice is essentially motivated by the electricity consumption that he obtains. So if he faces two contract structures $\mathbf{A}$ and $\mathbf{A}'$ with the property that $span(\mathbf{A}) = span(\mathbf{A}')$, he simply adjusts his demand for this contract in order to maintain the same state contingent electricity consumption. In other words, we can say that $\mathbf{A}$ is equivalent to $\mathbf{A}'$ in terms of electricity demand, i.e. $\mathbf{A} \sim_e \mathbf{A}'$. Since $\sim_e$ is an equivalence relation, we can even restrict our attention to contracts induced by a representative element of each indifference class of $\sim_e$. To identify this element, we introduce a simplifying assumption which say that the full rank matrix $\mathbf{A}$ can be decomposed into $\begin{bmatrix} \mathbf{A}_{S-K} \\ \mathbf{A}_K \end{bmatrix}$ where $\mathbf{A}_K$ is an invertible matrix of dimension K and $\mathbf{A}_{S-K}$

a matrix of dimension $(S-K,K)$[11]. Under this assumption, by simply changing the portfolio to $\boldsymbol{\theta}_{\mathbf{B}} = \mathbf{A}_K \boldsymbol{\theta}_{\mathbf{A}}$, any electricity allocation, $\mathbf{x} = \mathbf{A}\boldsymbol{\theta}$, can be obtained with the equivalent contract structure:

$$\begin{bmatrix} \mathbf{B} \\ \mathbf{I}_K \end{bmatrix} \text{with } \mathbf{B} = \mathbf{A}_{S-K} \left(\mathbf{A}_K\right)^{-1} \tag{19}$$

This observation clearly suggests that we can take a matrix $\begin{bmatrix} \mathbf{B} \\ \mathbf{I}_K \end{bmatrix}$ as a representative element of a family of contract structure given by:

$$\mathbf{A} = \begin{bmatrix} \mathbf{BC} \\ \mathbf{C} \end{bmatrix} \text{with } \mathbf{C} \text{ any invertible matrix of dimension } K \tag{20}$$

This observation has another consequence. If we restrict our attention to delivery contracts given by $\begin{bmatrix} \mathbf{B} \\ \mathbf{I}_K \end{bmatrix}$, we can say that the contingent electricity consumption, $\mathbf{x}$, satisfies:

$$\mathbf{x} = \begin{bmatrix} \mathbf{B} \\ \mathbf{I}_K \end{bmatrix} \boldsymbol{\theta} \Leftrightarrow \begin{cases} \boldsymbol{\theta} = (x_s)_{s=S-K+1}^{K} \\ \begin{bmatrix} \mathbf{I}_{S-K} & -B \end{bmatrix} \mathbf{x} = \mathbf{0} \end{cases} \tag{21}$$

This clearly says, for $K < S$, the potential electricity consumption that our consumer can obtain from the delivery contract structure is limited to the set of all $\mathbf{x}$ which satisfies

$$\begin{bmatrix} \mathbf{I}_{S-K} & -\mathbf{B} \end{bmatrix} \mathbf{x} = \mathbf{0} \tag{22}$$

This means the existence of a limited amount of delivery contracts reduces the trade capacity of the consumer and restricts the adjustment capacity of the electricity demand to the fluctuations of the supply. To summarize, we can say :

Proposition 5. *Under our assumptions on the base contract structure* $\mathbf{A}$*:*
(i) The equivalent contract structure as given in Eq.(19) provides the same electricity allocation at the same contingent prices. Only contract prices and portfolio adjust and are given by $(\mathbf{q_B}, \boldsymbol{\theta}_{\mathbf{B}}) = \left(\left(\mathbf{A_K^{-1}}\right)' \mathbf{q_A}, \mathbf{A}_K \boldsymbol{\theta}_{\mathbf{A}}\right)$
(ii) Reciprocally, the electricity allocation and prices induced by the contract structure as given in Eq.(19) remain the same for every element of the family given by Eq.(20). The associated contract price and portfolio are $(\mathbf{q_A}, \boldsymbol{\theta}_{\mathbf{A}}) = (\mathbf{C}'\mathbf{q_B}, \mathbf{C}^{-1}\boldsymbol{\theta}_{\mathbf{B}})$
(iii) A contract structure $\mathbf{A}$ *equivalent to the structure as given in Eq.(19) induces a restriction on electricity trades given by* $\begin{bmatrix} \mathbf{I}_{S-K} & -\mathbf{B} \end{bmatrix} \mathbf{x} = \mathbf{0}$

[11] The reader can be surprised by this assumption since even if $\mathbf{A}$ is of full rank, it is not obvious that the last K rows form an invertible matrix. This property can nevertheless be obtained by introducing required permutations. We however decide, for the simplicity of the discussion, to introduce this assumption instead of using permutation matrices which unnecessarily complicate notations.

From this proposition, we will mainly restrict our attention to the contract structure given by $\begin{bmatrix} \mathbf{B} \\ \mathbf{I}_K \end{bmatrix}$ in the rest of this paper. But point (iii) of proposition 5 shows another aspect of the introduction of intermittent renewables. In fact, as long as the set of contracts proposed to the consumer is not large enough, i.e. $K < S$, his potential state contingent consumptions are constrained by Eq.(22). It is therefore impossible to reach a first best allocation (unless this one satisfies this set of restrictions). In other words, even in our competitive setting, Pareto-optimality is out of reach and it is always interesting to see how changes in the contract structure affect welfare. To address this question, let us first introduce a notion of constrained efficiency:

Definition 2. *An electricity production plan and allocation* $(\tilde{\mathbf{y}}, \tilde{\kappa}, \tilde{\mathbf{x}}) \in \mathbb{R}_+^{S+1} \times \mathbb{R}_+^S$ *is constrained efficient iff it solves :*

$$SW(\mathbf{B}) = \max_{(\mathbf{y},\kappa,\mathbf{x})\in\mathbb{R}_+^{2S+1}} \mathcal{U}(\mathbf{x}) - \mathcal{C}(\mathbf{y}) - \mathcal{K}(\kappa) \ s.t. \begin{cases} \mathbf{x} - \mathbf{y} - \kappa\mathbf{g} = \mathbf{0} \\ \begin{bmatrix} \mathbf{I}_{S-K} & -\mathbf{B} \end{bmatrix} \mathbf{x} = \mathbf{0} \end{cases} \tag{23}$$

If we denote by $\boldsymbol{\lambda} = (\lambda_s)_{s=1}^S$ and $\boldsymbol{\mu} = (\mu_i)_{i=1}^{S-K}$ Lagrangian multipliers associated respectively with the first and second set of constraints, we obtain the following first order conditions

$$\begin{cases} \partial\mathcal{U}(\mathbf{x}) - \boldsymbol{\lambda} - \begin{bmatrix} \mathbf{I}_{S-K} \\ -\mathbf{B}' \end{bmatrix} \cdot \boldsymbol{\mu} = \mathbf{0} \\ -\partial\mathcal{C}(\mathbf{y}) + \boldsymbol{\lambda} = \mathbf{0} \\ -\frac{d\mathcal{K}(\kappa)}{d\kappa} + \mathbf{g}' \cdot \boldsymbol{\lambda} = 0 \end{cases} \tag{24}$$

If we identify, as usually, the Lagrangian multiplier, $\boldsymbol{\lambda}$, to a contingent price vector, $\mathbf{p}$, we immediately observe that the second and the third conditions of Eq.(24) are exactly the same as the profit maximization conditions for, respectively, the conventional sector (see Eq.(13)) and the intermittent sector (see Eq.(15)). A same observation can be made for the first condition of Eq.(24) but it requires a little transformation. In fact:

$$\partial\mathcal{U}(\mathbf{x}) - \boldsymbol{\lambda} - \begin{bmatrix} \mathbf{I}_{S-K} \\ -\mathbf{B}' \end{bmatrix} \cdot \boldsymbol{\mu} = \mathbf{0} \Leftrightarrow \begin{cases} \begin{bmatrix} \mathbf{B} \\ \mathbf{I}_K \end{bmatrix}' (\partial\mathcal{U}(\mathbf{x}) - \boldsymbol{\lambda}) = \mathbf{0} \\ \boldsymbol{\mu} = (\partial_{x_s}\mathcal{U}(\mathbf{x}) - \lambda_s)_{s=1}^{S-K} \end{cases} \tag{25}$$

The first condition of Eq.(25) becomes very close to the consumer's first order condition given by Eq.(10). One simply has to remember that (i) we work with an equivalent contract structure and (ii) by the no-arbitrage condition and the identification of $\boldsymbol{\lambda}$ to $\mathbf{p}$, we must have that $\mathbf{q} = \begin{bmatrix} \mathbf{B} \\ \mathbf{I}_K \end{bmatrix}' \boldsymbol{\lambda}$. The second condition defines $\boldsymbol{\mu}$ whose interpretation is non-trivial in the context of our framework. Remember that $\boldsymbol{\mu}$ is tied to the set of constraints on electricity allocation as defined by condition 2 of Eq.(23). One can therefore expect that $\boldsymbol{\mu} \neq \mathbf{0}$ otherwise all constraints on trades become ineffective. In other words, the first best consumption plan satisfies these restrictions on trades induced by the contract structure: a situation which surely does not resist to a slight perturbation of the utility function. In the next section in which we consider the effect of changing contracts, we will even sometimes assume that all components of $\boldsymbol{\mu}$ are different from $\mathbf{0}$ meaning, loosely speaking, that the direction to the first best is a non-zero linear combination of all the restrictions on trades (i.e. the different lines of $\left[\begin{array}{cc} \mathbf{I}_{S-K} & -\mathbf{B} \end{array} \right]$).

In any case the above largely suggests that:

Proposition 6. *Any competitive electricity production plan and allocation issued from definition 1 is a constrained efficient allocation given by definition 2 and reciprocally.*

6. Changing contracts

As long as welfare is concerned and at least one constraint on trade is effective (i.e. $\boldsymbol{\mu} \neq \mathbf{0}$), it is obvious that the introduction of an additional base contract to an existing contract structure $\mathbf{A}$ will be welfare improving as long as this one is linearly independent of the existing ones. The intuition is clear. If we move from a matrix $\mathbf{A}_0$ of dimension (S, K) to $\mathbf{A}_1$ of dimension $(S, K+1)$ by adding a new column, the different electricity consumption profiles $\mathbf{x} = \mathbf{A}_0\boldsymbol{\theta}$ initially available remains reachable by simply not purchasing the new contract. Hence $span(\mathbf{A_0}) \subset span(\mathbf{A_1})$. This clearly means, if one remembers the definition of a constraint efficient allocation (see definition 2), that moving from a contract structure $\mathbf{A}_0$ to $\mathbf{A}_1$ must be welfare improving.

Instead of adding a new base contract, we now identify which changes of the existing contracts can be Pareto improving. Let us start with a given contract structure $\mathbf{A}$ or its equivalent matrix $\mathbf{B}$. We can show (see proof of proposition 7) by a standard Envelope Theorem applied to the optimization problem of definition 2 that: $\forall s = 1, \ldots, S-K$, $k = 1, \ldots, K$,

$$\partial_{b_{sk}} SW(\mathbf{B}) = \mu_s x_{S-K+k} = (\partial_{x_s}\mathcal{U}(\mathbf{x}) - \lambda_s)\, x_{S-K+k} \tag{26}$$

Considering the first best electricity consumption does not satisfy the trade constraints, i.e. $\boldsymbol{\mu} \neq \mathbf{0}$, (see our discussion in the previous section), we identify state s and asset k for which $|\mu_s|\, x_{S-K+k}$ is maximal. The above equation says that the largest effect on welfare can be obtained by increasing or decreasing b_{sk}, whenever $\mu_s > 0$ or < 0. The question which remains open is the impact of changing b_{sk} on the base contract structure $\mathbf{A}$. Since several contract structures correspond to $\mathbf{B}$ (see (ii) of proposition 5), any change $d\mathbf{B}$ in this matrix induces several potential changes $d\mathbf{A}$ for $\mathbf{A}$. Knowing $\mathbf{B} = \mathbf{A}_{S-K}\left(\mathbf{A}_K\right)^{-1}$ (see (i) of proposition 5), one simply has to make sure that:

$$d\mathbf{A}_{S-K} = (d\mathbf{B})\,\mathbf{A}_K + (d\mathbf{B})\,(d\mathbf{A}_K) \text{ for any arbitrary choice of } d\mathbf{A}_K \tag{27}$$

For instance, if $(s, k)_0$ with $\mu_{s_0} > 0$ is the most improving component of $\mathbf{B}$, we obtain the same effect when we change the terms $a_{(sk)_0}$ and a_{S-K+k_0,s_0} in $\mathbf{A}$ by:

$$\left\{ \begin{array}{l} da_{(sk)_0} = (a_{S-K+k_0,s_0} + cst)\, db_{(sk)_0} \\ da_{S-K+k_0,s_0} = cst \end{array} \right. \quad \text{for any constant } cst \tag{28}$$

We summarize this discussion by:

Proposition 7. *Concerning the welfare effect of a change of the contract structure* $\mathbf{A}$*, we can say as long as* $\boldsymbol{\mu} \neq \mathbf{0}$ *that*
(i) any addition of a new contract linearly independent of the existing ones improves welfare.
(ii) the best welfare improving option is to change the coefficient $(s,k)_0$ *of the equivalent matrix* $\mathbf{B}$ *for which* $(s,k)_0 = \arg\max_{(s,k)} \{|\mu_s| x_{S-K+k}\}$*. This can be done by changing* $\mathbf{A}$ *as described in Eq.(28) (or in Eq.(27) for more general changes).*

We now move to the effect of changing $\mathbf{B}$ on equilibrium investment in renewable capacity, $\kappa^* = (\partial\mathcal{K})^{-1}(\mathbf{g}'\mathbf{p}^*)$ (see Eq.(15)). For this, it is important to understand how changes in $\mathbf{B}$ affect the contingent equilibrium prices $\mathbf{p}^*$ since

$$d\kappa^* = \frac{1}{\partial^2\mathcal{K}\left((\partial\mathcal{K})^{-1}(\mathbf{g}'\mathbf{p}^*)\right)}\mathbf{g}'\partial_{\mathbf{B}}\mathbf{p}^*\mathbf{dB} \tag{29}$$

For this exercise, we reformulate the definition of an equilibrium price. To get the intuition, let us start with the consumer's first order condition given by Eq.(10) by replacing $\mathbf{A}'$ by the equivalent contract structure $\begin{bmatrix} \mathbf{B}' & \mathbf{I}_K \end{bmatrix}$. By market clearing, (i) the consumption will be equal to the supply $\mathbf{S}(\mathbf{p})$ and (ii) the contract price must be given by $\mathbf{q} = \begin{bmatrix} \mathbf{B}' & \mathbf{I}_K \end{bmatrix}\mathbf{p}$. If we add to the story the idea that the supply $\mathbf{S}(\mathbf{p})$ must satisfy the restriction on trades imposed by the contract matrix, we can construct a function $f : \mathbb{R}^S_+ \times \mathbb{R}^{(S-K)K} \to \mathbb{R}^S$ given by:

$$f(\mathbf{p},\mathbf{B}) = \begin{cases} \begin{bmatrix} \mathbf{B}' & \mathbf{I}_K \end{bmatrix}(\partial\mathcal{U}(\mathbf{S}(\mathbf{p})) - \mathbf{p}) \\ \begin{bmatrix} \mathbf{I}_{S-K} & -\mathbf{B} \end{bmatrix}\mathbf{S}(\mathbf{p}) \end{cases} \tag{30}$$

The zero of Eq.(30) is our state-contingent equilibrium price vector $\mathbf{p}^*$. The effect of a change of $\mathbf{B}$ on $\mathbf{p}^*$ can therefore be obtained by applying the Implicit Function Theorem to $f(\mathbf{p},\mathbf{B}) = \mathbf{0}$.

Lemma 1. *We observe that:*
(i) $\partial_p f(\mathbf{p},\mathbf{B})$ *is invertible and by the Implicit Function Theorem:*

$$\partial_{\mathbf{B}}\mathbf{p}^*(\mathbf{B}) = [\partial_p f(\mathbf{p},\mathbf{B})]^{-1}\begin{bmatrix} \mathbf{I}_K \otimes \left(\left(\partial_{x_s}U(\mathbf{S}(\mathbf{p})) - p_s\right)_{s=1}^{S-K}\right)' \\ \left(-\left(\mathbf{S}_{S-K+k}(\mathbf{p})\right)_{k=1}^{K}\right)' \otimes \mathbf{I}_{S-K} \end{bmatrix} \tag{31}$$

($\otimes$ *denotes the tensor product)*
(ii) if only one component of $\left(\partial_{x_s}U(\mathbf{S}(\mathbf{p})) - p_s\right)_{s=1}^{S-K}$ *is different from* 0*, then* $\partial_{\mathbf{B}}\mathbf{p}^*(\mathbf{B})$ *is of rank* $S-1$*.*
(iii) if at least two components of $\left(\partial_{x_s}U(\mathbf{S}(\mathbf{p})) - p_s\right)_{s=1}^{S-K}$ *are different from* 0*, then* $\partial_{\mathbf{B}}\mathbf{p}^*(\mathbf{B})$ *is of full rank* S*. This implicitly requires that* $1 < K < S-1$[12].

[12]If $K = S-1$, the vector $\left(\partial_{x_s}U(\mathbf{S}(\mathbf{p})) - p_s\right)_{s=1}^{S-K}$ contains only one component while for $K=1$, $\partial_{\mathbf{B}}\mathbf{p}^*(\mathbf{B})$ only contains $S-1$ columns.

Let us now try to understand the effect of a change in $\mathbf{B}$ on investment in renewable capacity. From Eq.(29), we begin by observing the induced price change $\mathbf{dP} = \partial_{\mathbf{B}}\mathbf{p}^{*}d\mathbf{B}$ on investment in renewable capacity. The set of all reachable $\mathbf{dP}$ being a linear space, we can conclude, still from Eq.(29), that if there exists a $\mathbf{dP} \subset g^{\perp}$ with $g^{\perp}$ the $S-1$ dimensional hyperplane orthogonal to $\mathbf{g}$, then investment in renewable capacity can be improved by a contract change $\mathbf{dB}$. This occurs in case *(iii)* of Lemma 1 since $\partial_{\mathbf{B}}\mathbf{p}^{*}(\mathbf{B})$ is of full rank. As a surjective mapping, we can even define the most efficient direction of the induced price change which solves $\max_{\mathbf{dP}} \mathbf{g}' \frac{\mathbf{dP}}{\|\mathbf{dP}\|}$ and is given by any $\mathbf{dP}$ collinear to $\mathbf{g}$. If we now have in mind that $\mathbf{dP} : \mathbb{R}^{K(S-K)} \longrightarrow \mathbb{R}^{S}$, then there even exists, by the Rank Theorem, a subset of contract change associated to this most efficient direction of price change. This subset of $\mathbf{dB}$ is of dimension $K(S-K)-S$. We can therefore say:

Proposition 8. *If at least two components of $\left(\partial_{x_s}U\left(\mathbf{S}(\mathbf{p})\right)-p_s\right)_{s=1}^{S-K}$ are different from 0 and $1<K<S-1$, all the directions of price changes which improve investment in renewables can be reached, especially the one which is collinear to $\mathbf{g}$ and which "maximizes" the penetration of renewables. Moreover each of these improving directions can be obtained by a subset of dimension $(K(S-K)-S)$ of changes in $\mathbf{B}$.*

Now, what does at least two components of $\left(\partial_{x_s}U\left(\mathbf{S}(\mathbf{p})\right)-p_s\right)_{s=1}^{S-K}$ are different from 0 mean? To answer this question, let us recall Proposition 6 and Eq.(25). Proposition 6 says that a competitive equilibrium is a constrained efficient allocation obtained by solving the optimization program given by Definition 2. As for Eq.(25) which says at least two constraints on trade are effective in this optimization program, a rather generic case, is in fact depicted by the restriction of Proposition 8. Of course if this restriction is not fulfilled, then the result on investment in renewable capacity holds only if there exists a $\mathbf{dP}$ that does not belong to $g^{\perp}$.

Proposition 8 also suggests that there are many ways to change the contract structure in order to improve investment in renewables. This therefore asks two additional questions. First, can we improve both welfare and investment in renewables? The answer to this is quite obvious. Let us come back (see Eq.(29) and Eq.(26)) to the vectors $\partial_{B}\kappa^{*}$ and $\partial_{B}SW$ of dimension $(1, K(S-K))$. These two gradients point towards the best improving direction of investment in renewables and social welfare respectively. So as long as these two vectors do not point to exactly two opposite directions, we can conclude (at least intuitively) that there exists changes in the contract structure which improve both investment and welfare.

Let us now move to the second question. Can we ensure that an increase in investment in renewables reduces the production of the conventional sector in each state of nature? The answer is unfortunately no. Since the conventional sector adjusts its production level to the observed (*ex post*) price by equating the price and the marginal cost, production in state s only decreases if p_S decreases. A lower conventional production level in each state therefore requires lower prices in each state. But this reduces the expected returns, $\mathbf{g}'\mathbf{p}$, of unit investment in renewables and therefore total investment. This observation mainly drives our negative result.

More formally, by using the Gordan's form of Farkas' Lemma, we can say:

Proposition 9. *(i) As long as $\partial_B \kappa^*$ and $\partial_B SW$ are not collinear with a negative coefficient, there exists changes in* $\mathbf{B}$ *which strictly improve welfare and investment in renewables.*
(ii) It is nevertheless impossible to find a change in $\mathbf{B}$ *which both increases investment in renewables and reduces production of conventional electricity in each state.*

7. Conclusion

The deployment of intermittent renewables introduces pressure on the grid which calls for more flexibility on the electricity market. Focusing on demand-side flexibility, we have addressed the question if diversified retail contracts at different prices can ease the penetration of intermittent renewables. We have modeled intermittency by a contingent electricity market and diversified retail contracts by a set of base state-contingent electricity delivery contracts. First, we have shown existence and uniqueness of a competitive equilibrium of the contingent wholesale and retail markets. Secondly, assuming a limited number of base delivery contracts which constrain electricity allocations, we have been abled to find that the electricity market equilibrium and social welfare are constraint efficient. Finally, we have described the conditions under which changing the structure of the base contracts can improve welfare, the degree of integration of renewable capacity and both. We have also found that it is impossible to find a change in the contracts which both increases investment in renewable capacity and reduces production of conventional electricity in each state of nature.

The results of the paper firstly provide insights on how the role of retailers can be redefined, for example, in the context of liberalized retail electricity markets. The base delivery contracts modeled here can be a tool for retailers to propose diversified contracts that can trigger demand-side flexibility. Secondly, the results are also useful from the perspective of policy implications and highlight the importance of accounting for intermittency in order to achieve renewable capacity objectives.

Several extensions of this model can be expected. The first one has to do with the accounting of carbon emissions. The model can be used to design an economic model for transitioning to a decarbonized electricity mix by considering both intermittency of renewables and carbon emissions from fossil fuels. Ambec and Crampes (2019) examine how the presence of policy instruments (e.g., carbon tax, feed-in tariffs and renewable portfolio standards) affect the socially efficient energy mix with intermittent renewables. This literature can be complemented by investigating how, in the context of a contingent market with renewables, the decision strategy between an *ex-ante* Pigouvian tax and *ex-post* trade of carbon emissions permits matters.

A second extension can be to include supply flexibility in the model through storage of electricity. However, this question cannot be directly addressed with the static model considered here since storage is intrinsically a dynamic one. By extending the model to a dynamic framework, it may be interesting to determine, in the context of missing markets, the optimal decision strategy to store and deliver stored electricity given the intermittent nature of renewables. This is in line with a recent literature such as Pommeret and Schubert (2019). The authors focus on a social planner's problem and propose one of the first dynamic models of optimal transition from fossil-fueled technologies to renewables-based that includes intermittency of renewables together with storage.

Thirdly, throughout this paper, we have made the assumption that the wholesale and retail markets are perfectly competitive. It may therefore be worthwhile to investigate how the model behaves with market power. Joskow and Tirole (2007) and Rouillon (2015), by considering demand and supply intermittency respectively, show that when conventional producers own transmission and distribution networks, investment in renewables become less attractive. With the idea that sophisticated retail contracts can be a solution to manage supply intermittency, we can think of tapping into a contract instrument to address the question of market power distortions in the electricity market.

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Appendix

Appendix A. Proof of Proposition 1

(o)Existence and uniqueness of the contract choice $\boldsymbol{\theta}_c(\mathbf{q})$

Let us study the consumer program given by Eq.(1) of definition 1 for all $\mathbf{q} \in Q$. First observe that the global utility $\mathcal{U}(\mathbf{A}\boldsymbol{\theta}_\mathbf{c}) + m$ is increasing in m. We can therefore say that the optimal solution necessarily belongs to:

$$B = \left\{(\boldsymbol{\theta}, m) \in \mathbb{R}^{K+1} : \ \mathbf{A}\boldsymbol{\theta} \geq \mathbf{0} \text{ and } \mathbf{q}'\boldsymbol{\theta} + m = m_0\right\} \tag{A.1}$$

This set is obviously non-empty and closed. So if we show that B is also bounded, hence compact, we know that this program has a solution. To verify this property, let us first observe that $\forall(\boldsymbol{\theta}, m) \in B$, m is bounded from above. In fact since $\mathbf{A}\boldsymbol{\theta} \geq \mathbf{0}$, we can say by the no-arbitrage condition (see Eq.(7)) that $\boldsymbol{\beta}_1'\mathbf{A}\boldsymbol{\theta} = \mathbf{q}'\boldsymbol{\theta} \geq \mathbf{0}$ (remember that $\boldsymbol{\beta}_1 \gg 0$). It follows that $m = m_0 - \mathbf{q}'\boldsymbol{\theta}$ is bounded from above by m_0. Let us now show that $\forall(\boldsymbol{\theta}, m) \in B$, $\boldsymbol{\theta}$ is bounded. Assume the contrary, i.e. there exists a sequence $(\boldsymbol{\theta}_n, m_n) \in B$ with the property that $\|\boldsymbol{\theta}_n\| \to \infty$ and define $\boldsymbol{\vartheta}_n = \frac{\boldsymbol{\theta}_n}{\|\boldsymbol{\theta}_n\|}$. Since $\boldsymbol{\vartheta}_n$ belongs to the unit circle of $\mathbb{R}^K$ which is a compact set, $\boldsymbol{\vartheta}_n$ admits a converging subsequence whose limit is $\boldsymbol{\vartheta}_0$. Because $(\boldsymbol{\theta}_n, m_n) \in B$, we can also say that $\forall n$, $\mathbf{A}\boldsymbol{\vartheta}_n \geq \mathbf{0}$ and $\mathbf{q}_n'\boldsymbol{\vartheta} + \frac{m_n}{\|\boldsymbol{\theta}_n\|} \leq \frac{m_0}{\|\boldsymbol{\theta}_n\|}$ and since m_n is bounded from above and m_0 finite, we deduce that $\mathbf{A}\boldsymbol{\vartheta}_0 \geq \mathbf{0}$ and $\mathbf{q}'\boldsymbol{\vartheta}_0 \leq 0$. By the no-arbitrage condition, neither one component of $\mathbf{A}\boldsymbol{\vartheta}_0$ nor of $\mathbf{q}'\boldsymbol{\vartheta}_0$ can be strictly positive otherwise $\boldsymbol{\vartheta}_0$ is an arbitrage portfolio. It follows in particular that $\mathbf{A}\boldsymbol{\vartheta}_0 = 0$ and since $\mathbf{A}$ is of full rank, this implies that $\boldsymbol{\vartheta}_0 = 0$. But this is the desired contradiction since by construction $\|\boldsymbol{\vartheta}_0\| = 1$. Finally, since $m = m_0 - \mathbf{q}'\boldsymbol{\theta}$ with $\boldsymbol{\theta}$ bounded, we can say that m is not only bounded from above but also from below.

Let us now move to the uniqueness issue. We have seen from Eq. (9) that the optimal portfolio choice can be obtained by solving the equivalent program given by:

$$\max_{\boldsymbol{\theta} \in \mathbb{R}^K} \left\{ \underbrace{\mathcal{U}(\mathbf{A}\boldsymbol{\theta}) - \mathbf{q}'\boldsymbol{\theta}}_{=f(\boldsymbol{\theta},\mathbf{q})} \text{ s.t. } \mathbf{A}\boldsymbol{\theta} \geq \mathbf{0} \right\} \tag{A.2}$$

Seeing that the set of feasible solutions ($\mathbf{A}\boldsymbol{\theta} \geq \mathbf{0}$) is convex, if $f(\boldsymbol{\theta})$ is strictly concave, we know that the solution is unique and is given by the continuous function $\boldsymbol{\theta}_c : Q \to \mathbb{R}^K$. So let us verify that the Hessian of $f(\boldsymbol{\theta}, \mathbf{q})$ with respect to $\boldsymbol{\theta}$ is negative definite. By computation $\forall \mathbf{v} \in \mathbb{R}^K$ and $\mathbf{v} \neq 0$

$$\begin{aligned} \mathbf{v}'\partial^2_{\boldsymbol{\theta},\boldsymbol{\theta}} f(\boldsymbol{\theta}, \mathbf{q})\mathbf{v} &= \mathbf{v}'\left(\mathbf{A}'\partial^2\mathcal{U}(\mathbf{A}\boldsymbol{\theta})\mathbf{A}\right)\mathbf{v} \\ &= (\mathbf{A}\mathbf{v})'\,\partial^2\mathcal{U}(\mathbf{A}\boldsymbol{\theta})(\mathbf{A}\mathbf{v}) \end{aligned} \tag{A.3}$$

Let us now remember that $\mathbf{A}$ is a (S, K) matrix of full rank with $K < S$. It follows that for $\mathbf{v} \neq 0$, $\mathbf{h} = \mathbf{A}\mathbf{v} \neq 0$ and since $\mathcal{U}$ is strictly concave, we can conclude that $\mathbf{v}'\partial^2 f(\boldsymbol{\theta})\mathbf{v} < 0$

(i) $\boldsymbol{\theta}_c : Q \to \mathbb{R}^K$ is differentiable and its Jacobian $\partial\boldsymbol{\theta}_c(\mathbf{q}) = \left(\mathbf{A}'\partial\mathcal{U}(\mathbf{A}\boldsymbol{\theta}(\mathbf{q}))A\right)^{-1}$ is negative definite

Let us first show that the constraints $\mathbf{A}\boldsymbol{\theta}_c(\mathbf{q}) \geq \mathbf{0}$ are non binding at an optimal solution of Eq.(A.2). Assume the contrary. This means $\exists \mathbf{q}_0 \in Q$ and $\boldsymbol{\theta}_c(\mathbf{q}_0)$ with the property that for at least one s, the s^{th} component $\left(\mathbf{A}\boldsymbol{\theta}_c(\mathbf{q}_0)\right)_s = 0$. Moreover, since the optimization problem given by Eq.(A.2) is differentiable, it should also satisfy the Karush-Kuhn-Tucker conditions. So if $\boldsymbol{\lambda}_0 \geq 0$ denotes the Lagrangian multipliers, we should have:

$$\mathbf{A}'\partial\mathcal{U}(\mathbf{A}\boldsymbol{\theta}_c(\mathbf{q}_0)) - \mathbf{q}_0 + \mathbf{A}'\boldsymbol{\lambda}_0 = 0 \tag{A.4}$$

But let us now define a sequence $\boldsymbol{\theta}_n \to \boldsymbol{\theta}_c(\mathbf{q}_0)$ such that $\forall n$, $\mathbf{A}\boldsymbol{\theta}_n \gg 0$ and construct the sequence

$$\mathbf{t}_n = \mathbf{A}'\left(\underbrace{\frac{\partial\mathcal{U}(\mathbf{A}\boldsymbol{\theta}_n)}{\|\partial\mathcal{U}(\mathbf{A}\boldsymbol{\theta}_n)\|} + \frac{\boldsymbol{\lambda}_0}{\|\partial\mathcal{U}(\mathbf{A}\boldsymbol{\theta}_n)\|}}_{=\mathbf{y}_n}\right) - \frac{\mathbf{q}_0}{\|\partial\mathcal{U}(\mathbf{A}\boldsymbol{\theta}_n)\|} \tag{A.5}$$

Since $\mathbf{A}\boldsymbol{\theta}_n \to \mathbf{A}\boldsymbol{\theta}_0$ which contains a zero component, we know, by assumption on the boundary behavior of the utility function, that $\|\partial\mathcal{U}(\mathbf{A}\boldsymbol{\theta}_n)\| \to \infty$. Now observe that $\frac{\partial\mathcal{U}(\mathbf{A}\boldsymbol{\theta}_c(\mathbf{q}_n))}{\|\partial\mathcal{U}(\mathbf{A}\boldsymbol{\theta}_c(\mathbf{q}_n))\|} \to \mathbf{z} \geq \mathbf{0}$ at least for a subsequence and $\mathbf{z}$ belong to the unit circle of $\mathbb{R}^S$, i.e. with at least one strictly positive component, say, $s' \in S$. Because $\boldsymbol{\lambda}_0 \geq 0$, we deduce that $\exists N$, $\forall n > N$, $(\mathbf{y}_n)_{s'} \geq z_{s'} > 0$. Let us now remember that $\mathbf{A} \geq \mathbf{0}$ and $\forall s$, $\exists k$, $a_{sk} > 0$. This means $\forall n > N$ at least one component of $\mathbf{A}'\mathbf{y}_n$, say component k, verifies $(\mathbf{A}'\mathbf{y}_n)_k \geq a_{sk} z_{s'} > 0$. Moreover $\frac{\mathbf{q}_{0,k}}{\|\partial\mathcal{U}(\mathbf{A}\boldsymbol{\theta}_c(\mathbf{q}_n))\|} \to \mathbf{0}$ which means that $\forall\varepsilon \in (0, a_{sk} z_{s'})$, there exists $N' > N$ with the property $\forall n > N'$ $(\mathbf{A}'\mathbf{y}_n)_k - \frac{\mathbf{q}_{0,k}}{\|\partial\mathcal{U}(\mathbf{A}\boldsymbol{\theta}_c(\mathbf{q}_n))\|} \geq \varepsilon > 0$. In other words, the k^{th} condition of Eq.(A.4) cannot be satisfied at the limit, otherwise this contradicts optimality.

From the previous observation, we deduce that the necessary and sufficient condition for optimality of Eq.(A.2), is given by:

$$\phi(\boldsymbol{\theta}_c, \mathbf{q}) = \mathbf{A}'\partial\mathcal{U}(\mathbf{A}\theta_c) - \mathbf{q} = 0 \tag{A.6}$$

Now observe that $\partial_{\boldsymbol{\theta}_c}\phi(\boldsymbol{\theta}_c, \mathbf{q}) = \partial^2_{\boldsymbol{\theta},\boldsymbol{\theta}} f(\boldsymbol{\theta}, \mathbf{q})$ (see Eq.(A.3)) is negative definite and therefore of full-rank. It follows by the Implicit Function Theorem that $\boldsymbol{\theta}_c : Q \to \mathbb{R}^K$ is differentiable and its Jacobian is:

$$\partial\boldsymbol{\theta}_c(\mathbf{q}) = \left(\mathbf{A}'\partial^2\mathcal{U}(\mathbf{A}\boldsymbol{\theta}(\mathbf{q}))\,\mathbf{A}\right)^{-1} \tag{A.7}$$

Moreover $\partial\boldsymbol{\theta}_c(\mathbf{q})$ is the inverse of a symmetric and negative definite matrix and therefore shares this last property.

(ii) If $\forall n$, $\mathbf{q}_n \in Q$ and $\mathbf{q}_n \to \mathbf{q}_0$ with $\mathbf{q}_0 \in bd(Q)$ then $\|\boldsymbol{\theta}(\mathbf{q}_n)\| \to \infty$

Assume the contrary, i.e. $\exists K > 0$, $\forall n$, $\|\boldsymbol{\theta}(\mathbf{q}_n)\| < K$, this means for at least one subsequence, $\boldsymbol{\theta}(\mathbf{q}_n) \to \theta_0$. By continuity of the optimization problem given by Eq.(A.2), $\boldsymbol{\theta}_0$ should therefore be a solution at price $\mathbf{q}_0$. But remember that $\mathbf{q}_n \to \mathbf{q}_0$ with $\mathbf{q}_0 \in bd(Q)$. This means by Eq.(6) that there exists, for $\mathbf{q}_0$, an arbitrage portfolio $\boldsymbol{\theta}_1$ with the property that $\mathbf{A}\boldsymbol{\theta}_1 \geq 0$ and $(\mathbf{q}_0)'\boldsymbol{\theta}_1 \leq 0$ with at least one strict inequality. It follows that the portofolio $\boldsymbol{\theta}_2 = \boldsymbol{\theta}_0 + \boldsymbol{\theta}_1$ satisfies the constraint of problem (A.2) since $\mathbf{A}\boldsymbol{\theta}_2 = \mathbf{A}\boldsymbol{\theta}_0 + \mathbf{A}\boldsymbol{\theta}_1 \geq 0$ and, under the assumption that $\partial\mathcal{U}(x) \gg 0$,

$$\mathcal{U}(\mathbf{A}\boldsymbol{\theta}_2) - (\mathbf{q}_0)'\boldsymbol{\theta}_2 = \mathcal{U}(\mathbf{A}\boldsymbol{\theta}_0 + \mathbf{A}\boldsymbol{\theta}_1) - (\mathbf{q}_0)'\boldsymbol{\theta}_0 - (\mathbf{q}_0)'\boldsymbol{\theta}_1 > \mathcal{U}(\mathbf{A}\boldsymbol{\theta}_0) - (\mathbf{q}_0)'\boldsymbol{\theta}_0 \tag{A.8}$$

which contradicts the fact that $\boldsymbol{\theta}_0$ is a solution at price $\mathbf{q}_0$.

(iii) If $\forall n$, $\mathbf{q}_n \in Q$ and $\|\mathbf{q}_n\| \to \infty$ then $\exists s_0 \in S$, $(\mathbf{A}\boldsymbol{\theta}(\mathbf{q}_n))_{s_0} \to 0$

From the optimality condition given by Eq.(A.6), we know $\forall n$, $\mathbf{q}_n = \mathbf{A}'\partial\mathcal{U}(\mathbf{A}\boldsymbol{\theta}(\mathbf{q_n}))$. So if $\|\mathbf{q}_n\| \to \infty$, we can then say that $\|\partial\mathcal{U}(\mathbf{A}\boldsymbol{\theta}(\mathbf{q_n}))\| \to \infty$. From our assumption on the boundary behavior of the utility function, we conclude that $\exists s_0 \in S$, $(\mathbf{A}\boldsymbol{\theta}(\mathbf{q_n}))_{s_0} \to 0$.

Appendix B. Proof of Proposition 2

(i) $\partial\mathbf{D}(\mathbf{p}) = \mathbf{A}\partial\boldsymbol{\theta}_c(\mathbf{A}'\mathbf{p})\mathbf{A}'$ is symmetric and negative semi-definite matrix

Computing $\partial\mathbf{D}(\mathbf{p})$ is a simple exercise. This matrix is symmetric since $\partial\boldsymbol{\theta}_c(\mathbf{q})$ is symmetric. But it is only negative semi-definite even if $\partial\boldsymbol{\theta}_c(\mathbf{q})$ is negative definite since $dim(\ker(\mathbf{A}')) = S - K > 0$. This induces that $\forall\mathbf{h} \in \ker(\mathbf{A}')$, $\mathbf{h}'\partial\mathbf{D}(\mathbf{p})\,\mathbf{h} = 0$

(ii) If $\forall n$, $\mathbf{p}_n \in \mathbb{R}^S_{++}$ and $\mathbf{p}_n \to \mathbf{p}_0$ with some $\mathbf{p}_{0,s} = 0$ then $\|\mathbf{D}(\mathbf{p}_n)\| \to \infty$

If $\forall n$, $\mathbf{p}_n \in \mathbb{R}^S_{++}$ and $\mathbf{p}_n \to \mathbf{p}_0$ with some $p_{0,s} = 0$, we know that $\mathbf{q}_n = \mathbf{A}'\mathbf{p}_n \in Q$ but at the limit $\mathbf{q}_0 \in bd(Q)$. It follows, from (ii) of proposition 1, that $\|\boldsymbol{\theta}(\mathbf{q}_n)\| \to \infty$. Now observe that $\|\mathbf{D}(\mathbf{p}_n)\| = \|\boldsymbol{\theta}(\mathbf{q}_n)\| \left\|\mathbf{A}\frac{\boldsymbol{\theta}(\mathbf{q}_n)}{\|\boldsymbol{\theta}(\mathbf{q}_n)\|}\right\|$. Since $\frac{\boldsymbol{\theta}(\mathbf{q}_n)}{\|\boldsymbol{\theta}(\mathbf{q}_n)\|} \in S^k$, the unit sphere and A is of full rank, $\mathbf{A}\frac{\boldsymbol{\theta}(\mathbf{q}_n)}{\|\boldsymbol{\theta}(\mathbf{q}_n)\|} \to z \neq 0$. It follows that $\|\mathbf{D}(\mathbf{p}_n)\| \to \infty$.

(iii) If $\forall n$, $\mathbf{p}_n \in \mathbb{R}^S_{++}$ and $\|\mathbf{p}_n\| \to \infty$ then $\exists s_0 \in S$, $(\mathbf{D}(\mathbf{p}_n))_{s_0} \to 0$

Notice that when $\|\mathbf{p}_n\| \to \infty$, then $\|\mathbf{q}_n\| = \|\mathbf{A}'\mathbf{p}_n\| \to \infty$ since $\mathbf{p}_n \geq \mathbf{0}$, $\mathbf{A} \geq \mathbf{0}$ and $\forall s$, $\exists k$, $a_{sk} > 0$. This result then follows directly from (iii) of proposition 1.

Appendix C. Proof of Proposition 3

(i) $\partial\mathbf{S}(\mathbf{p}) = \mathcal{D} + \frac{1}{\partial^2\mathcal{K}\left((\partial\mathcal{K})^{-1}(\mathbf{g}'\mathbf{p})\right)}\mathbf{g}'\mathbf{g}$ with $\mathcal{D}$ a diagonal matrix of generic term $\frac{1}{\partial^2 c\left((\partial c_s)^{-1}(p_s)\right)}$

This follows directly from Eq.(14) ad Eq.(17).

(ii) $\partial\mathbf{S}(\mathbf{p})$ is positive definite

$\partial\mathbf{Y}(\mathbf{p})$ is a diagonal matrix $\mathcal{D}$ with positive terms and is therefore positive definite. $\partial\mathbf{I}(\mathbf{p})$ is positive semi-definite (see Eq.(18)). Their sum is therefore positive definite.

(iii) If $\forall n,\ \mathbf{p}_n \in \mathbb{R}^S_{++}$ and $\|\mathbf{p}_n\| \to \infty$ then $\forall s \in S,\ (\mathbf{S}(\mathbf{p}))_s \to +\infty$

If $\forall n,\ \mathbf{p}_n \in \mathbb{R}^S_{++}$ and $\|\mathbf{p}_n\| \to \infty$, we know from the property of the intermittent production that $\forall s \in S,\ (\mathbf{I}(\mathbf{p}))_s \to +\infty$. This is sufficient to show that $\forall s \in S,\ (\mathbf{S}(\mathbf{p}))_s \to +\infty$ since $\mathbf{S}(\mathbf{p}) = \mathbf{Y}(\mathbf{p}) + \mathbf{I}(\mathbf{p})$ and $\mathbf{Y}(\mathbf{p}) \geq \mathbf{0}$.

(iv) If $\forall n,\ \mathbf{p}_n \in \mathbb{R}^S_{++}$ and $\mathbf{p}_n \to \mathbf{p}_0$ with some $p_{0,s} = 0$, $\mathbf{S}(\mathbf{p}_n) \to \mathbf{S}(\mathbf{p}_0) \geq \mathbf{0}$ and finite

If $\forall n,\ \mathbf{p}_n \in \mathbb{R}^S_{++}$ and $\mathbf{p}_n \to \mathbf{p}_0$ with some $p_{0,s} = 0$, we know that for the conventional sector $\lim_{p_s \to 0} y_s(p_s) = 0$. This results in $\mathbf{Y}(\mathbf{p}_n) \to \mathbf{Y}(\mathbf{p}_0) \geq \mathbf{0}$ and finite. As for the intermittent sector, with some $\mathbf{p}_{0,s} = 0$, we have $\mathbf{I}(\mathbf{p}_n) \to \mathbf{I}(\mathbf{p}_0) \geq \mathbf{0}$ and finite. Consequently, $\mathbf{S}(\mathbf{p}_n) \to \mathbf{S}(\mathbf{p}_0) \geq \mathbf{0}$ and finite.

Appendix D. Proof of Proposition 4

(i) Existence

The proof is essentially based on a homotopy argument. An intuitive presentation can be found in Eaves and Schmedders (1999) (for a more detailed argument see Villanacci et al. (2002) ch.7 or Hirsch (1976) ch.5). Following this presentation, a complex equation system $\mathbf{f}(\mathbf{p}) = \mathbf{0}$, here the function $\mathbf{f} : \mathbb{R}^S_{++} \to \mathbb{R}^S$ with $\mathbf{f}(\mathbf{p}) = \mathbf{S}(\mathbf{p}) - \mathbf{D}(\mathbf{p})$, has a solution if there exists (i) a simple equation system $\mathbf{g}(\mathbf{p}) = \mathbf{0}$, here the function $\mathbf{g} : \mathbb{R}^S_{++} \to \mathbb{R}^S$ with $\mathbf{g}(\mathbf{p}) = \mathbf{p} - \hat{\mathbf{p}}$, $\hat{\mathbf{p}} \gg \mathbf{0}$ given, and (ii) a homotopy $\mathbf{H} : [0,1] \times \mathbb{R}^S_{++} \to \mathbb{R}^S$ given by $\mathbf{H}(\mathbf{p}, \lambda) = \lambda\mathbf{f}(\mathbf{p}) + (1-\lambda)\,\mathbf{g}(\mathbf{p})$ with the property that :

- $\mathbf{g}(\mathbf{p})$ admits a unique and regular solution. This is the case here since (i) $\mathbf{g}(\mathbf{p}) = \mathbf{0} \Leftrightarrow \mathbf{p} = \hat{\mathbf{p}}$ and (ii) $\partial\mathbf{g}|_{\mathbf{p}=\hat{\mathbf{p}}} = \mathbf{I}_S$, the identity matrix of dimension S, a matrix obviously of full rank.
- $\mathbf{0}$ is a regular value of $\mathbf{H}$, meaning that for all $(\mathbf{p}, \lambda) \in H^{-1}(\mathbf{0})$, $\partial\mathbf{H}|_{(\mathbf{p},\lambda)}$ is a surjection. This is for instance the case if $\partial\mathbf{H}$ is of full rank. Here, the sub-matrix $\partial_p\mathbf{H}$ of $\partial\mathbf{H}$ is given by:

$$\partial_p\mathbf{H} = \lambda\mathbf{I}_S + (1-\lambda)\left(\partial\mathbf{S}(\mathbf{p}) - \partial\mathbf{D}(\mathbf{p})\right) \tag{D.1}$$

  Since $\partial\mathbf{S}(\mathbf{p})$ is positive definite (see point (ii) of proposition 3) and $\partial\mathbf{D}(\mathbf{p})$ negative semi-definite (see point (i) of proposition 2), $\partial_p\mathbf{H}$ is also positive definite and therefore of rank S. It follows for all $(\mathbf{p}, \lambda) \in H^{-1}(0)$, $\partial\mathbf{H}|_{(\mathbf{p},\lambda)}$ is a surjection.
- $\mathbf{H}^{-1}(\mathbf{0})$ is a compact subset of $[0,1] \times \mathbb{R}^S_{++}$

It simply remains to check this last point. Assume the contrary there exists a sequence $(\mathbf{p}_n, \lambda_n) \in \mathbf{H}^{-1}(\mathbf{0})$ with the property that either $\|\mathbf{p}_n\| \to \infty$ or $\mathbf{p}_n \to \mathbf{p}_0$ with some $p_{0,s} = 0$. If $\lambda_n \to \lambda_0 = 0$, the only point in $\mathbf{H}^{-1}(\mathbf{0})$ is $(\mathbf{p}_0, \lambda)$. So let us assume in the rest of the argument that $\lambda_0 > 0$. In the first case, we know by (iii) of proposition 2 that $\exists s_0 \in S,\ (\mathbf{D}(\mathbf{p}_n))_{s_0} \to 0$ and by (iii) of proposition 3 that $\forall s \in S,\ (\mathbf{S}(\mathbf{p}_n))_s \to +\infty$. This implies that $\exists s_0 \in S,\ (\mathbf{H}(\mathbf{p}_n, \lambda_n))_{s_0} \to +\infty$. It therefore exists, for each $K > 0$, a rank N such that $\forall n > N$, $(\mathbf{H}(\mathbf{p}_n, \lambda_n))_{s_0} > K > 0$ which is the desired contradiction. In the second case, we know by (ii) of proposition 2 that $\|\mathbf{D}(\mathbf{p}_n)\| \to \infty$. But remember that $\mathbf{p}_n \to \mathbf{p}_0$ finite, this means that $\mathbf{S}(\mathbf{p}_0)$ as well as $\mathbf{g}(\mathbf{p})$ are finite. Hence $\exists s_0 \in S,\ (\mathbf{H}(\mathbf{p}_n, \lambda_n))_{s_0} \to -\infty$, which is again the desired contradiction.

(i) Uniqueness

This argument is mainly based on the degree theory (see Hirsch (1976) ch.5, or Villanacci et al. (2002) ch.7). In fact, from the previous point, we also deduce that the two maps **f** and **g** have the same degree, i.e. $\deg(\mathbf{f}) = \deg(\mathbf{g})$. Moreover, if one defines for a given regular map **h** the quantity $ind_{\mathbf{h}}(\mathbf{p}) = \frac{\det(\partial \mathbf{h}|_{\mathbf{p}})}{|\det(\partial \mathbf{h}|_{\mathbf{p}})|}$, we know that $\deg(\mathbf{h}) = \sum_{\mathbf{p}\in\mathbf{h}^{-1}(0)} ind_{\mathbf{h}}(\mathbf{p})$. It follows, by computation, that the degree of our simple map **g** is 1, hence $\sum_{\mathbf{p}\in\mathbf{f}^{-1}(0)} ind_{\mathbf{f}}(\mathbf{p}) = 1$. Let us now remember that $\partial\mathbf{f}(\mathbf{p}) = \partial\mathbf{S}(\mathbf{p}) - \partial\mathbf{D}(\mathbf{p})$ is a positive definite matrix. Its determinant is therefore always positive. This implies in particular that $\forall \mathbf{p} \in \mathbf{f}^{-1}(0)$, $ind_{\mathbf{f}}(\mathbf{p}) = 1$ and since these quantities sum to 1, the solution is unique.

Appendix E. Proof of Proposition 4

(i) Let $(\boldsymbol{\theta}_c^*, m^*, \boldsymbol{\theta}_r^*, \mathbf{y}^*, \kappa^*, \mathbf{p}^*, \mathbf{q}^*)_{\mathbf{A}}$ be an equilibrium with the contract structure $\mathbf{A} = \begin{bmatrix} \mathbf{A}_{S-K} \\ \mathbf{A}_K \end{bmatrix}$ and $\mathbf{A}_K$ invertible satisfying definition 1. Now define

$$(\boldsymbol{\vartheta}^*, m^*, \boldsymbol{\vartheta}^*, \mathbf{y}^*, \kappa^*, \mathbf{p}^*, \mathbf{q}^*)_{\mathbf{B}} = \left(\mathbf{A}_K\boldsymbol{\theta}_{c,\mathbf{A}}^*, m_{\mathbf{A}}^*, \mathbf{A}_K\boldsymbol{\theta}_{r,\mathbf{A}}^*, \mathbf{y}_{\mathbf{A}}^*, \kappa_{\mathbf{A}}^*, \mathbf{p}_{\mathbf{A}}^*, \left(\mathbf{A}_{\mathbf{K}}^{-1}\right)' \mathbf{q}_{\mathbf{A}}^*\right) \tag{E.1}$$

and let us verify that this vector also satisfies 1 with contract structure of $\begin{bmatrix} \mathbf{A}_{S-K}\left(\mathbf{A}_K\right)^{-1} \\ \mathbf{I}_K \end{bmatrix}$. With regard to point (i) of definition 1, we observe that we obtain the new budget constraint by simply operating a change of variable by replacing $\boldsymbol{\theta}$ by $\boldsymbol{\theta} = \left(\mathbf{A}_K\right)^{-1}\boldsymbol{\vartheta}$, i.e.

$$\begin{cases} \mathbf{A}\boldsymbol{\theta} \geq \mathbf{0} \\ \left(\mathbf{q}_{\mathbf{A}}^*\right)'\boldsymbol{\theta} + m = m_0 \end{cases} \Leftrightarrow \begin{cases} \begin{bmatrix} \mathbf{A}_{S-K}\left(\mathbf{A}_K\right)^{-1} \\ \mathbf{I}_K \end{bmatrix}\boldsymbol{\vartheta} \geq \mathbf{0} \\ \left(\underbrace{\left(\mathbf{A}_{\mathbf{K}}^{-1}\right)'\mathbf{q}_A^*}_{=\mathbf{q}_{\mathbf{B}}^*}\right)'\boldsymbol{\vartheta} + m = m_0 \end{cases} \tag{E.2}$$

The solution of this new program $(\boldsymbol{\vartheta}^*, m^*)_{\mathbf{B}} = \left(\mathbf{A}_K\boldsymbol{\theta}_{c,\mathbf{A}}^*, m_{\mathbf{A}}^*\right)$ is therefore the same up to, of course, this change of variable. If we now move to (ii) of definition 1, the reader knows from our early discussion (see section 2), that the retailer maximizes his profit and that the contract market clears if the following relation is satisfied $\mathbf{q}^* = \mathbf{A}'\mathbf{p}^*$. It is a matter of fact to verify, since $\mathbf{p}_{\mathbf{B}}^* = \mathbf{p}_{\mathbf{A}}^*$, that:

$$\mathbf{q}_{\mathbf{A}}^* = \mathbf{A}'\mathbf{p}_{\mathbf{A}}^* \Leftrightarrow \left(\mathbf{A}_{\mathbf{K}}^{-1}\right)'\mathbf{q}_{\mathbf{A}}^* = \left(\mathbf{A}_{\mathbf{K}}^{-1}\right)'\mathbf{A}'\mathbf{p}_{\mathbf{A}}^* \Leftrightarrow \mathbf{q}_{\mathbf{B}}^* = \begin{bmatrix} \left(\mathbf{A}_{S-K}\left(\mathbf{A}_K\right)^{-1}\right)' & \mathbf{I}_k \end{bmatrix}\mathbf{p}_{\mathbf{B}}^* \tag{E.3}$$

Moreover (iii) and (iv) of definition 1 are not affected by the contract change, the electricity supply remains therefore unchanged. So if the retailer's demand remains unchanged, the proof is finished. This new demand, at price $\mathbf{p}_{\mathbf{B}}^* = \mathbf{p}_{\mathbf{A}}^*$, is, from Eq.(11) given by:

$$\mathbf{D}_{\mathbf{B}}(\mathbf{p}_{\mathbf{B}}^*) = \begin{bmatrix} \mathbf{A}_{S-K}\left(\mathbf{A}_K\right)^{-1} \\ \mathbf{I}_K \end{bmatrix}\boldsymbol{\vartheta}^* = \mathbf{A}\boldsymbol{\theta}^* = \mathbf{D}_{\mathbf{A}}\left(\mathbf{p}_{\mathbf{A}}^*\right) \tag{E.4}$$

(ii) Conversely, let $(\boldsymbol{\vartheta}_c^*, m^*, \boldsymbol{\vartheta}_r^*, \mathbf{y}^*, \kappa^*, \mathbf{p}^*, \mathbf{q}^*)_{\mathbf{B}}$ be an equilibrium with the contract structure $\begin{bmatrix} \mathbf{B} \\ \mathbf{I}_K \end{bmatrix}$ satisfying definition 1 and let $\mathbf{C}$ be a K dimensional invertible matrix. Define:

$$(\boldsymbol{\theta}^*, m^*, \boldsymbol{\theta}^*, \mathbf{y}^*, \kappa^*, \mathbf{p}^*, \mathbf{q}^*)_{\mathbf{A}} = \left(\mathbf{C}^{-1}\boldsymbol{\vartheta}_B^*, m_{\mathbf{B}}^*, \mathbf{C}^{-1}\boldsymbol{\vartheta}_B^*, \mathbf{y}_{\mathbf{B}}^*, \kappa_{\mathbf{B}}^*, \mathbf{p}_{\mathbf{B}}^*, \mathbf{C}'\mathbf{q}_{\mathbf{B}}^*\right) \tag{E.5}$$

and let us verify that this vector satisfies definition 1 with $\mathbf{A} = \begin{bmatrix} \mathbf{BC} \\ \mathbf{C} \end{bmatrix}$. First notice that by replacing $\boldsymbol{\vartheta}$ by $\boldsymbol{\vartheta} = \mathbf{C}\boldsymbol{\theta}$, the budget constraints become

$$\left\{ \begin{array}{l} \begin{bmatrix} \mathbf{B} \\ \mathbf{I}_K \end{bmatrix} \boldsymbol{\vartheta} \geq \mathbf{0} \\ (\mathbf{q}^*_{\mathbf{B}})' \boldsymbol{\vartheta} + m = m_0 \end{array} \right. \Leftrightarrow \left\{ \begin{array}{l} \mathbf{A}\boldsymbol{\theta} \geq \mathbf{0} \\ \Bigg(\underbrace{\mathbf{C}'\mathbf{q}^*_{\mathbf{B}}}_{\mathbf{q}^*_{\mathbf{A}}} \Bigg)' \boldsymbol{\theta} + m = m_0 \end{array} \right. \tag{E.6}$$

It follows that the new solution of (i) of definition 1 is, at price $\mathbf{q}^*_{\mathbf{A}}$, given by $(\boldsymbol{\theta}^*_{\mathbf{A}}, m^*_{\mathbf{A}}) = \left(\mathbf{C}^{-1}\boldsymbol{\vartheta}^*_B, m^*_{\mathbf{B}}\right)$. It is also immediate to verify that the relation between the contract and the state contingent prices are maintained since:

$$\mathbf{q}^*_{\mathbf{B}} = \begin{bmatrix} \mathbf{B}' & \mathbf{I}_K \end{bmatrix} \mathbf{p}^*_{\mathbf{B}} \Leftrightarrow \mathbf{C}'\mathbf{q}^*_{\mathbf{B}} = \mathbf{A}'\mathbf{p}^*_{\mathbf{B}} \Leftrightarrow \mathbf{q}^*_{\mathbf{A}} = \mathbf{A}'\mathbf{p}^*_{\mathbf{A}} \tag{E.7}$$

Finally we also notice that the electricity demand at price $\mathbf{p}^*_{\mathbf{A}} = \mathbf{p}^*_{\mathbf{B}}$ remains the same since:

$$\mathbf{D}_{\mathbf{A}}(\mathbf{p}^*_{\mathbf{A}}) = \begin{bmatrix} \mathbf{BC} \\ \mathbf{C} \end{bmatrix} \mathbf{C}^{-1}\boldsymbol{\vartheta}^*_B = \begin{bmatrix} \mathbf{B} \\ \mathbf{I}_K \end{bmatrix} \boldsymbol{\vartheta}^*_{\mathbf{B}} = \mathbf{D}_{\mathbf{B}}(\mathbf{p}^*_{\mathbf{B}}) \tag{E.8}$$

We can therefore conclude that $(\boldsymbol{\theta}^*, m^*, \boldsymbol{\theta}^*, \mathbf{y}^*, \kappa^*, \mathbf{p}^*, \mathbf{q}^*)_{\mathbf{A}}$ with $\mathbf{A} = \begin{bmatrix} \mathbf{BC} \\ \mathbf{C} \end{bmatrix}$ for all choice of $\mathbf{C}$, a K dimensional invertible matrix.

(iii) directly follows from the computation given by Eq.(21)

Appendix F. Proof of Proposition 6

This result follows directly from our identification of the First Order Conditions.

Appendix G. Proof of Proposition 7

From our discussion in section 5, we only need to verify that Eq.(26) holds. So let us consider $SW(\mathbf{B})$ introduced in definition 2 and denote by $(\boldsymbol{\lambda}(\mathbf{B}), \boldsymbol{\mu}(\mathbf{B}))$, the Lagrangian multipliers associated to the constraints. In order to compute the derivative of $SW(\mathbf{B})$, we adopt the following convention: we differentiate per assets (per column of $\mathbf{B}$) and do this for each asset. $\partial_B SW(\mathbf{B}, \mathbf{g})$ is therefore a vector of dimension $(1, (S-K)K)$ which satisfies by using the FOC (see Eq.(24)):

$$\begin{aligned} \partial SW(\mathbf{B}) &= (\partial\mathcal{U}(\mathbf{x}(\mathbf{B})))' \partial\mathbf{x}(\mathbf{B}) - (\partial\mathcal{C}(\mathbf{x}(\mathbf{B})))' \partial\mathbf{y}(\mathbf{B}) - \tfrac{d\mathcal{K}(\kappa(\mathbf{B}))}{d\kappa} \partial\kappa(\mathbf{B}) \\ &= (\boldsymbol{\lambda}(\mathbf{B}))' (\partial\mathbf{x}(\mathbf{B}) - \partial\mathbf{y}(\mathbf{B}) - \mathbf{g}\partial\kappa(\mathbf{B})) + (\boldsymbol{\mu}(\mathbf{B}))' \begin{bmatrix} \mathbf{I}_{S-K} & -\mathbf{B} \end{bmatrix} \partial\mathbf{x}(\mathbf{B}) \end{aligned} \tag{G.1}$$

Using the first set of constraints of Eq.(23) as an identity, we know that:

$$\partial\mathbf{x}(\mathbf{B}) - \partial\mathbf{y}(\mathbf{B}) - \mathbf{g} \cdot \partial\kappa(\mathbf{B}) = \mathbf{0} \tag{G.2}$$

It follows that:

$$\partial SW(\mathbf{B}) = (\boldsymbol{\mu}(\mathbf{B}))' \begin{bmatrix} \mathbf{I}_{S-K} & -\mathbf{B} \end{bmatrix} \partial\mathbf{x}(\mathbf{B}) \tag{G.3}$$

By using now the second set of constraints of Eq.(23) as an identity, we get after computation that:

$$\begin{bmatrix} \mathbf{I}_{S-K} & -\mathbf{B} \end{bmatrix} \partial\mathbf{x}(\mathbf{B}) - \left((x_{S-K+k}(\mathbf{B}))_{k=1}^K \right)' \otimes \mathbf{I}_{S-K} = \mathbf{0} \tag{G.4}$$

where $\otimes$ denotes the tensor product. We can therefore say that:

$$\begin{aligned} \partial SW(\mathbf{B}) &= (\boldsymbol{\mu}(\mathbf{B}))' \left(\left((x_{S-K+k}(\mathbf{B}))_{k=1}^K \right)' \otimes \mathbf{I}_{S-K} \right) \\ &= \left(x_{S-K+k}(\mathbf{B}) (\boldsymbol{\mu}(\mathbf{B}))' \right)_{k=1}^K \end{aligned} \tag{G.5}$$

Appendix H. Proof of Lemma 1

(i) Computation of $\partial_{\mathbf{B}}\mathbf{p}^*(\mathbf{B})$

Let us apply the Implicit Function Theorem to $f(\mathbf{p},\mathbf{B})=\mathbf{0}$ given by Eq.(30). To apply this theorem, we first need to verify that $\partial_{\mathbf{p}}f(\mathbf{p},\mathbf{B})$ is a square matrix of full rank, here S. This will be done by showing that $\ker(\partial_{\mathbf{p}}f(\mathbf{p},\mathbf{B}))=\{\mathbf{0}\}$. By computation:

$$\partial_{\mathbf{p}}f(\mathbf{p},\mathbf{B})\,\mathbf{h}=\mathbf{0}\Leftrightarrow\begin{cases}\begin{bmatrix}\mathbf{B}' & \mathbf{I}_K\end{bmatrix}\left(\partial^2\mathcal{U}(\mathbf{S}(\mathbf{p}))\,\partial_{\mathbf{p}}\mathbf{S}(\mathbf{p})-\mathbf{I}_S\right)\mathbf{h}=\mathbf{0}\\ \begin{bmatrix}\mathbf{I}_{S-K} & -\mathbf{B}\end{bmatrix}\partial_{\mathbf{p}}\mathbf{S}(\mathbf{p})\mathbf{h}=\mathbf{0}\end{cases}\tag{H.1}$$

Since $\partial_{\mathbf{p}}\mathbf{S}(\mathbf{p})$ is invertible, in fact even positive definite (see (ii) of proposition 3), we obtain by setting $\mathbf{h}_1=\partial_{\mathbf{p}}\mathbf{S}(\mathbf{p},\mathbf{g})\mathbf{h}$:

$$\begin{cases}\begin{bmatrix}\mathbf{B}' & \mathbf{I}_K\end{bmatrix}\left(\partial^2\mathcal{U}(\mathbf{S}(\mathbf{p},\mathbf{g}))-(\partial_{\mathbf{p}}\mathbf{S}(\mathbf{p},\mathbf{g}))^{-1}\right)\mathbf{h}_1=\mathbf{0}\\ \begin{bmatrix}\mathbf{I}_{S-K} & -\mathbf{B}\end{bmatrix}\mathbf{h_1}=\mathbf{0}\end{cases}\tag{H.2}$$

Now let us observe that $\ker\left(\begin{bmatrix}\mathbf{B}' & \mathbf{I}_K\end{bmatrix}\right)\perp\ker\left(\begin{bmatrix}\mathbf{I}_{S-K} & -\mathbf{B}\end{bmatrix}\right)$. More precisely :

$$\begin{cases}\mathbf{v}_1\in\ker\left(\begin{bmatrix}\mathbf{B}' & \mathbf{I}_K\end{bmatrix}\right)\\ \mathbf{v}_2\in\ker\left(\begin{bmatrix}\mathbf{I}_{S-K} & -\mathbf{B}\end{bmatrix}\right)\end{cases}\Leftrightarrow\begin{cases}\mathbf{v}_1=\begin{bmatrix}\mathbf{I}_{S-K}\\ -\mathbf{B}'\end{bmatrix}\mathbf{x}_1\text{ with }\mathbf{x}_1\in\mathbb{R}^{S-K}\\ \mathbf{v}_2=\begin{bmatrix}\mathbf{B}\\ I_K\end{bmatrix}\mathbf{x}_2\text{ with}\mathbf{x}_2\in\mathbb{R}^{K}\end{cases}\Rightarrow\mathbf{v}_1'\mathbf{v}_2=0\tag{H.3}$$

We deduce from Eq.(H.2) that:

$$\mathbf{h}_1'\left(\partial^2\mathcal{U}(\mathbf{S}(\mathbf{p},\mathbf{g}))-(\partial_{\mathbf{p}}\mathbf{S}(\mathbf{p},\mathbf{g}))^{-1}\right)\mathbf{h}_1=0\tag{H.4}$$

Now remember that $\partial^2\mathcal{U}$ is, by assumption, negative definite; the same being true for $-(\partial_{\mathbf{p}}\mathbf{S}(\mathbf{p},\mathbf{g}))^{-1}$ since $\partial_{\mathbf{p}}\mathbf{S}(\mathbf{p},\mathbf{g})$ is positive definite (see (ii) of proposition 3). It follows that $\mathbf{h}_1=\mathbf{0}$ and therefore, $\ker(\partial_{\mathbf{p}}f(\mathbf{p},\mathbf{g},\mathbf{B}))=\{\mathbf{0}\}$.

We can now move to the construction of $\partial_{\mathbf{B}}f(\mathbf{p},\mathbf{B})$. To compute this derivative with respect to the coefficient of $\mathbf{B}$, we keep the same convention as in the proof of proposition 7. Let us start with the first part of this function f given by $\begin{bmatrix}\mathbf{B}' & \mathbf{I}_K\end{bmatrix}\cdot(\partial\mathcal{U}(\mathbf{S}(\mathbf{p}))-\mathbf{p})$ whose generic term is:

$$\left(\sum_{j=1}^{S-K}b_{j,i}\left(\partial_{x_j}\mathcal{U}(\mathbf{S}(\mathbf{p}))-p_j\right)+\left(\partial_{x_{S-K+i}}\mathcal{U}(\mathbf{S}(\mathbf{p}))-p_{S-K+i}\right)\right)_{i=1}^{K}\tag{H.5}$$

If we denote by $\mathbf{e}_k$ a vector of $\mathbb{R}^K$ containing 1 at rank k and 0 elsewhere, the derivative of Eq.(H.5) with respect to $b_{s,k}$ is $(\partial_{x_s}\mathcal{U}(\mathbf{S}(\mathbf{p}))-p_s)\,\mathbf{e}_k$. It follows that the derivative with respect to $(b_{s,k})_{s=1}^{S-K}$ will be $\mathbf{e}_k\otimes\left((\partial_{x_s}\mathcal{U}(\mathbf{S}(\mathbf{p}))-p_s)_{s=1}^{S-K}\right)'$ and finally the $(K,(S-K)\,K)$ matrix of the derivatives with respect to $\mathbf{B}$ is:

$$\mathbf{I}_K\otimes\left((\partial_{x_s}\mathcal{U}(\mathbf{S}(\mathbf{p}))-p_s)_{s=1}^{S-K}\right)'\tag{H.6}$$

Let us now move to the second part of f given by $\begin{bmatrix}\mathbf{I}_{S-K} & -\mathbf{B}\end{bmatrix}\cdot\mathbf{S}(\mathbf{p})$ whose generic term is:

$$\left(\mathbf{S}_i(\mathbf{p})-\sum_{j=1}^{K}b_{i,,j}\mathbf{S}_{S-K+j}(\mathbf{p})\right)_{i=1}^{S-K}\tag{H.7}$$

If $\boldsymbol{\epsilon}_s$ now denotes a vector of $\mathbb{R}^{S-K}$ containing 1 at rank s and 0 elsewhere, the derivative of Eq.(H.7) with respect to $b_{s,k}$ is $(-\mathbf{S}_{S-K+k}(\mathbf{p},\mathbf{g}))\,\boldsymbol{\epsilon}_s$ and, with a same argument, the $(K,(S-K)\,K)$ matrix of the derivatives with respect to $\mathbf{B}$ becomes:

$$\left(-(\mathbf{S}_{S-K+k}(\mathbf{p}))_{k=1}^{K}\right)'\otimes\mathbf{I}_{S-K}\tag{H.8}$$

Since Eqs.(H.6) and (H.8) are the two parts of $\partial_{\mathbf{B}} f(\mathbf{p},\mathbf{B})$ we can say that:

$$\partial_{\mathbf{B}} f(\mathbf{p},\mathbf{B}) = \begin{bmatrix} \mathbf{I}_K \otimes \left((\partial_{x_s}\mathcal{U}(\mathbf{S}(\mathbf{p})) - p_s)_{s=1}^{S-K} \right)' \\ \left(-(\mathbf{S}_{S-K+k}(\mathbf{p}))_{k=1}^{K} \right)' \otimes \mathbf{I}_{S-K} \end{bmatrix} \tag{H.9}$$

From Eqs.(H.1) and (H.9), we finally conclude that:

$$\partial_{\mathbf{B}}\mathbf{p}^*(\mathbf{B}) = \begin{bmatrix} \begin{bmatrix} \mathbf{B}' & \mathbf{I}_K \end{bmatrix} \left(\partial^2\mathcal{U}(\mathbf{S}(\mathbf{p}))\,\partial_{\mathbf{p}}\mathbf{S}(\mathbf{p}) - \mathbf{I}_S\right) \\ \begin{bmatrix} \mathbf{I}_{S-K} & -\mathbf{B} \end{bmatrix} \partial_{\mathbf{p}}\mathbf{S}(\mathbf{p}) \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{I}_K \otimes \left((\partial_{x_s}\mathcal{U}(\mathbf{S}(\mathbf{p})) - p_s)_{s=1}^{S-K} \right)' \\ \left(-(\mathbf{S}_{S-K+k}(\mathbf{p}))_{k=1}^{K} \right)' \otimes \mathbf{I}_{S-K} \end{bmatrix} \tag{H.10}$$

(ii) Rank of $\partial_{\mathbf{B}}\mathbf{p}^*(\mathbf{B})$

Let us first consider the two particular cases given by $K = S-1$ and $K = 1$. If $K = S-1$, $\partial_{\mathbf{B}} f(\mathbf{p},\mathbf{B})$ writes:

$$\partial_{\mathbf{B}} f(\mathbf{p},\mathbf{B}) = \begin{bmatrix} (\partial_{x_1} U(\mathbf{S}(\mathbf{p})) - p_1)\,\mathbf{I}_{S-1} \\ \left(-(\mathbf{S}_s(\mathbf{p}))_{s=2}^{S} \right)' \end{bmatrix} \tag{H.11}$$

If $(\partial_{x_1} U(\mathbf{S}(\mathbf{p})) - p_1) = 0$, $rank\,(\partial_{\mathbf{B}} f(\mathbf{p},\mathbf{B})) = 1$ and therefore $rank\,(\partial_{\mathbf{B}}\mathbf{p}^*(\mathbf{B})) = 1$. Otherwise $rank\,(\partial_{\mathbf{B}} f(\mathbf{p},\mathbf{B})) = S-1$ and $rank\,(\partial_{\mathbf{B}}\mathbf{p}^*(\mathbf{B})) = S-1$. If $K = 1$, $\partial_{\mathbf{B}} f(\mathbf{p},\mathbf{B})$ becomes

$$\partial_{\mathbf{B}} f(\mathbf{p},\mathbf{B}) = \begin{bmatrix} \left((\partial_{x_s} U(\mathbf{S}(\mathbf{p})) - p_s)_{s=1}^{S-1} \right)' \\ -S_S(\mathbf{p})\mathbf{I}_{S-1} \end{bmatrix} \tag{H.12}$$

which is obviouly a matrix of rank $S-1$.

Let us now move to the case $1 < K < S-1$ and let us observe that the derivative of $f(\mathbf{p},\mathbf{B})$ with respect to $(b_{s,k})_{s=1}^{S-K}$, i.e. the $k-th$ vertical block of $(S-K)$ columns of $\partial_{\mathbf{B}} f(\mathbf{p},\mathbf{B})$, is given by:

$$\partial_{(b_{s,k})_{s=1}^{S-K}} f(\mathbf{p},\mathbf{B}) = \begin{bmatrix} \mathbf{e}_k \otimes \left((\partial_{x_s} U(\mathbf{S}(\mathbf{p})) - p_s)_{s=1}^{S-K} \right)' \\ -(\mathbf{S}_{S-K+k}(\mathbf{p}))\,\mathbf{I}_{S-K} \end{bmatrix} \tag{H.13}$$

We can now make a first observation. Since the supply $\mathbf{S}(\mathbf{p})$ is strictly positive at equilibrium, the lower part of the previous matrix is the identity of $(S-K)$ up to multiplication by non-zero constant. So if we select the first block of $(S-K)$ columns of $\partial_{\mathbf{B}} f(\mathbf{p},\mathbf{B})$, we have $(S-K)$ linearly independent vectors. Now assume that $(\partial_{x_s} U(\mathbf{S}(\mathbf{p})) - p_s)_{s=1}^{S-K} \neq \mathbf{0}$. Thus, at least one term is non-zero, say the $s_0 - th$. Now observe that the upper part of $\partial_{(b_{s,k})_{s=1}^{S-K}} f(\mathbf{p},\mathbf{B})$ is mostly composed of zeros except in line k and this line changes with the order number of the block. So if we select from block 2 to K the $s_0 - th$ column, we obtain again $(K-1)$ linear independent vectors. This shows point (ii) of the Lemma. Moreover, if at least one other component of $(\partial_{x_s} U(\mathbf{S}(\mathbf{p})) - p_s)_{s=1}^{S-K}$ is non-zero, say the $s_1 - th$, we can add to the previously selected columns, the $s_1 - th$ column of the second block and therefore conclude that $rank\,(\partial_{\mathbf{B}}\mathbf{p}^*(\mathbf{B})) = S$

Appendix I. Proof of Proposition 8

The proof directly follows from our discussion

Appendix J. Proof of Proposition 9

Remember that the Gordan's form of Farkas' Lemma states that either (i) $\exists \mathbf{x}\ \mathbf{A}\mathbf{x} \gg \mathbf{0}$ or (ii) $\exists \mathbf{y} \geq \mathbf{0}$, and $\mathbf{y} \neq \mathbf{0}$, $\mathbf{y}'\mathbf{A} = \mathbf{0}$. It follows:

(i) if we set $\mathbf{A} = \begin{bmatrix} \partial_{\mathbf{B}}\kappa^{*}\left(\mathbf{p}^{*}(\mathbf{B})\right) \\ \partial_{\mathbf{B}}SW(\mathbf{B}) \end{bmatrix}$ and assume that $\nexists \mathbf{y} \geq \mathbf{0}$, and $\mathbf{y} \neq \mathbf{0}$, $y_1 \partial_B \kappa^{*}\left(p^{*}(B)\right) + y_2 \partial_B SW(B) = 0$ or, in other words, that the two vectors are collinear with a positive coefficient, we can say that $\exists d\mathbf{B}$ with the property that :

$$\begin{cases} \partial_{\mathbf{B}}\kappa^{*}\left(\mathbf{p}^{*}(\mathbf{B})\right) d\mathbf{B} > 0 \\ \partial_{\mathbf{B}}SW(\mathbf{B}) d\mathbf{B} > 0 \end{cases} \tag{J.1}$$

In other words, there exists changes in **B** which improve both investment in renewables and social welfare (ii) since $\mathbf{g} \gg \mathbf{0}$ and $\partial_p \mathbf{Y}(\mathbf{p}) = \mathcal{D}$, a diagonal matrix of generic term $\frac{1}{\partial^2 c\left((\partial c_s)^{-1}(p_s)\right)} > 0$ (see (i) of proposition 3), we can say that $\mathbf{g}' = \alpha' \ \partial_p \mathbf{Y}(\mathbf{p})$ with $\alpha_s = g_S \partial^2 c \left((\partial c_s)^{-1} (p_s) \right)$ for $s = 1, \ldots, S$, or, in other words, $\exists \mathbf{y} = \begin{pmatrix} 1 \\ \alpha \end{pmatrix} \geq \mathbf{0}$ and $\mathbf{y} \neq \mathbf{0}$, $\mathbf{y}' \begin{bmatrix} \mathbf{g}' \\ -\partial_p \mathbf{Y}(\mathbf{p}) \end{bmatrix} = 0$. This implies $\nexists \mathbf{x}$, $\begin{bmatrix} \mathbf{g}' \\ -\partial_p \mathbf{Y}(\mathbf{p}) \end{bmatrix} \mathbf{x} \gg \mathbf{0}$. So, if we replace $\mathbf{x}$ by $\partial_{\mathbf{B}} \mathbf{p}^{*}(\mathbf{B}) d\mathbf{B}$, we get :

$$\nexists d\mathbf{B} \text{ which satisfies } \begin{cases} \mathbf{g}' \partial_{\mathbf{B}} \mathbf{p}^{*}(\mathbf{B}) d\mathbf{B} > 0 \\ \partial_p \mathbf{Y}(\mathbf{p})) d\mathbf{B} < 0 \end{cases} \tag{J.2}$$

In other words, it is impossible by a contract change to improve investment in renewables and to decrease conventional electricity production in each state of nature.